Effect of Quad nonlinearity, field errors and misalignment on E-field & Pitch Corrections

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April 29, 2019

How does quad nonlinearity, field errors and misalignment alter effects of electric field and pitch?

How large are those effects?

Can we correct for those effects?

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{m^2}{p^2} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

$$C_e \sim -2 \frac{\Delta p}{p} \langle \frac{\vec{\beta} \times \vec{E}}{Bc} \rangle$$

Linearity

E-field correction

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{m^2}{p^2}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

$$C_e \sim -2\frac{\Delta p}{p}\langle\frac{\vec{\beta}\times\vec{E}}{Bc}\rangle$$

We measure the equilibrium radius of the trajectory (horizontal closed orbit).

If quad fields are linear then

$$\langle E_r \rangle = k x_c$$

$$\frac{\Delta p}{p} = \frac{x_c}{\eta_0} \quad \Rightarrow \quad C_E = -2\beta^2 n(1-n)\frac{\langle x_e^2 \rangle}{r_0^2}$$

Effect of nonlinearities

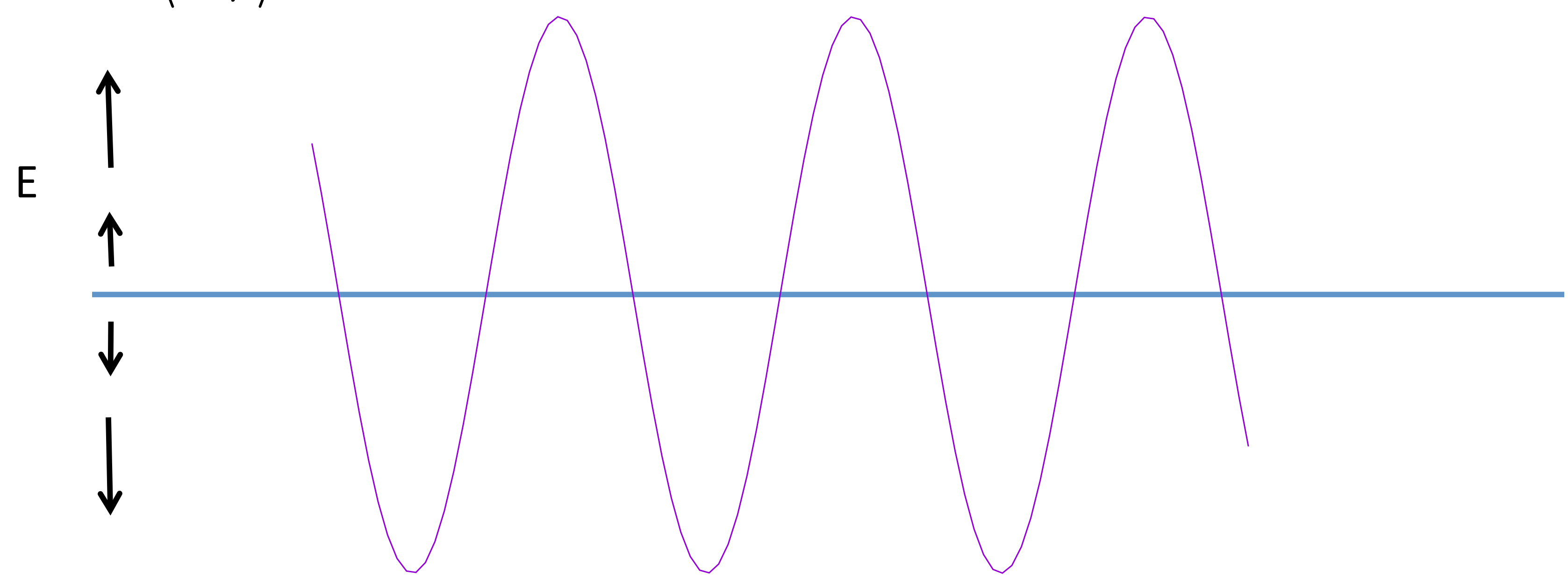
Quad E- field rolls off at large displacement vanishing at the plates.

- Effective defocusing is reduced for large amplitudes
- Horizontal betatron tune increases (quad index decreases)
- And the effective dispersion (η) decreases

Reducing the E-field shift of ω_a

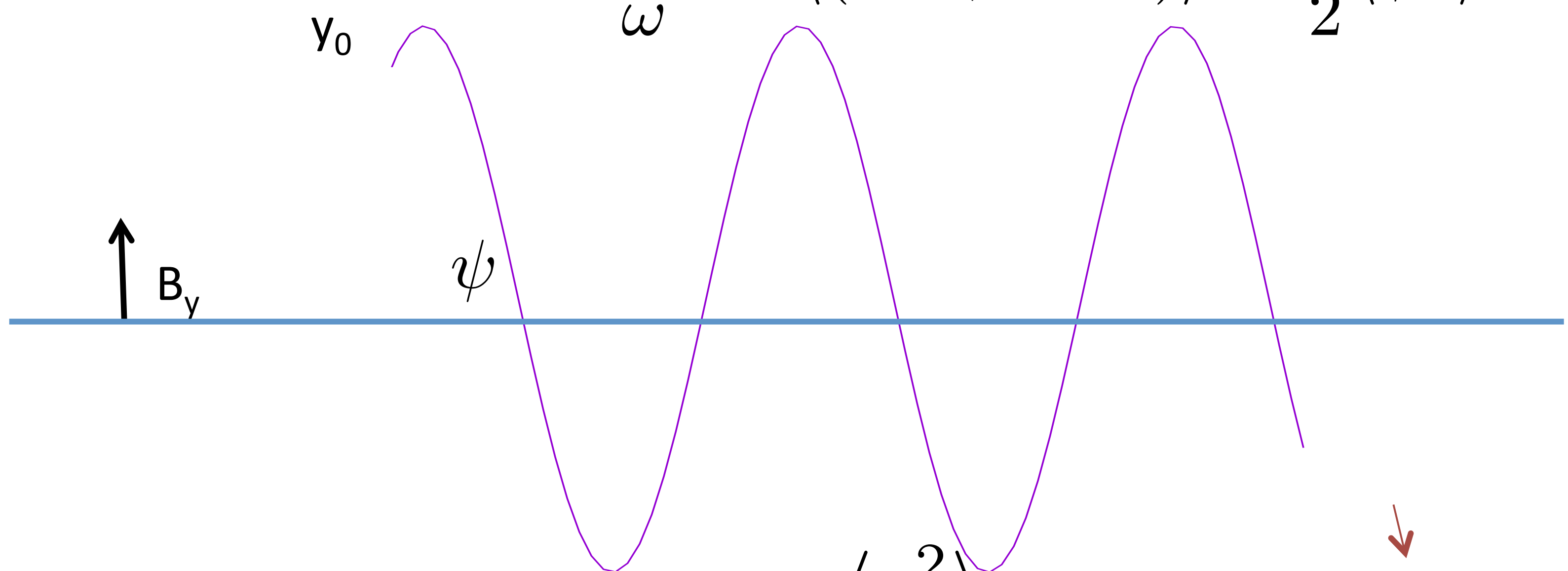
$$\frac{\Delta p}{p} > \frac{x_c}{\eta_0} \qquad \langle E_r \rangle < k x_c \qquad C_E = -2\beta^2 n(1-n)\frac{\langle x_e^2 \rangle}{r_0^2}$$

And $\langle E_r \rangle$ will depend on betatron amplitude



Pitch systematic

$$\frac{\Delta\omega}{\omega} = \langle (1 - \hat{\beta} \times \hat{\mathbf{B}}) \rangle \approx \frac{1}{2}\langle \psi^2 \rangle$$



Linearity $$\langle \psi^2 \rangle = \frac{\langle y^2 \rangle}{\beta_y^2}$$

Nonlinearity: Effective vertical focusing decreases and β_y increases with amplitude

$$C_p = -\frac{\langle y^2 \rangle}{2\beta_y^2} = -\frac{n\langle y^2 \rangle}{2R_0^2}$$

Reducing the pitch correction for large amplitudes

Analytic estimates of nonlinearity indicate a few percent of total correction for both E-field and pitch at large amplitdues

In addition to nonlinearity

- Voltage errors on individual quad plates can distort closed orbit, $\langle E_r \rangle$ and, η and β
- Misalignment of quad plates can distort closed orbit, introduce additional nonlinearity, alter focusing

