Simulation of E-field and pitch systematics

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Recall

$$\vec{\omega}_a \sim \vec{\omega}_{diff} = -\frac{qe}{m}\left[a_\mu \vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

Simulation of the effect of E-field or pitch on $\vec{\omega}_a$ at 1ppm would require tracking for 1 million periods or 29 x 10^6 turns!

Instead, to get E-field effect ,compute the average $\langle \vec{\beta} \times \vec{E} \rangle$ along the trajectory

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\int \frac{\vec{\beta}\times\vec{E}}{c}\frac{dt}{T}\right]$$

Then the E-field correction is

$$\vec{C}_e(T) \sim 2\frac{\Delta p}{p}\frac{1}{T}\int_0^T \frac{\vec{\beta}\times\vec{E}}{Bc}dt$$

Add a line to the tracking code to sum $\langle\vec{\beta}\times\vec{E}\rangle$ along the trajectory of the muon
=> E-field correction as a function of time, $C_e(t)$

How does E-field correction depend on betatron amplitude?
Consider 4 distinct trajectories,
initialized at t=0 with

$$x' = 0, \Delta p/p = 0.2\%$$

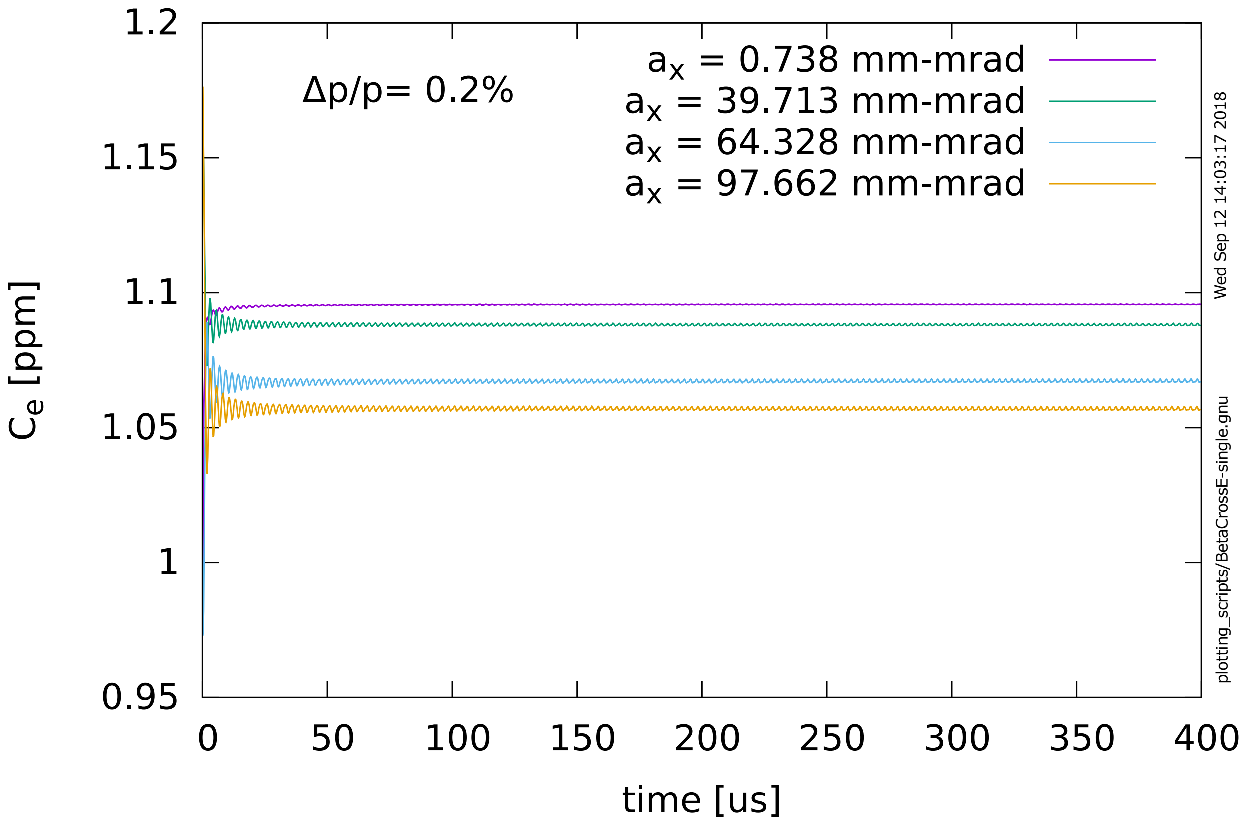
The initial displacements and betatron amplitudes are

