Muon campus simulation: the Bmad wedge

Samuel Grant

23rd Jan 2024

Argonne NATIONAL LABORATORY

The muon campus

Protons separated and kicked out

π^+ decay to μ^+ after a few turns

μ^+ injected into $g-2$ storage ring

8.89 GeV/c protons beam on target

3.1 GeV/c π^+ and μ^+ (and protons) travel along M2 & M3

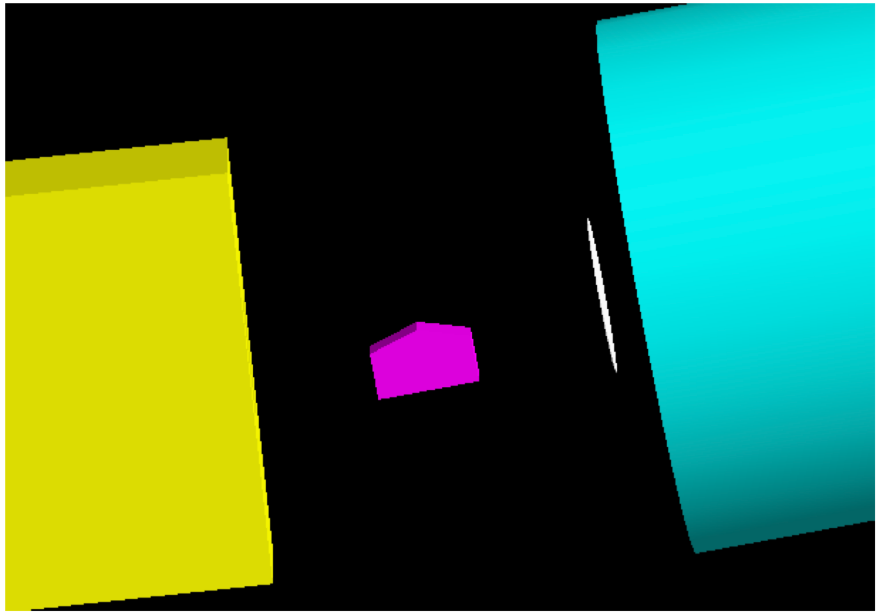
μ^+ extracted to the M4/M5 line

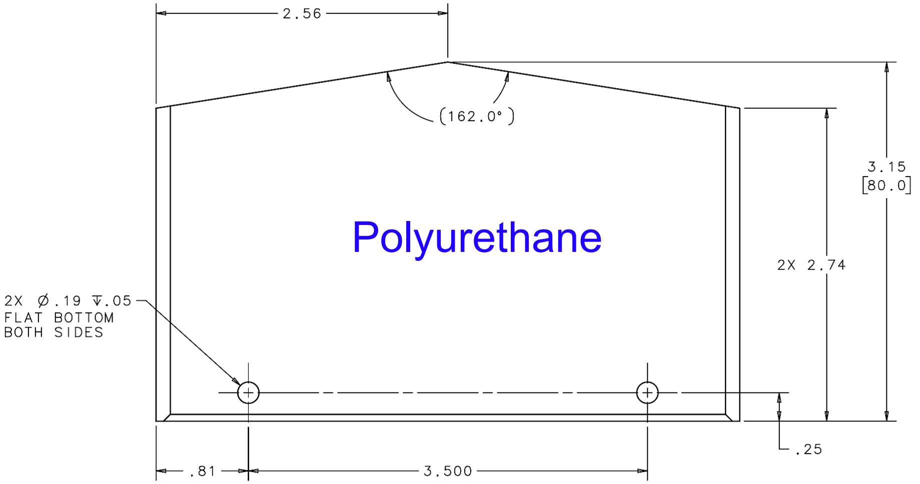
Triangular wedge: increase the number of μ^+ with energies within 0.2% of 3.094 GeV/c
DocDB-24444