Quantify impact of nonlinearity, field and alignment errors with simulation

- Establish integration along trajectory as proxy for spin phase advance

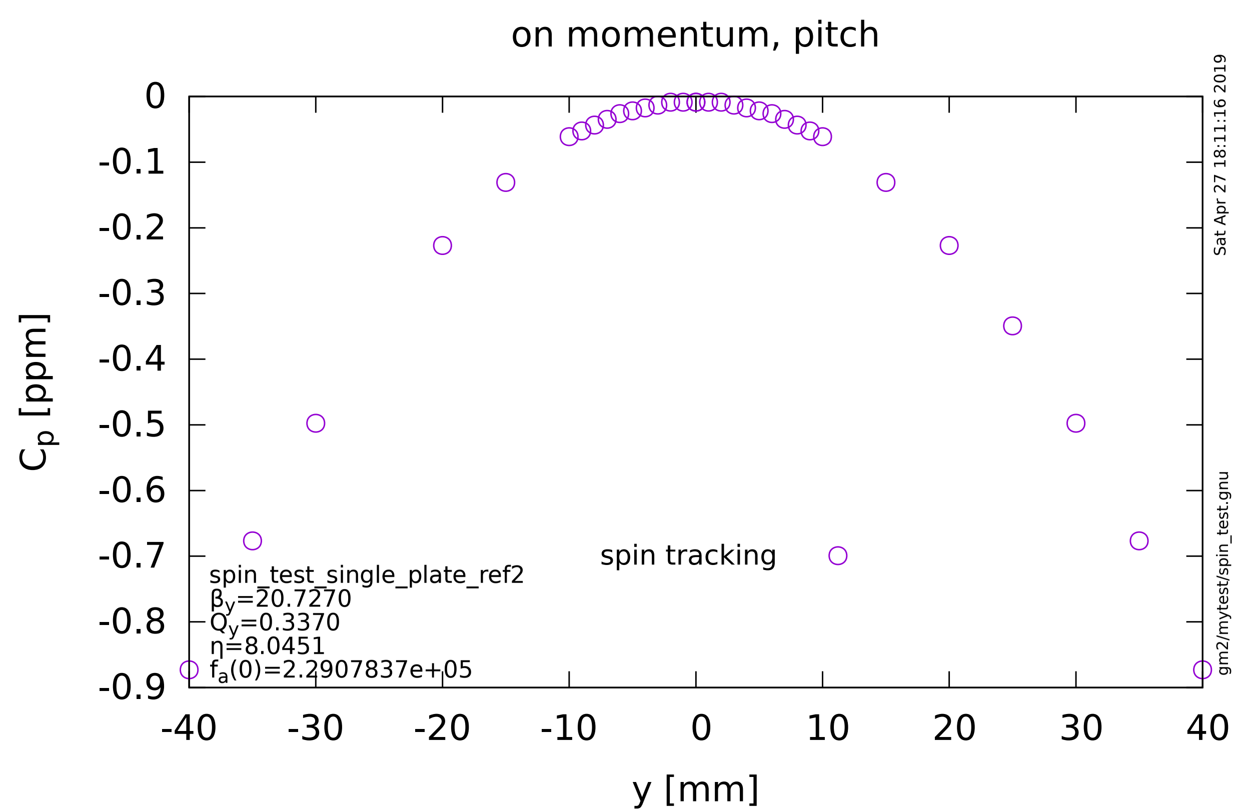
$$\vec{C}_e(T) \sim 2\frac{\Delta p}{p}\frac{1}{T}\int_0^T \frac{\vec{\beta}\times\vec{E}}{Bc}dt$$

$$C_p = \frac{1}{T}\int_0^T (1-\hat{\beta}\times\hat{\mathbf{B}})dt$$

- Compute dependence of E-field and pitch corrections on x_e and y_0

Is integration a reliable measure of the contributions from E-field and pitch?