1. x = 0.02 m, a_x = 0.738 mm-mrad
2. x = 0.0 m, a_x = 39.7 mm-mrad
3. x = -0.01 m, a_x = 64.3 mm-mrad
4. x= -0.02 m, a_x = 97.66 mm-mrad

Note that the betatron coordinates are related to x and x' according to

$$\begin{aligned} x_\beta &= x - \eta(\Delta p/p) \\ x'_\beta &= x' - \eta'(\Delta p/p) \end{aligned}$$

And $a_x = (\gamma x_\beta^2 + 2\alpha x_\beta x'_\beta + \beta {x'_\beta}^2)^{1/2}$ is the betatron amplitude



- $C_e(t)$ oscillates with betatron frequency at early time
- $C_e(t)$ decreases with increasing betatron amplitude
- Contribution from betatron amplitude < 40 ppb

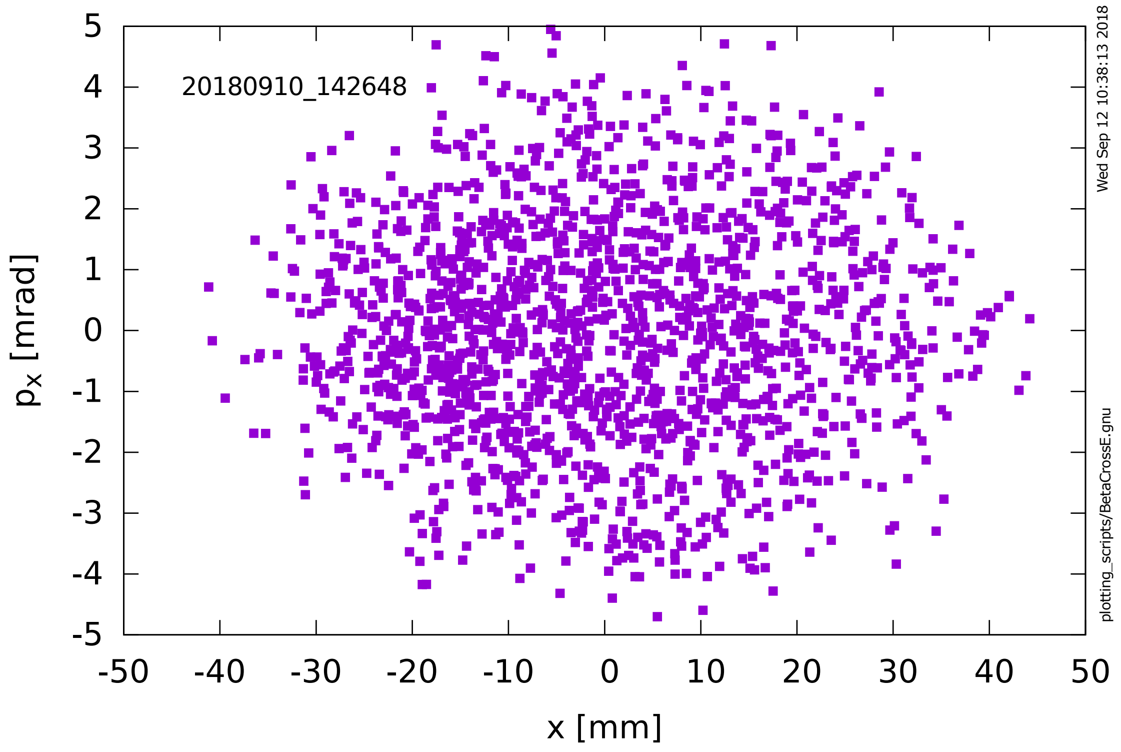
Realistic distribution

Propagate 100k muons into the ring

Assume kicker pulse shape as measured, B_{kick} =292 G

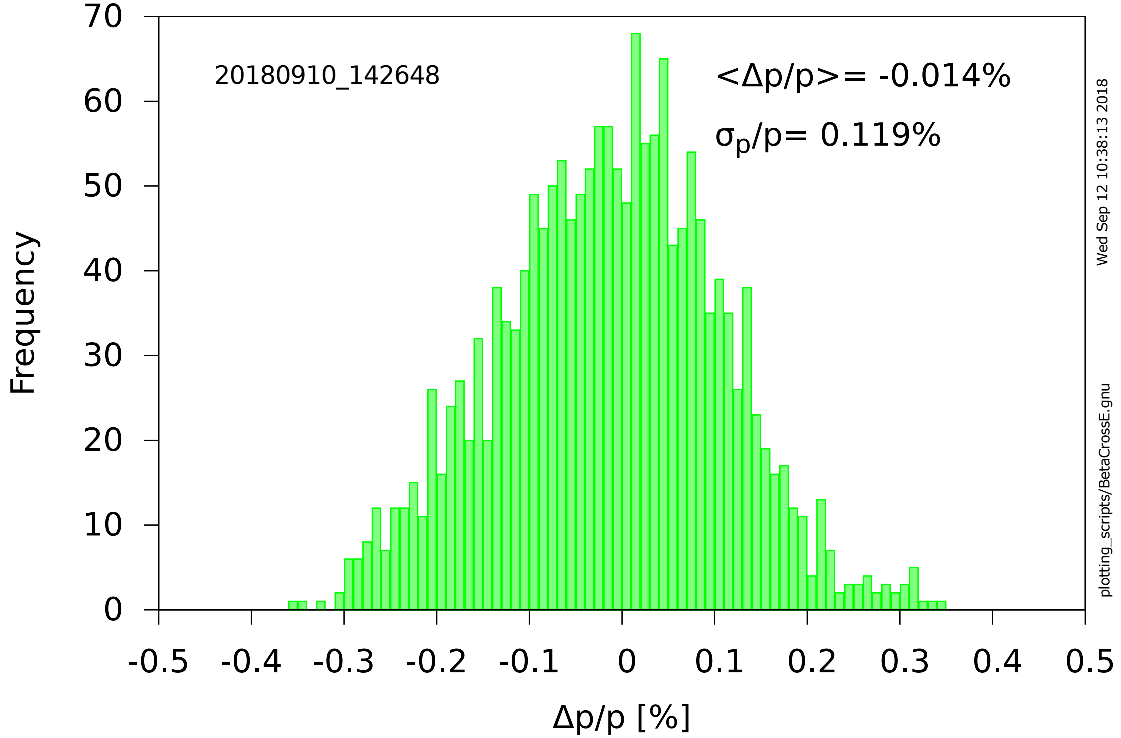
~ 2% are captured

(Muon decay is turned off)