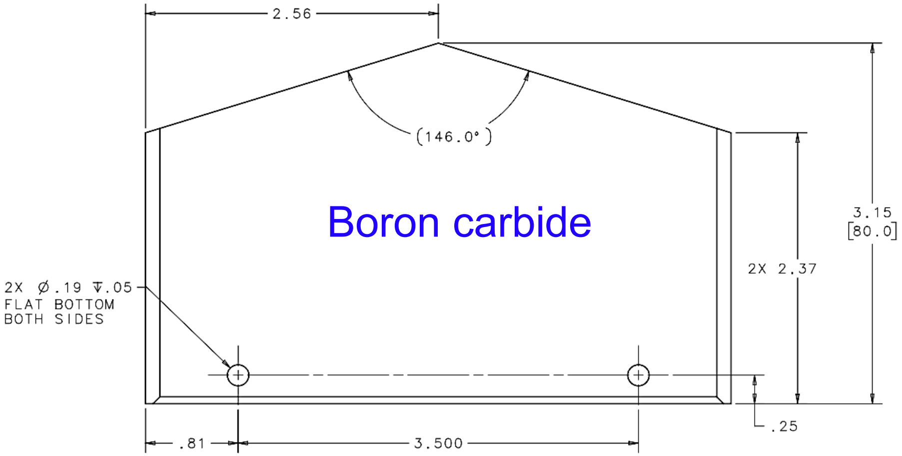
The wedge

- Triangular wedge inserted in mid-2019 (based on the elog)
- Two different materials/geometries:
  - **Polyurethane** (insert for a run or so, at the very beginning)
  - **Boron Carbide** (inserted 06/03/2019, elog-51019)

G4beamline representation







Thanks to Jim Morgan for the drawings!

Simulating the wedge

My understanding of the problem:

- The wedge is already present in a G4beamline model.
- G4beamline cannot accurately modelling the delivery ring closed orbit, instead we use Bmad from the delivery ring to injection.
- **We would ideally implement the wedges using Bmad**, rather than passing the phase space to G4beamline to model the wedges and then back to Bmad.

Developing a Bmad wedge:

- The wedge element needs to have a varying thickness in x-y and needs to “integrate the Highland-Lynch-Dahl formulas for scattering and the Bethe-Bloch formula for energy loss”.
- This type of element did not exist in Bmad, the closest thing was a “foil”: a flat sheet of a fixed thickness.
- I contacted the developer David Sagan for help, he agreed to update the existing foil element to give it varying thickness parameter.
- We also went down a multiple scattering rabbit hole and decided to give the foil the option to use the improved Lynch-Dahl approximation of the scattering angle width.
- I have been doing some testing with the updated foil element.

Bmad GitHub, which contains the manual, for more information: https://github.com/bmad-sim/bmad-ecosystem

Varying wedge thickness

- The updated foil element has a thickness which varies with x, t(x), defined as

$$t(x) = t_0(1 + x\frac{1}{t_0}\frac{dt}{dx})$$

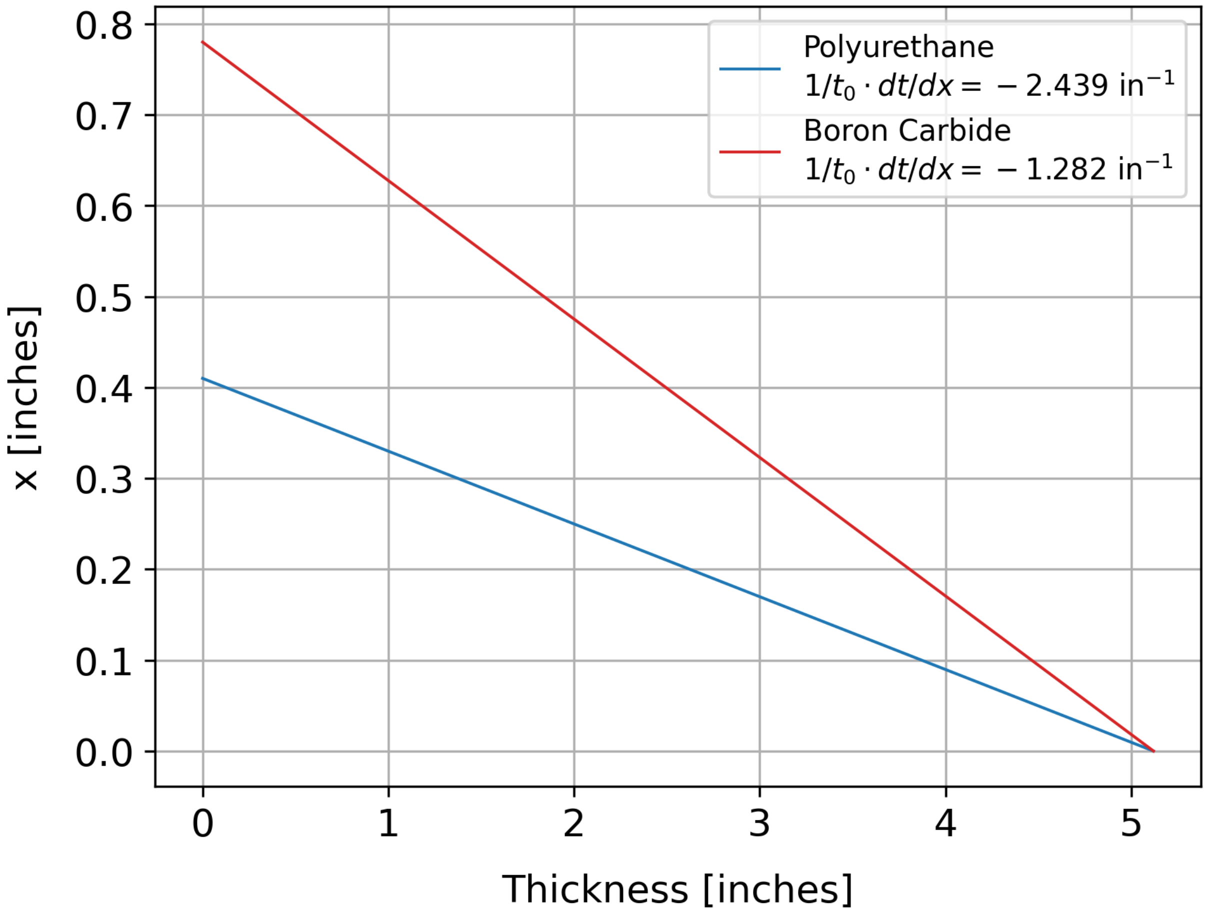
- t_0 is the maximum thickness and $d(t/t_0)/dx$ is the new varying thickness parameter.