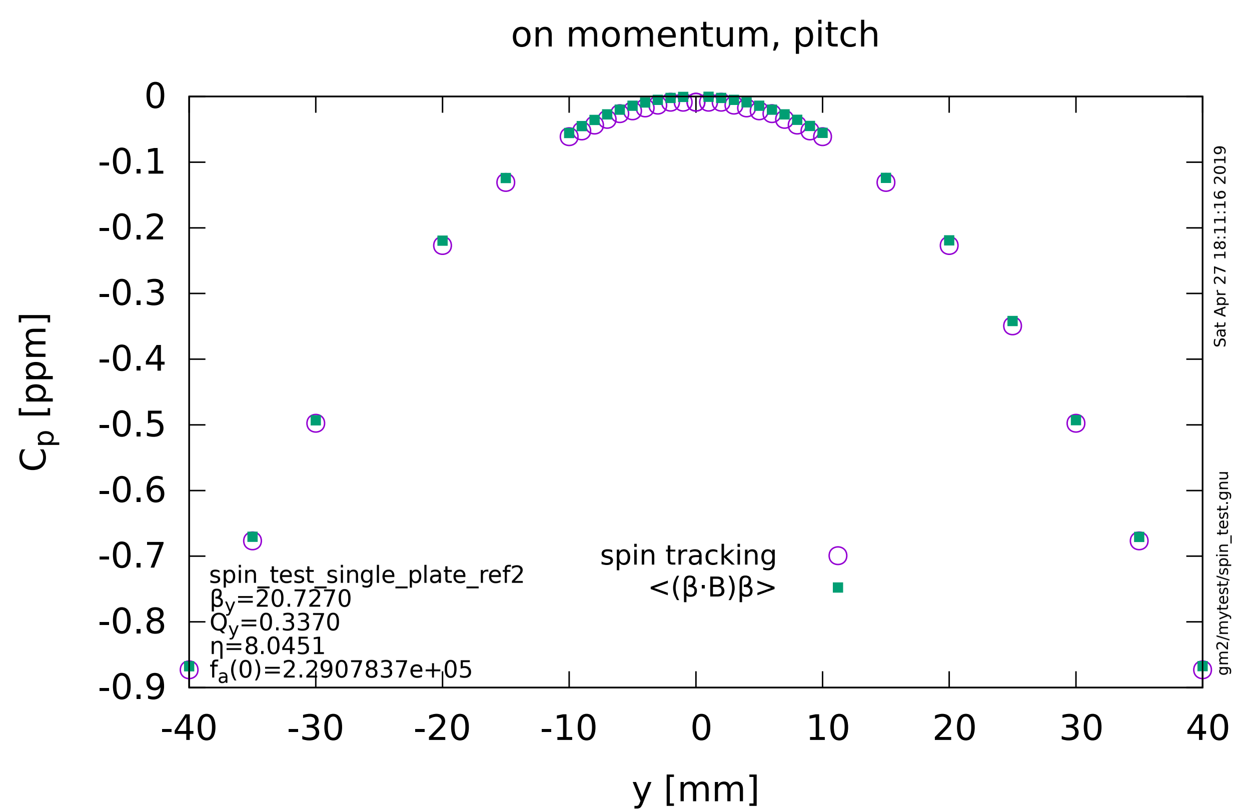
Spin tracking using BMT



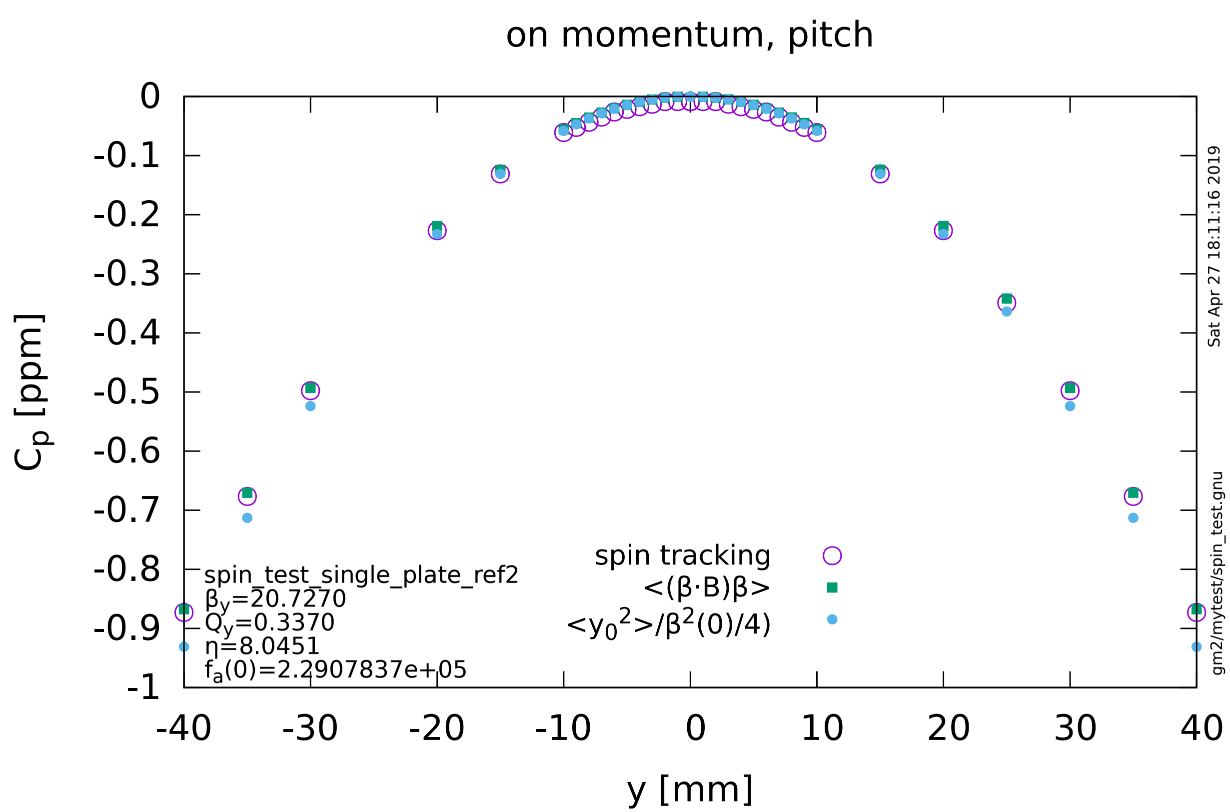
Pitch correction vs vertical amplitude

Spin tracking

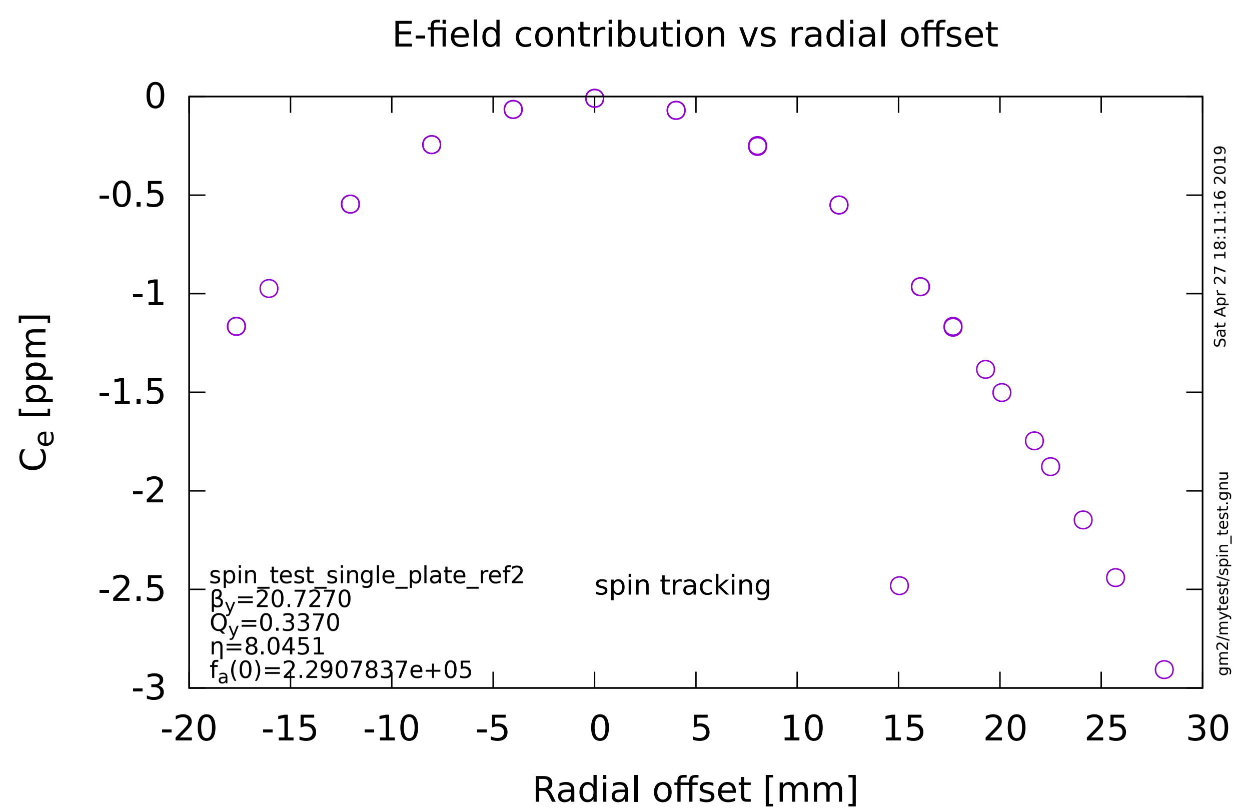
Integration $\vec{\beta} \cdot \vec{B}$



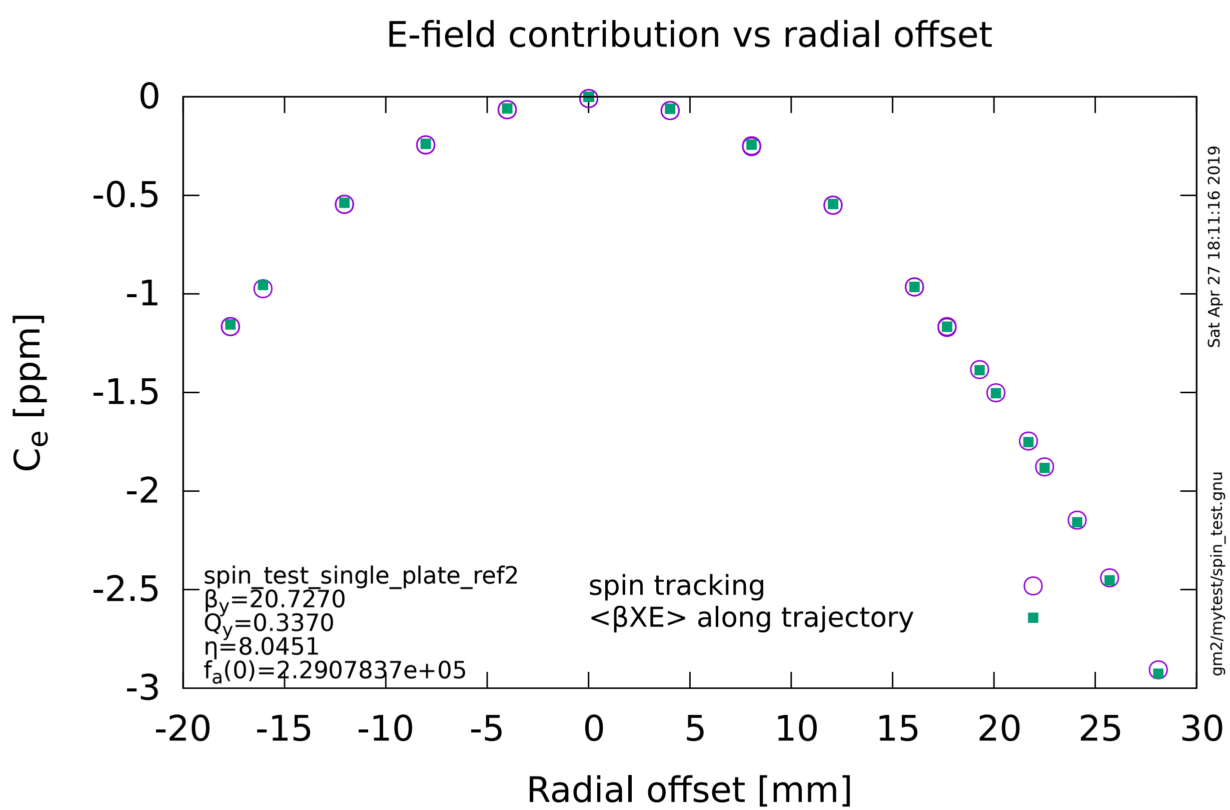
- Spin tracking
- Integration $\vec{\beta} \cdot \vec{B}$
- Linear method