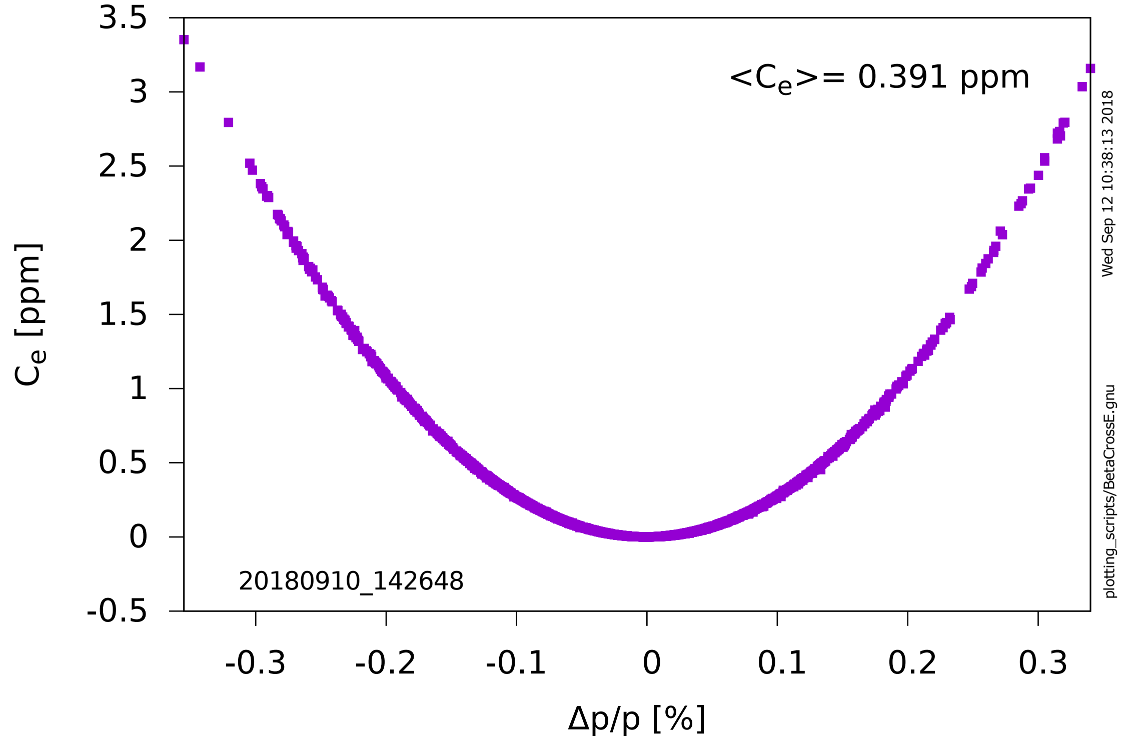


x-p_x phase space after 4000 turns

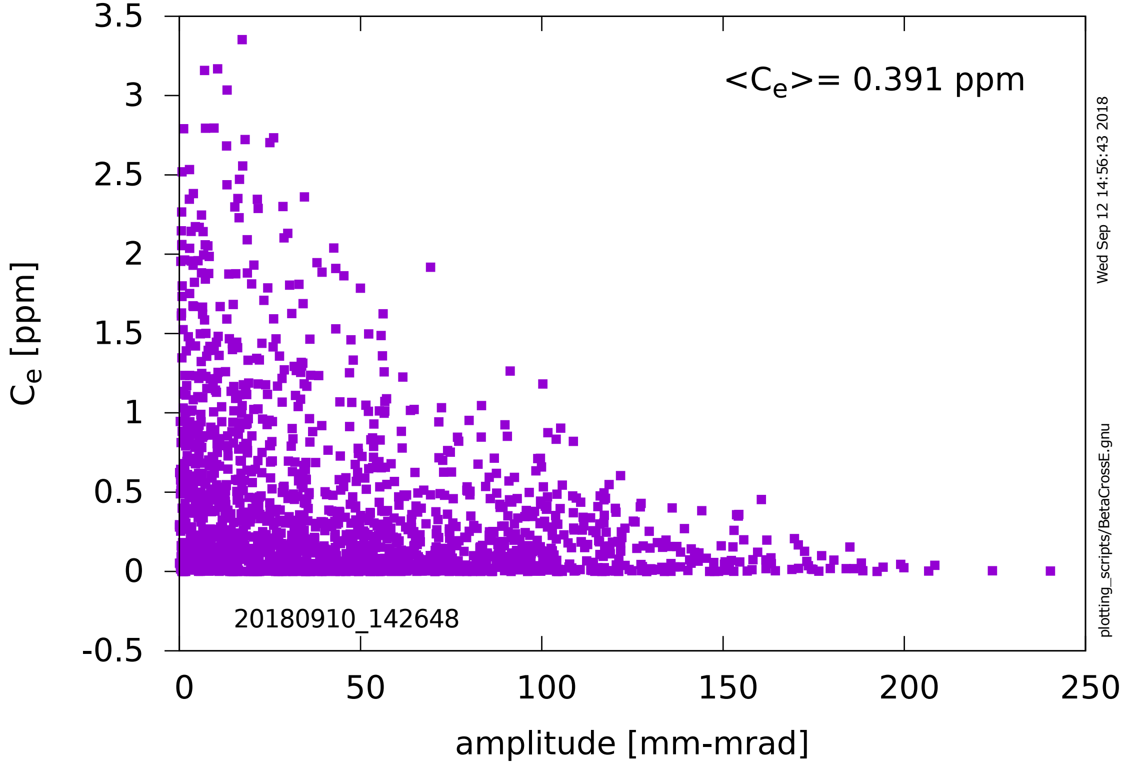
Momentum distribution after 4000 turns



E-field correction as a function of