- $t=t_0$ when x=0, t=0 when x is maximum.

- **I estimated varying thickness parameters for the two wedge geometries**, shown on slide 3, on the right-hand plot.

- The effective geometry is a right-angle triangle rather than an isosceles, but I don't think it should matter.

- This expression can also be rotated in x-y.



See section 4.20 of the most recent Bmad manual for more information

Multiple Coulomb scattering

Several approximations are available for the multiple scattering angle width, σ…

- **Rossi-Greisen** [1]: $\sigma = 15\ \mathrm{MeV} \cdot \frac{\sqrt{X/X_0}}{p\beta}$
  - X is the path length, X_0 is the radiation length, p is the momentum, and β is the speed factor.
  - Crude, ignores dependance on path length and atomic number.

- **Highland** [2] (PDG corrected): $\sigma = 13.6\ \mathrm{MeV} \cdot \frac{\sqrt{X/X_0}}{p\beta} \cdot [1 + 0.038 \log(X/X_0)]$
  - Deals with path length dependence.

- **Highland-Lynch-Dahl** [3] (PDG corrected): $\sigma = 13.6\ \mathrm{MeV} \cdot z \cdot \frac{\sqrt{X/X_0}}{p\beta} \cdot [1 + 0.038 \log(Xz^2/X_0\beta^2)]$
  - Accounts for multiply charged particles, $|z| > 1$, with $\beta < 1$
  - Default in Bmad, seems to be the default in Geant4

- **Lynch-Dahl** [3] (Geant3 corrected): $\sigma^2 = \frac{\chi_c^2}{1+F^2} \cdot [\frac{1+\nu}{\nu} \log(1+v) - 1], \quad \nu = 0.5\Omega/(1-F), \quad \Omega = \chi_c^2/0.167\chi_a^2$
  - χ_c and χ_a are the characteristic angle and screening angle from Moliere theory, F is the fraction of scatters the sample, Ω is the mean number of scatters.
  - Removes dependence on the number of radiation lengths, which "is a poor measure of the scattering".
  - This is now implemented as an option in the Bmad foil element.
  - Note: missing square in the original paper, see the Geant3 manual [4] for the corrected version.