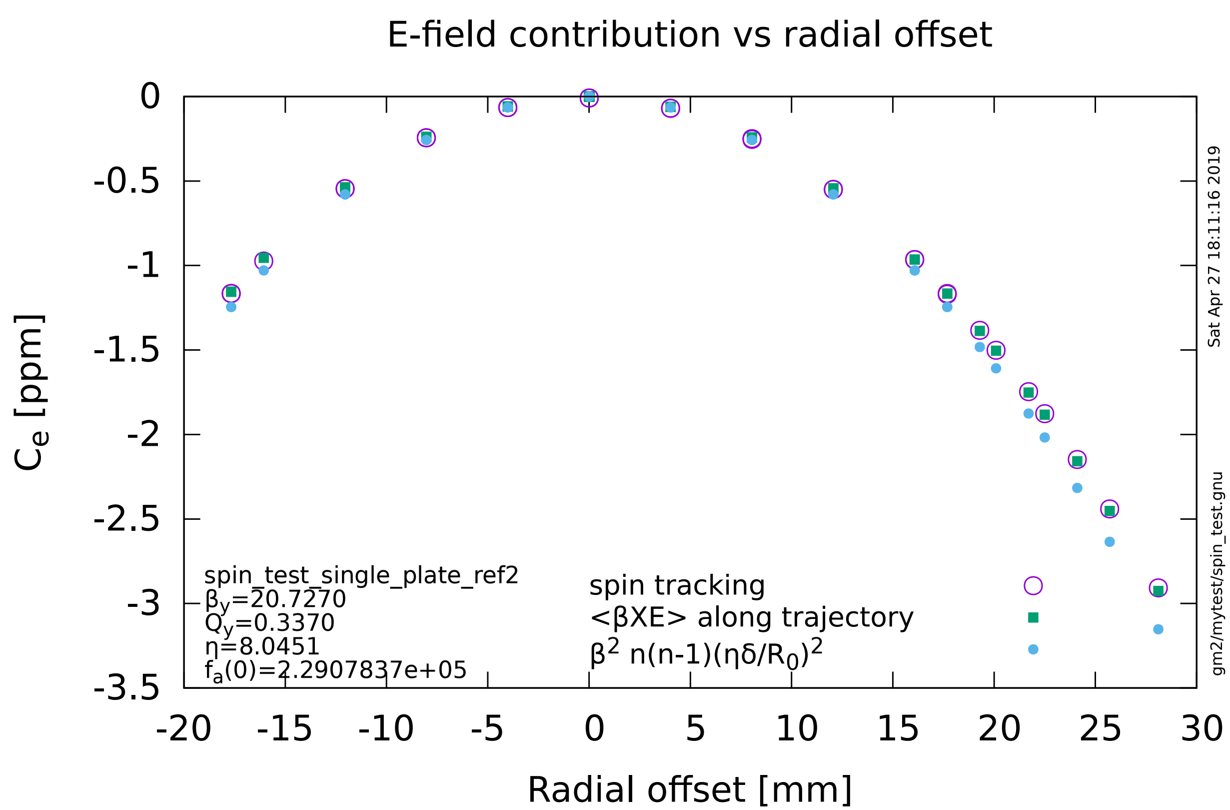
Spin tracking



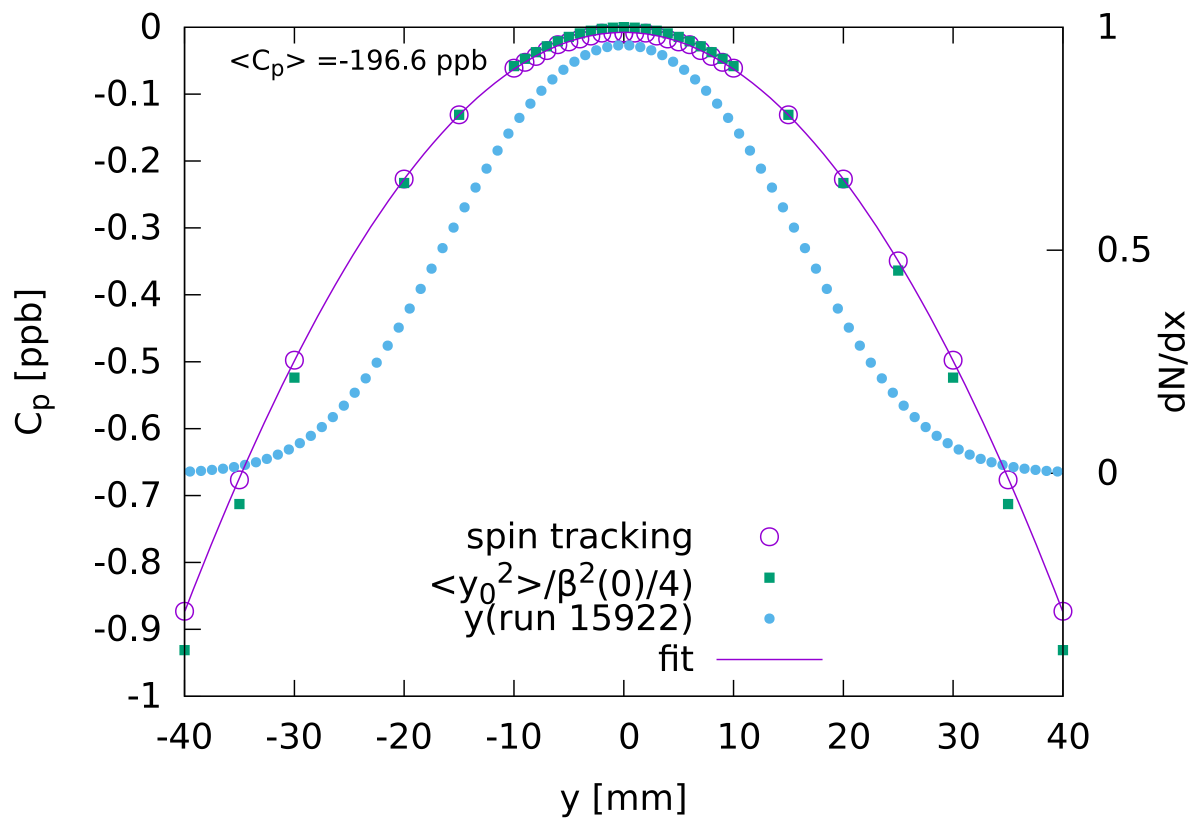
Spin tracking
Integration $\vec{\beta} \times \vec{E}$



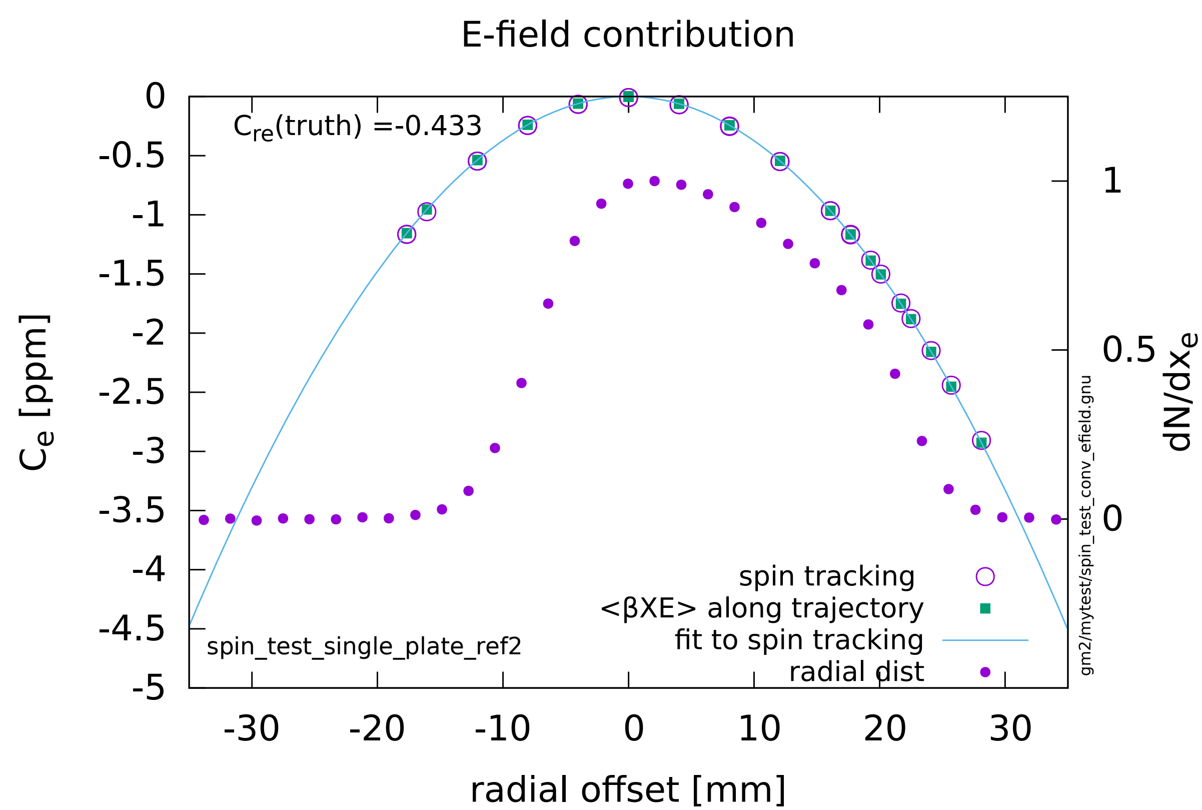
Spin tracking
Integration $\vec{\beta} \times \vec{E}$
Linear method