momentum offset for the distribution



E-field correction vs betatron amplitude for the distribution



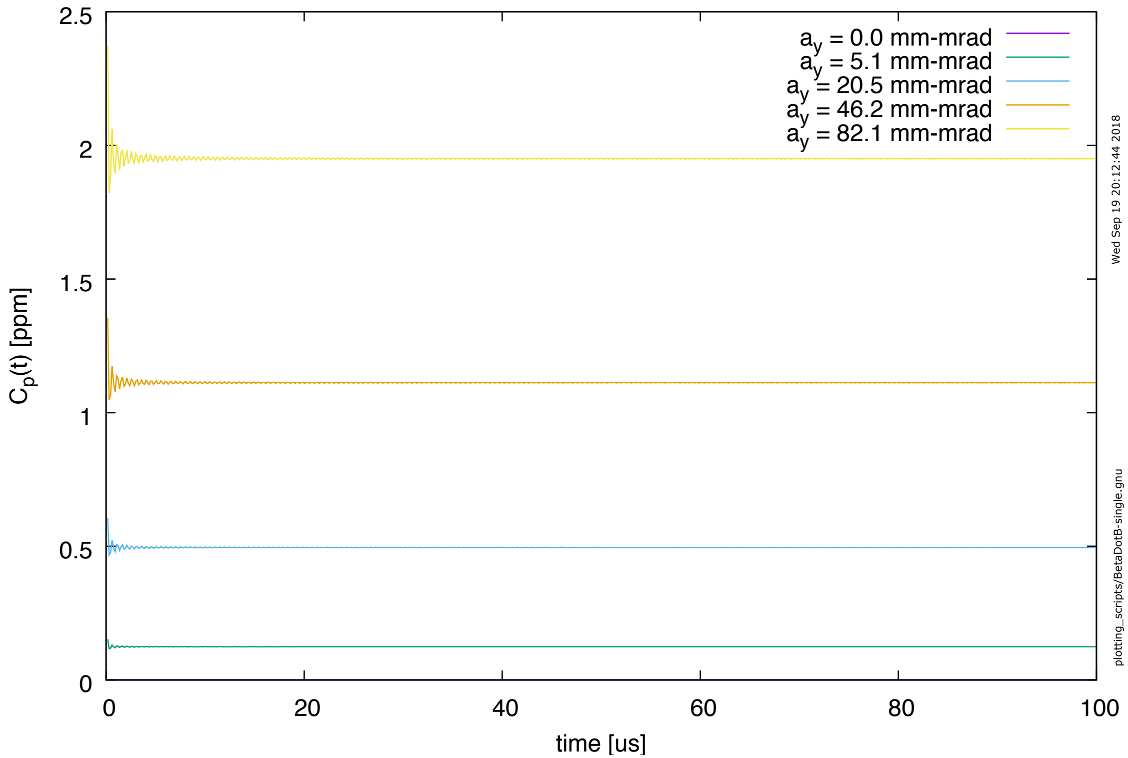
Pitch correction

$$\vec{C}_p(T) = \frac{1}{B}\frac{1}{T}\int_0^T (\vec{\beta}\cdot\vec{B})\vec{\beta}dt$$

How does pitch correction depend on betatron amplitude?

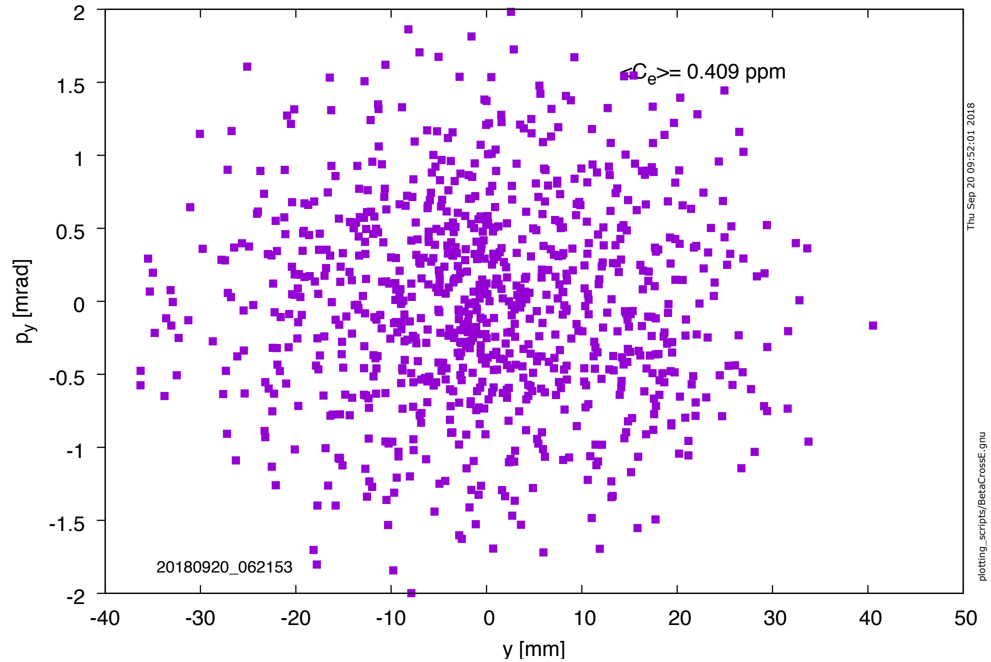
The initial displacements and amplitudes are

