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Differential Decay

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CONTENTS

Start with equation for boost. Boost muon momentum and spin from rest frame. Boosting in the 3 direction

$$A_{boost} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta_3 & 0 & 0 & \gamma \end{pmatrix} \tag{1}$$

The momentum four vector of the pion in its rest frame is

$$p = (m_\pi, 0, 0, 0)$$

The sum of the muon 4 momentum p_μ and the neutrino four momentum p_ν

$$p_\mu + p_\nu = p_\pi = E_\mu + E_\nu, \mathbf{p}_\mu + \mathbf{p}_\nu$$

$$\rightarrow \mathbf{p}_\mu = -\mathbf{p}_\nu$$

Since $E_\nu = |\mathbf{p}_\nu c| = |\mathbf{p}_\mu c|$ and $E_\mu = \sqrt{(m_\mu c^2)^2 + (\mathbf{p}_\mu c)^2}$ then

$$|\mathbf{p}_\mu c| + \sqrt{(m_\mu c^2)^2 + (\mathbf{p}_\mu c)^2} = m_\pi c^2$$

It follows that

$$(m_\mu c^2)^2 + (p_\mu c)^2 = (m_\pi c^2)^2 + (p_\mu c)^2 - 2m_\pi p_\mu c^3$$

and

$$|\mathbf{p}_\mu| = \frac{(m_\pi^2 - m_\mu^2)c}{2m_\pi}$$

Also

$$\begin{aligned} E_\mu^2 &= \mathbf{p}_\mu^2 c^2 + m_\mu^2 c^4 \\ &= \frac{(m_\pi^2 - m_\mu^2)^2 c^4}{4m_\pi^2} + m_\mu^2 c^4 \\ &= \frac{1}{4} m_\pi^2 c^4 + \frac{1}{2} m_\mu^2 c^4 + \frac{m_\mu^4 c^2}{4m_\pi^2} \\ &= \frac{(m_\pi^2 + m_\mu^2)^2 c^4}{4m_\pi^2} \\ \rightarrow E_\mu &= \frac{(m_\pi^2 + m_\mu^2) c^2}{2m_\pi} \end{aligned}$$

The direction of the momentum of the muon is not specified. If we suppose polar coordinates with z in the direction of the boost then in the pion rest frame,

$$p_\mu = [E_\mu, |\mathbf{p}_\mu| c(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)]$$

and in the boosted frame.

$$p'_\mu = [\gamma(E_\mu - \beta_3|\mathbf{p}_\mu c|\cos\theta, |\mathbf{p}_\mu c|(\cos\phi\sin\theta, \sin\phi\sin\theta), -\gamma(\beta_3 E_\mu - |\mathbf{p}_\mu c|\cos\theta)]$$