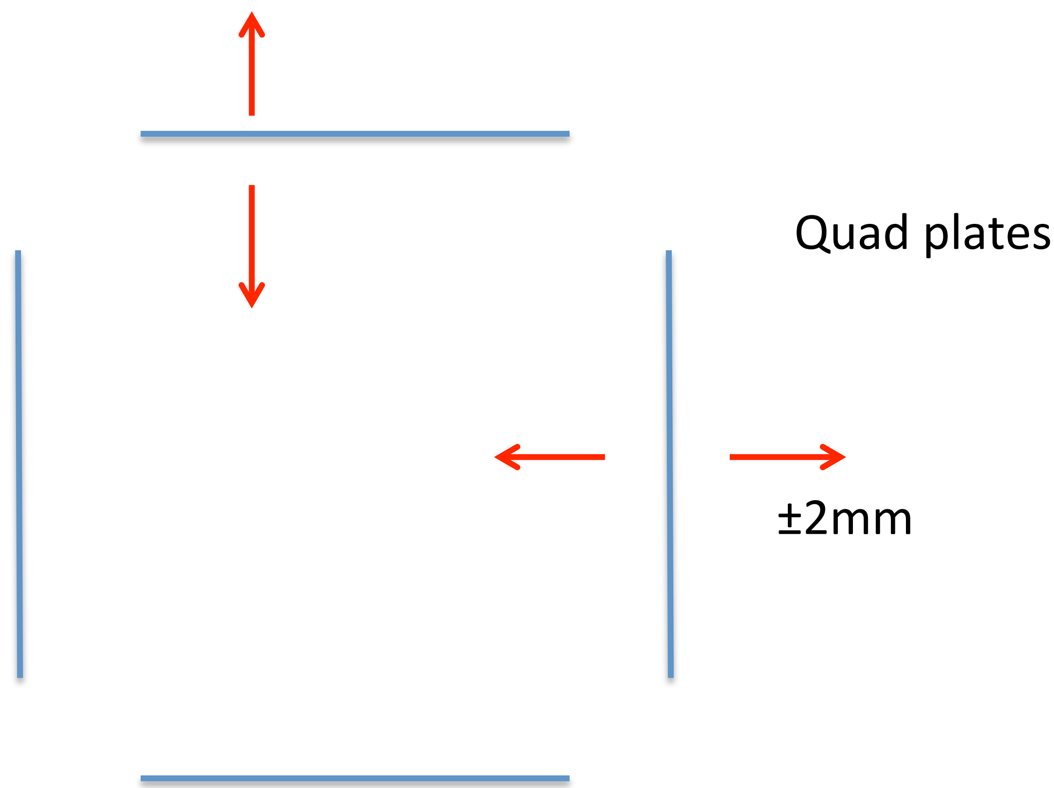
Convolution of measured distribution with $C_p(y)$



Convolution of measured distribution with $C_e(x_e)$



Explore effect of field and alignment errors and nonlinearity



$\Delta V_p = \pm\ 10\%\ V_0$

$\Delta B_r = \pm 20$ ppm

Create configurations that span the space of possibilities

- Vary alignment of each of 2X4X4 quad plates $\Delta x = \pm 2$mm, $\Delta y = \pm 2$mm
- Vary voltage on each of 32 quad plates $\Delta V_p = \pm\ 10\%\ V_0$

  Note that both misalignment and voltage errors enhance nonlinearities
- Vary radial magnetic field $\Delta B_r = \pm 20$ ppm

The corrections depend on

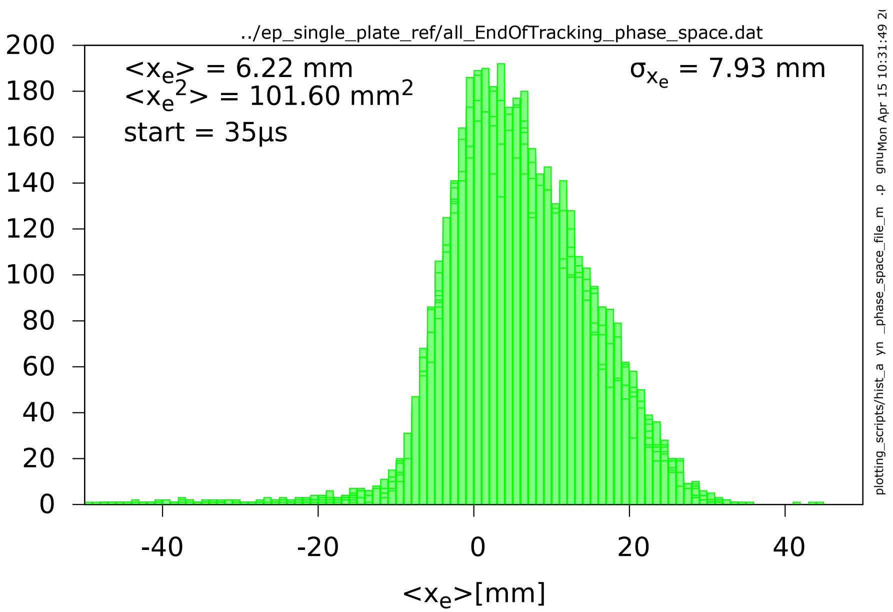
- $P(y)$ and $C_p(y)$ for pitch
- $P(x_e)$ and $C_e(x_e)$ for E-field

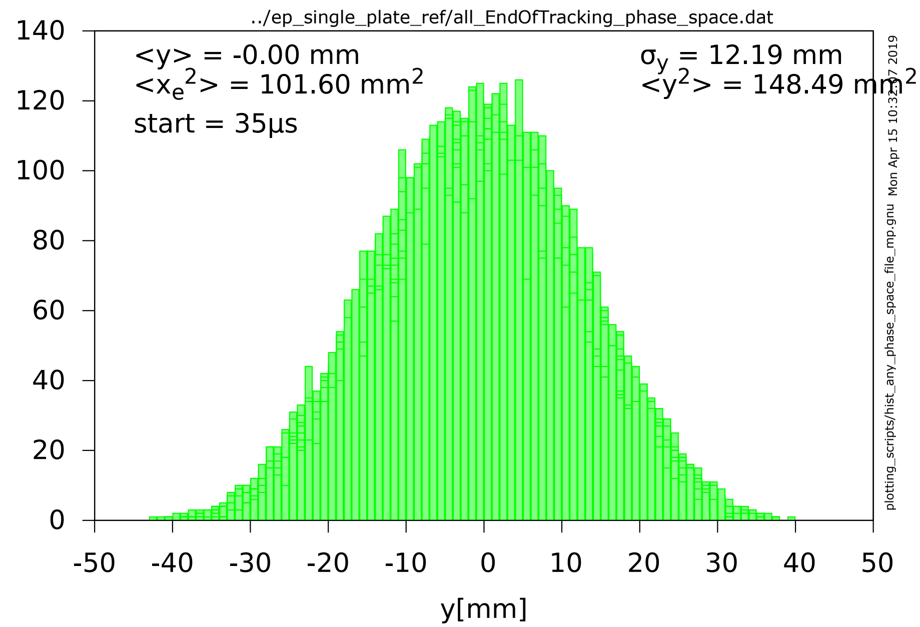
And all four quantities depend on the configuration.