1. y = 0 , a_y = 0.0 mm-mrad
2. y = 1 cm, a_y = 5.12 mm-mrad
3. y = 2 cm, a_y = 64.3 mm-mrad
4. y = 3 cm, a_y = 46.2 mm-mrad
5. y = 4 cm, a_y = 82.1 mm-mrad

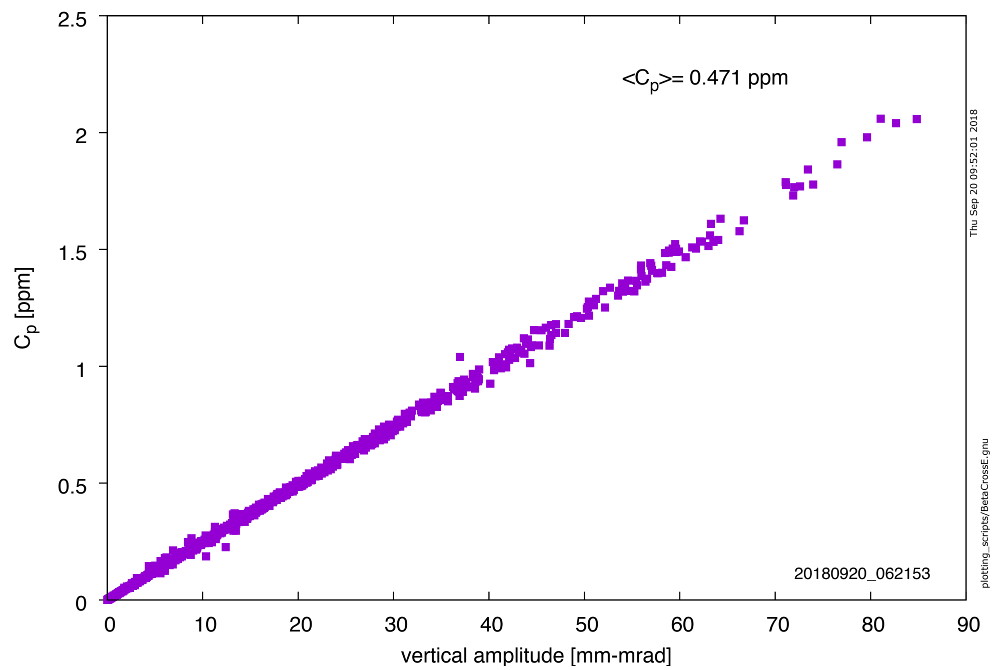


Realistic distribution

Vertical phase space after 500 turns



Pitch correction vs vertical amplitude



Summary

- Effect of E-field and pitch is computed in tracking simulation by integrating (summing along the trajectory)

$$\vec{C}_e(T) \sim 2\frac{\Delta p}{p}\frac{1}{T}\int_0^T \frac{\vec{\beta}\times\vec{E}}{Bc}dt$$

$$\vec{C}_p(T) = \frac{1}{B}\frac{1}{T}\int_0^T (\vec{\beta}\cdot\vec{B})\vec{\beta}dt$$

- Enables estimate of contributions from
    - Betatron oscillation
    - Details of phase space distribution
    - Quadrupole multipoles, fringe field, …
    - Quadrupole alignment
    - ?