Energy loss

- Integrate the Bethe-Bloch formula for the energy loss through the foil, Bmad uses the following:

The particle energy loss per unit length dE/dx through a foil is calculated using the `Bethe-Bloch` formula

$$-\left\langle \frac{dE}{dx} \right\rangle = \frac{4\pi}{m_e c^2} \cdot \frac{nz^2}{\beta^2} \cdot \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \cdot \left[\ln \left(\frac{2m_e c^2 \beta^2}{I \cdot (1-\beta^2)} \right) - \beta^2 \right] \tag{24.74}$$

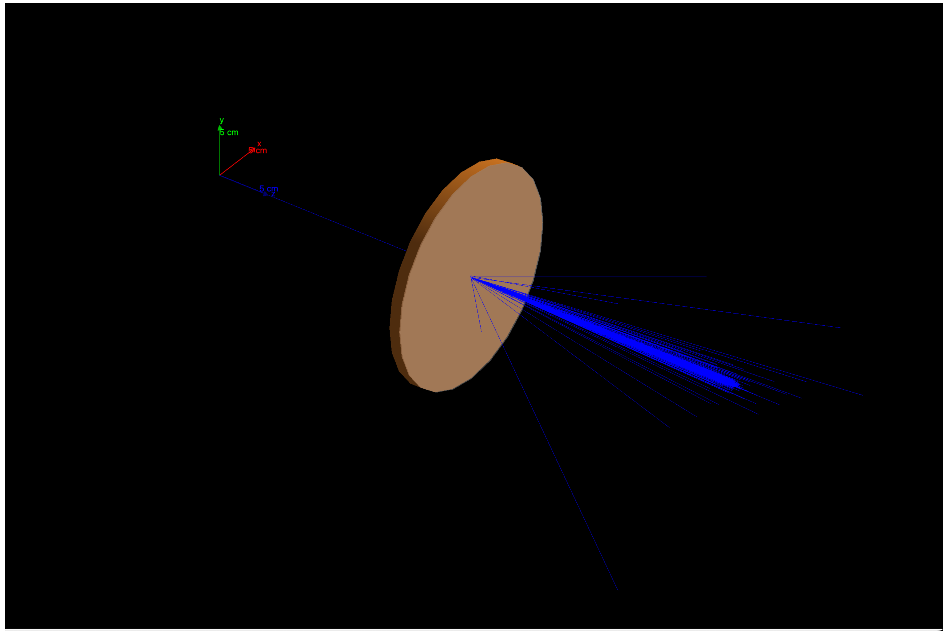
where n is the material electron density, I is the mean excitation energy, z is the particle charge, c is the speed of light, ϵ_0 is the vacuum permittivity, $\beta = v/c$, is the normalized velocity, and e and m_e the electron charge and rest mass respectively.

- Other forms are available, with different corrections.