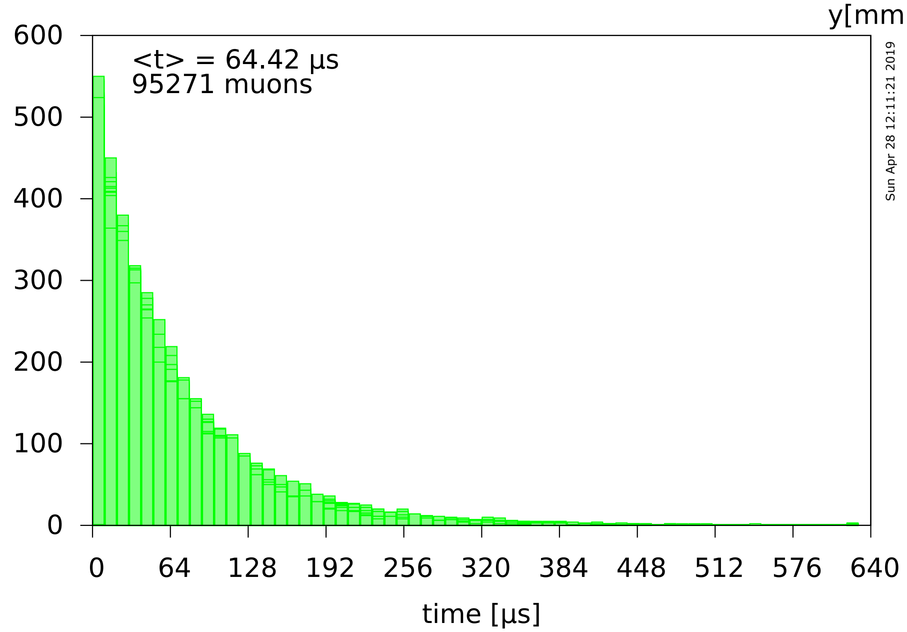
For each configuration

- Track through injection channel into ring to generate 'realistic' distribution
- Kicker B=175 G
- Quads at 18.3 kV
- Quad scrape 13.1kV -> 18.3kV
- Muon decay is turned on
- Compute Cp and Ce (by integration along trajectory) for each muon
- Include all muons that decay at t > 35 us

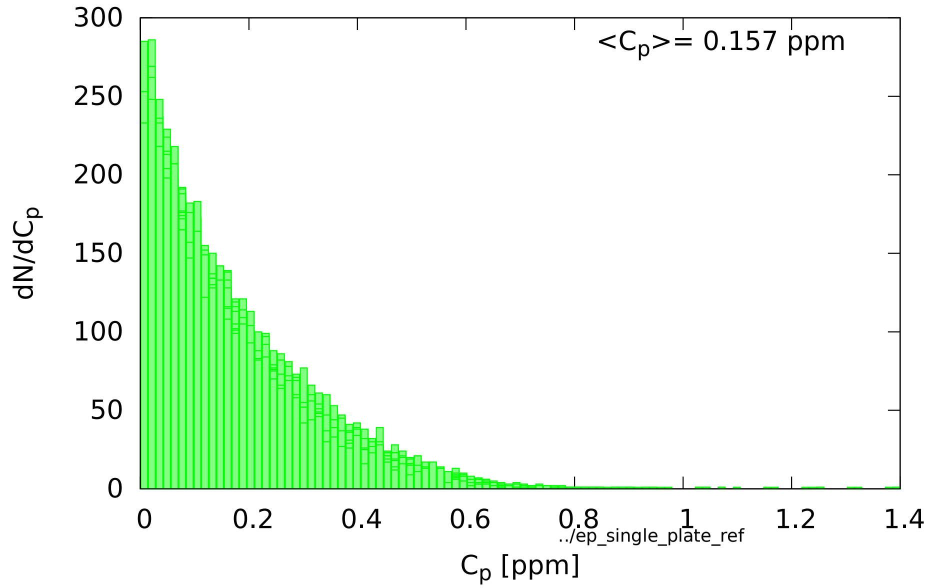
Reference configuration corresponds to surveyed alignment and nominal voltage

