

$$(1) \quad \vec{\omega}_a = -\frac{q}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

Energy dependence of $\vec{\omega}_a$

$$\begin{aligned} d\vec{\omega}_a &= -\frac{q}{m} d(\gamma^2 - 1)^{-1} \frac{\vec{\beta} \times \vec{E}}{c} \\ \frac{d\vec{\omega}_a}{d\gamma} &= \frac{q}{m} \frac{2\gamma}{(\gamma^2 - 1)^2} \frac{\vec{\beta} \times \vec{E}}{c} \\ \Delta\omega_a &= \frac{q}{m} \frac{2\gamma\Delta\gamma}{(\gamma^2 - 1)^2} \frac{\vec{\beta} \times \vec{E}}{c} \end{aligned}$$

Since $\gamma \gg 1$

$$\Delta\omega_a \approx \frac{q}{m} \frac{2\Delta\gamma}{\gamma^3} \frac{\vec{\beta} \times \vec{E}}{c}$$

The electric field is roughly proportional to $\Delta\gamma$. Let $E = E_0\Delta\gamma$. Note that the electric field changes sign with change in sign of energy offset. This is assuming that $\Delta\gamma = 0$ corresponds to $E = 0$.

$$\begin{aligned} \Delta\omega_a &\approx \frac{q}{mc} \frac{2\Delta\gamma}{\gamma^3} E_0\Delta\gamma \\ &\approx \frac{qE_0}{mc} \frac{2(\Delta\gamma)^2}{\gamma^3} \end{aligned}$$