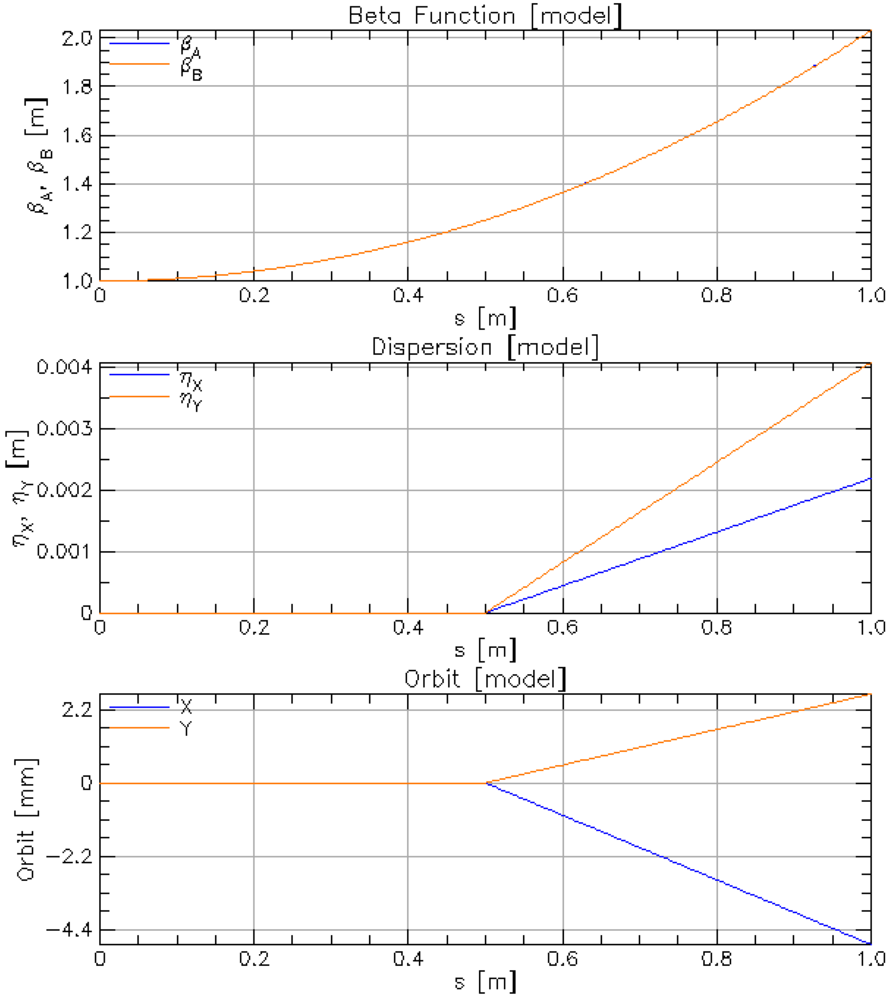
Testing with a simple foil

- Tested a 3000 MeV/c μ+ beam through a copper foil using both Bmad and G4beamline
- Scanned a range of foil thicknesses
- Compared the scattering angle and energy loss with theory
- I just use single particle tracking in Bmad for now (still learning)