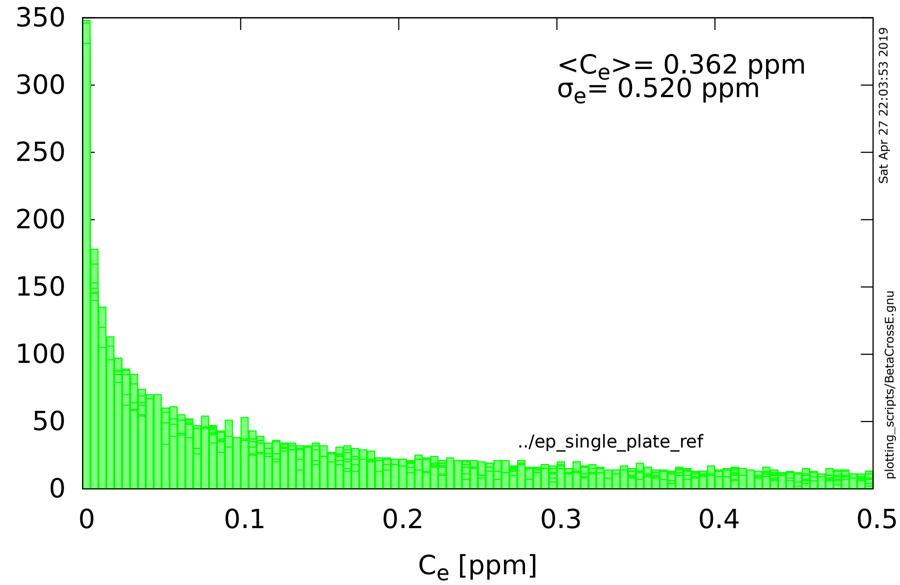


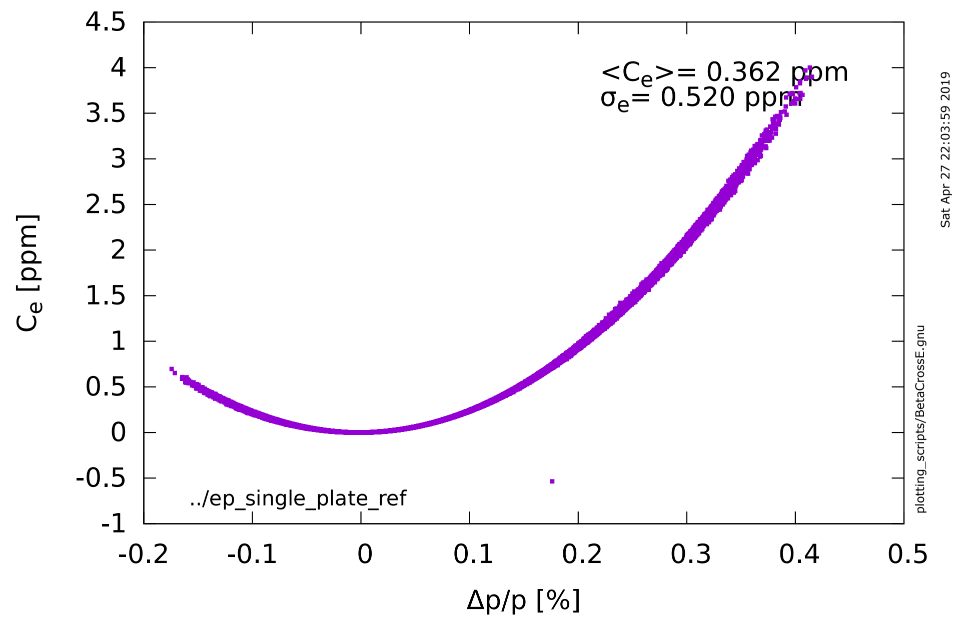




Pitch and E-field corrections for reference distribution



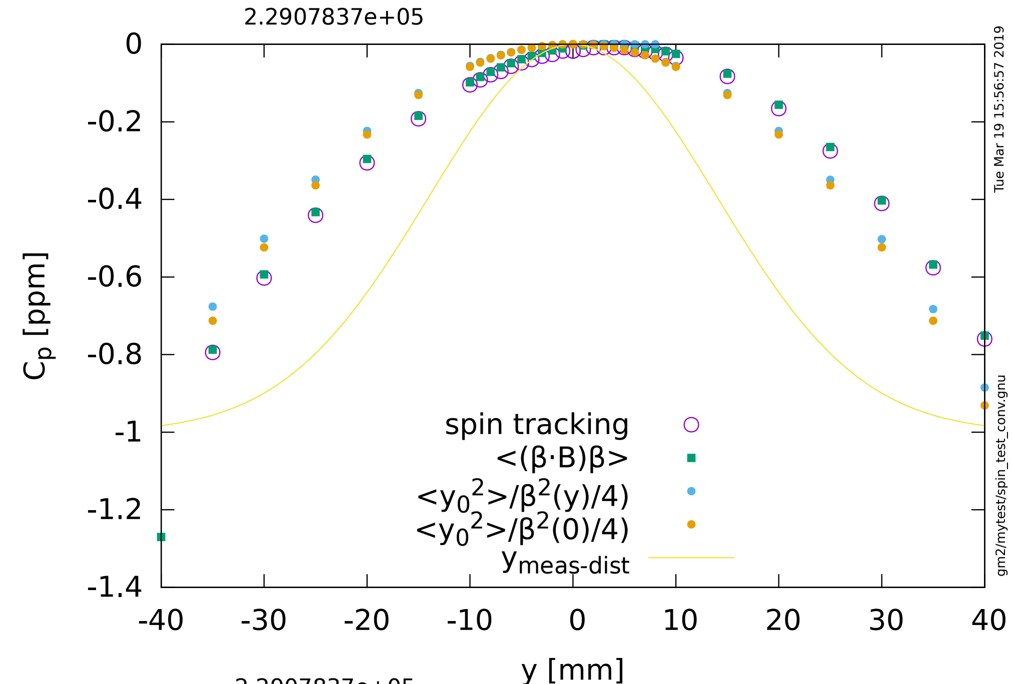


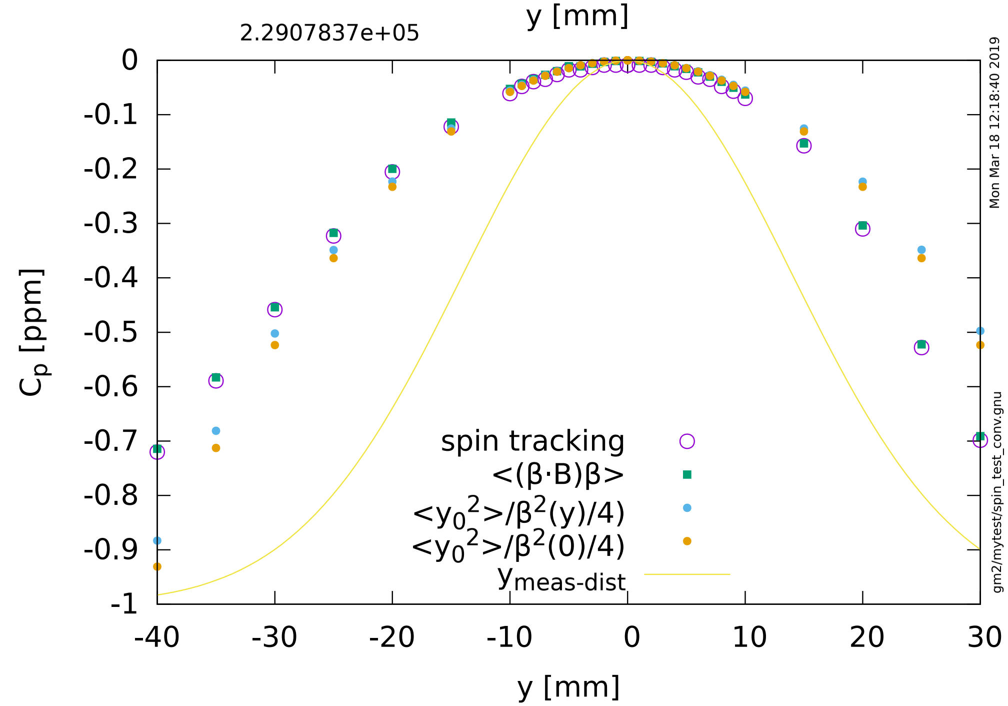


Examples of errors that impact pitch correction

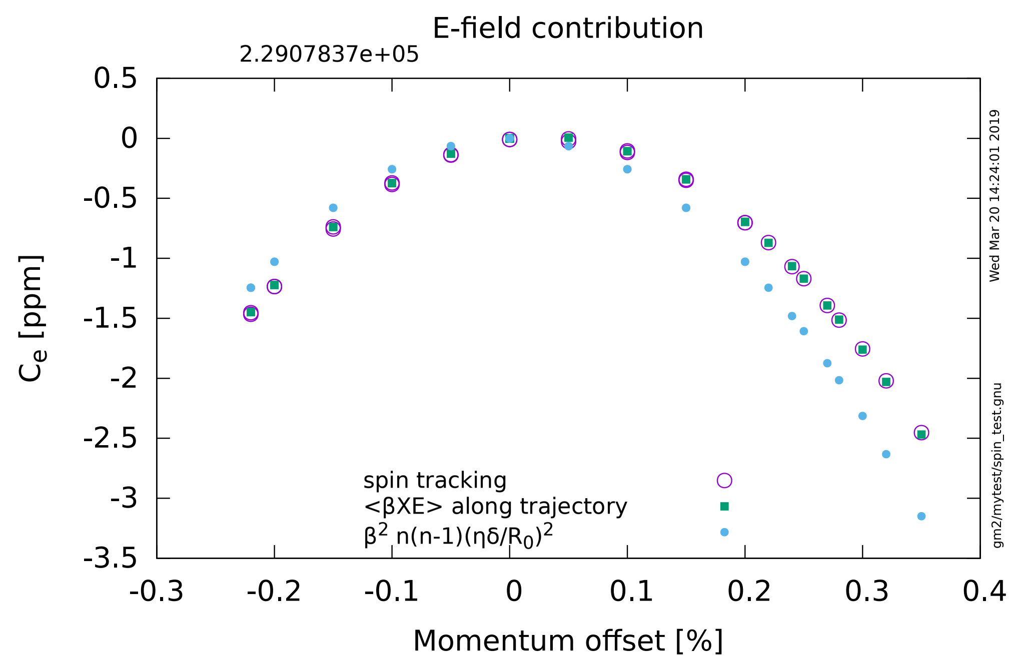
B_{radial} =50 ppm

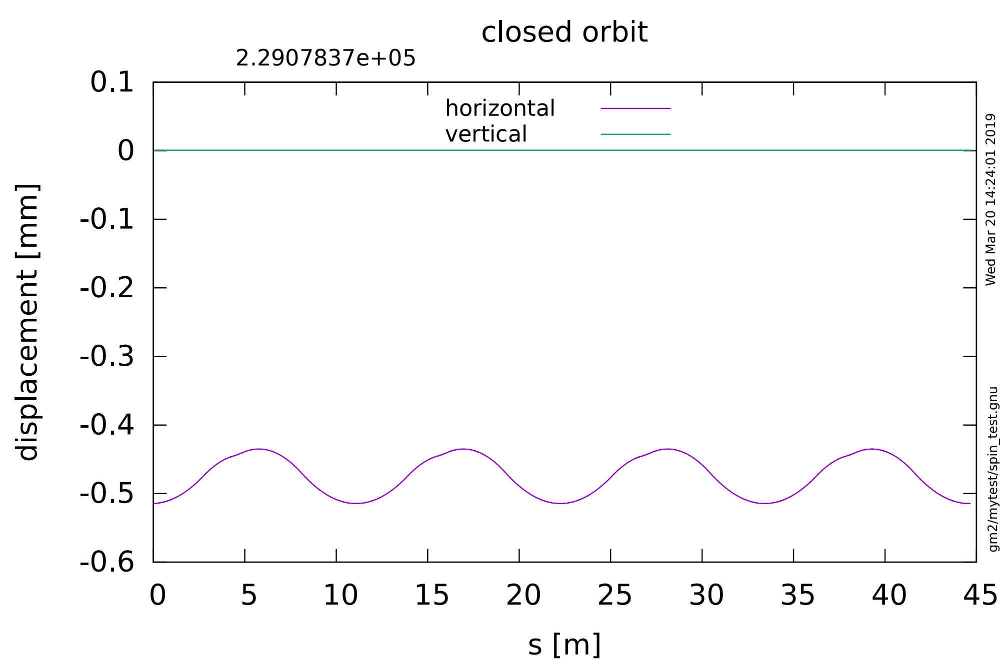
V [kV]	Inner	Bottom	Outer	Top
Q1	18.3	21.8	18.3	14.8
Q2	18.3	18.3	18.3	18.3
Q3	18.3	14.8	18.3	21.8
Q4	18.3	18.3	18.3	18.3