G4beamline GUI

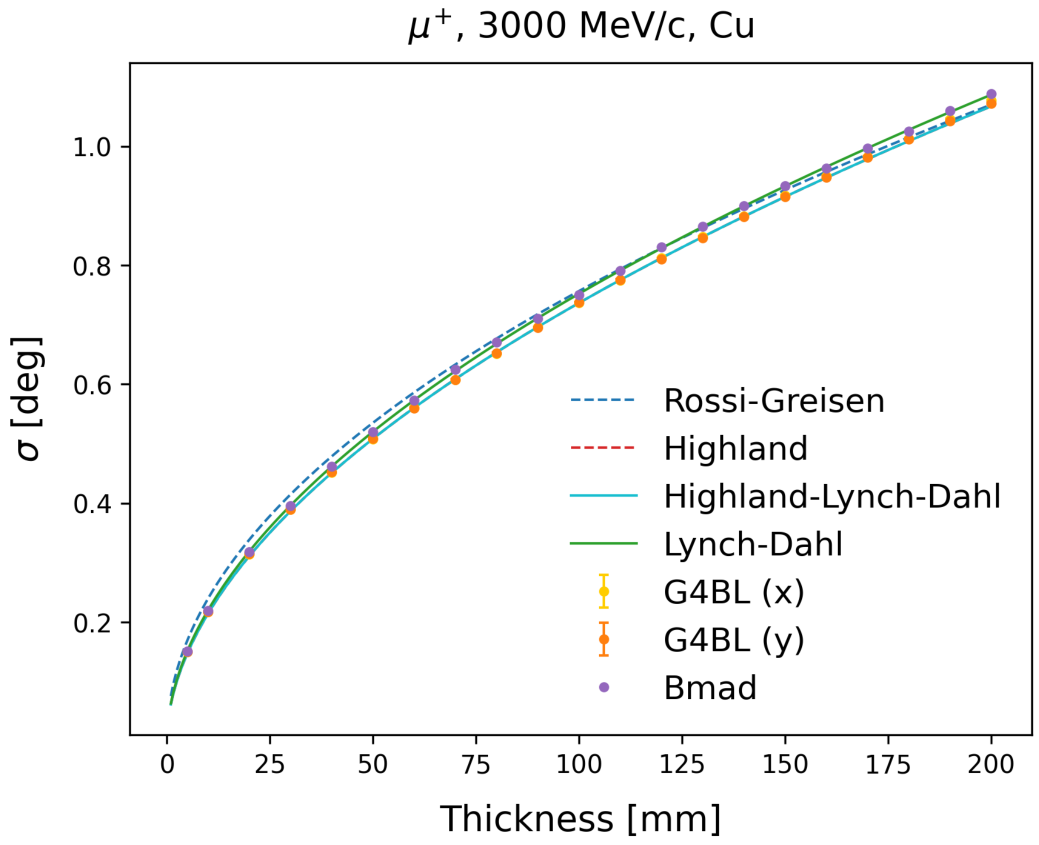


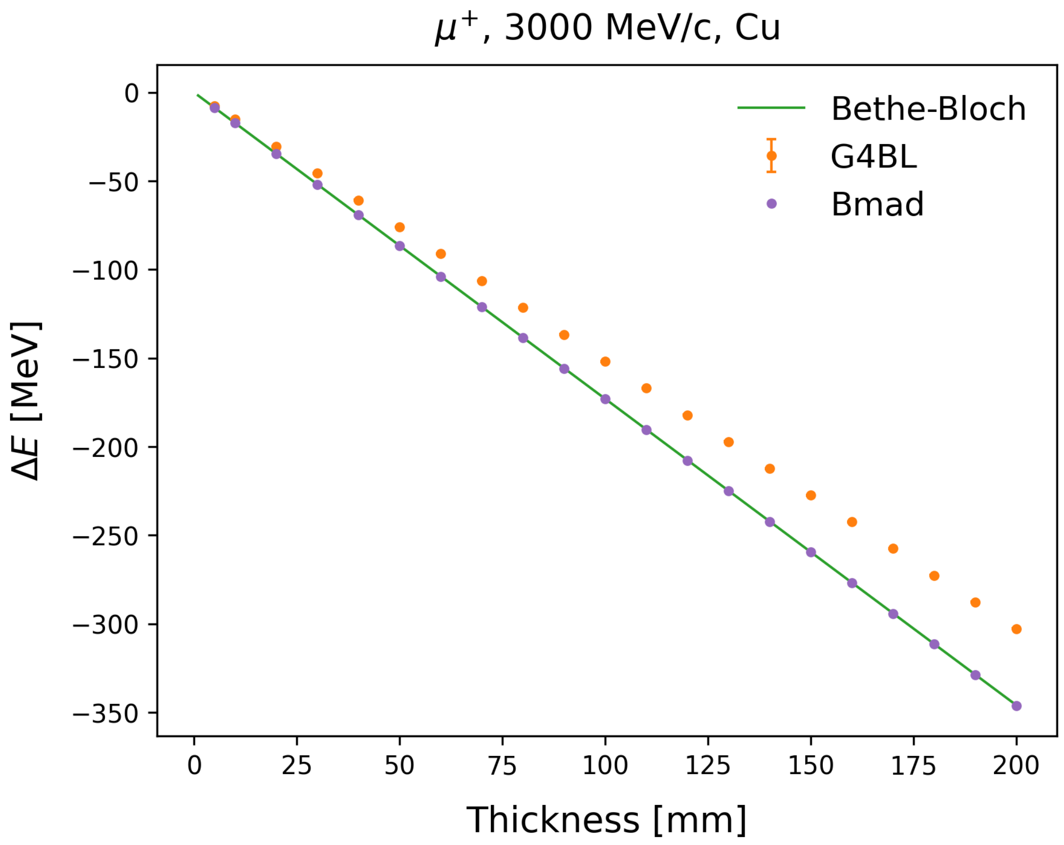
Bmad Tao interface



Testing with a simple foil

- The Bmad scattering width is consistent with Lynch-Dahl (when using that mode); G4beamline is consistent with Highland-Lynch-Dahl.
- The Bmad energy loss is consistent with the form of Bethe-Bloch quoted in the Bmad manual. I'm not certain what form G4beamline uses, it may be worth double-checking since it seems inconsistent with Bmad (I will also check that I haven't made a simple mistake here).





Summary

- A foil element with varying thickness (a wedge) is now available in Bmad! Thanks again to David Sagan.
- The Lynch-Dahl approximation for multiple Coulomb scattering was also added to the foil element.
- The varying thickness parameter was estimated for the two wedge geometries.
- Initial tests show that the multiple scattering angle and energy loss through the foil are consistent with theory.
- More testing is needed with a wedge rather than a flat foil: I'm still trying to learn Bmad so this is slow going.
- However, I think we are ready to start trying to track the beam from the delivery ring through this element. I will need help from Eremey for this.

References

[1] B. Rossi and K. Greisen, *Rev. Mod. Phys*., 13(240), 1941.
[2] V. L. Highland, *Nucl. Instr. and Meth.,* 129(497), 1975.
[3] G. R. Lynch and O. I. Dahl. *Nucl. Instrum. Methods Phys. Res. B*, 58(1), 1991.
[4] R. Brun et al., GEANT: detector description and simulation tool, CERN, 1993