All quads displaced 4mm radially out effects E-field correction



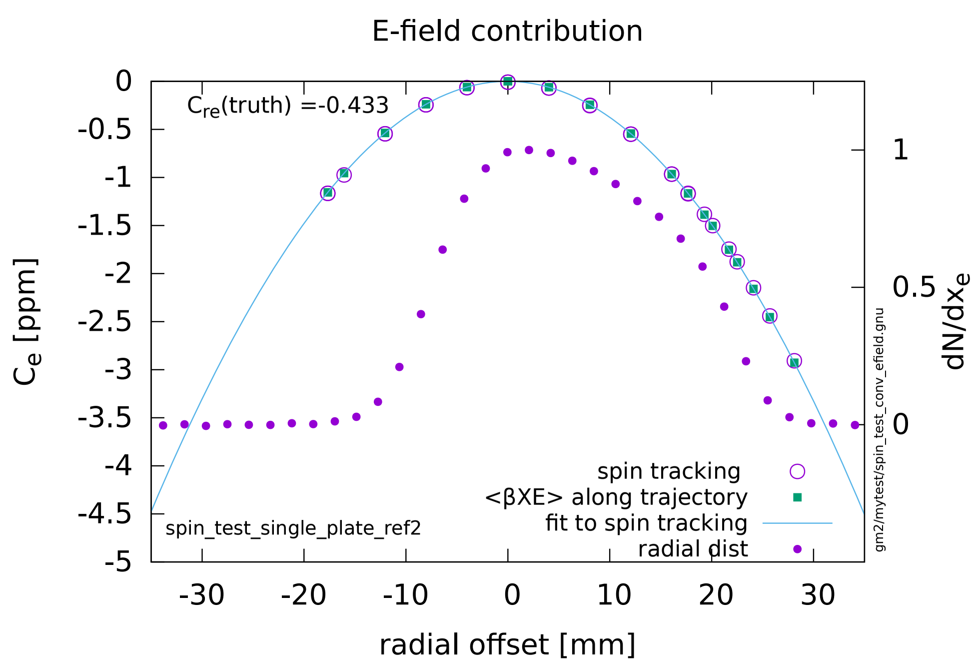


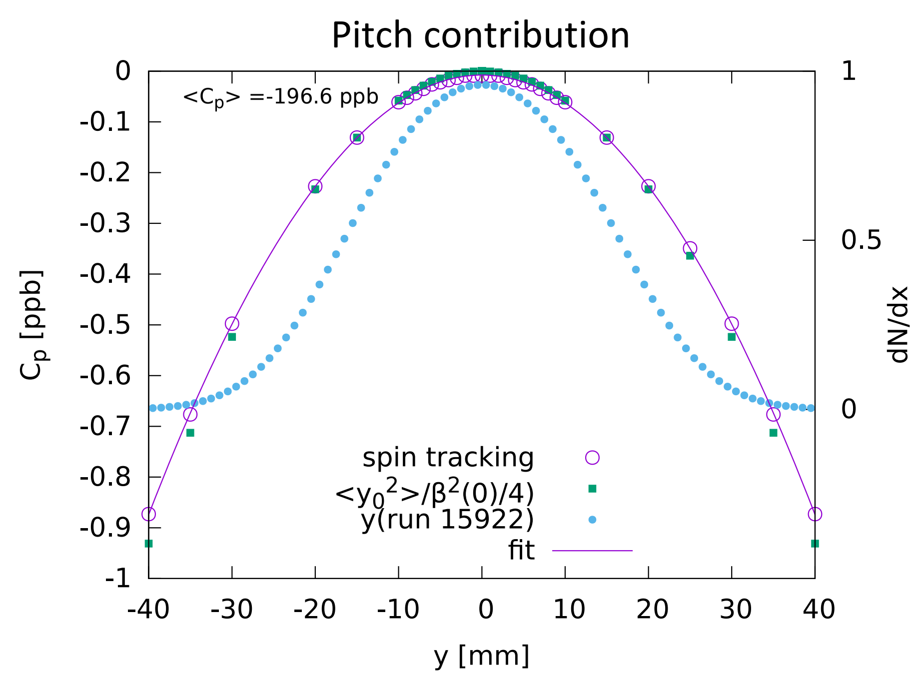
For each configuration

- Compute ‘real’ average C_p and C_e of all trajectories
- Convolute simulated distributions of x_e and y with $C_p(y)$ and $C_e(x_e)$ of the reference configuration

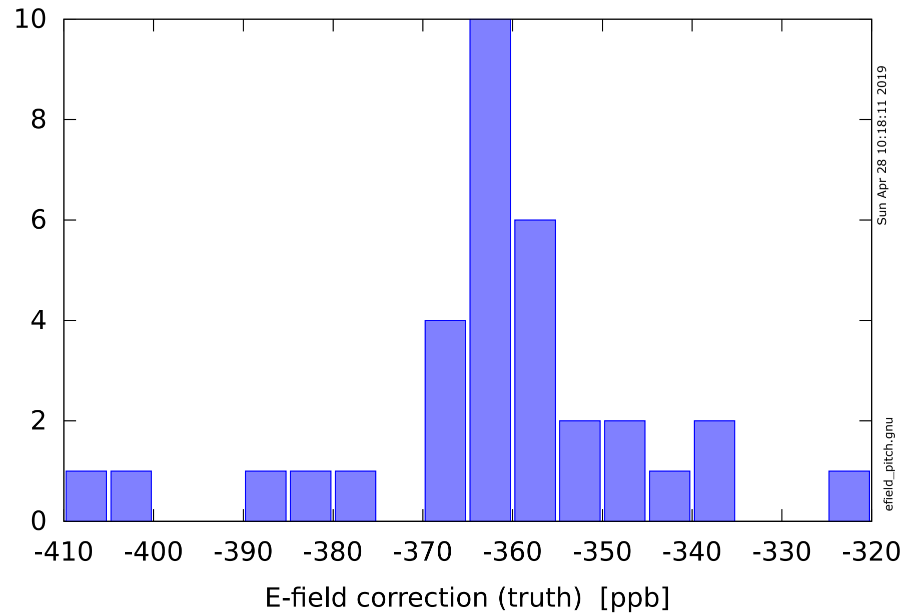
The convolution represents our ‘measurement’

Discrepancy between the ‘measurement’ and the ‘real’ C_p and C_e is the uncertainty due to alignment and field errors and nonlinearity.

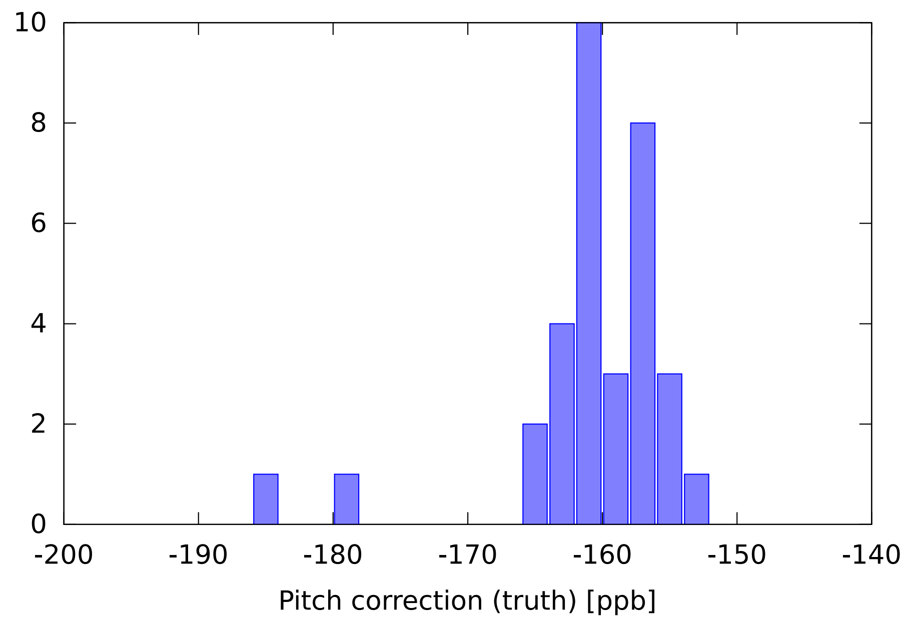




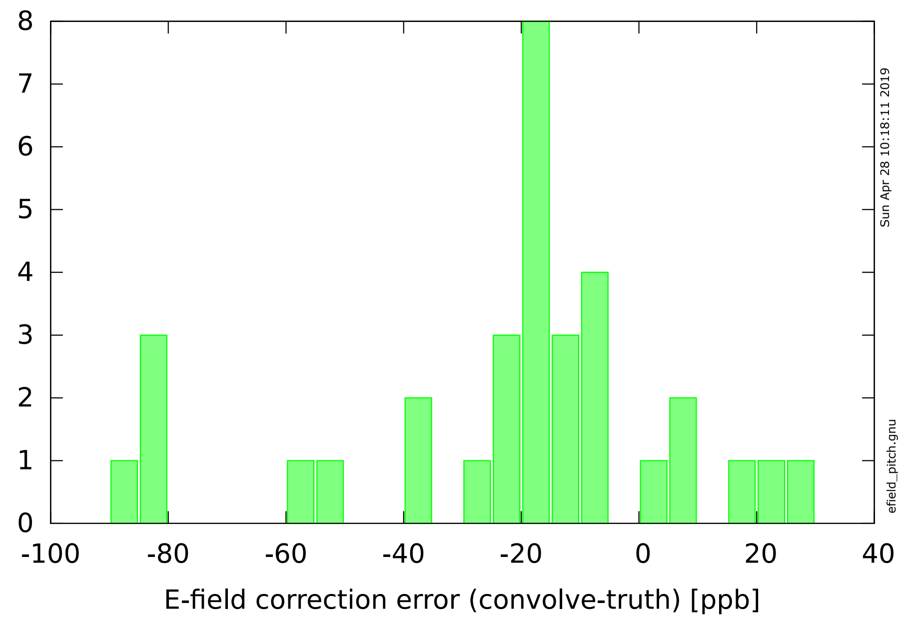
E-field contribution to ω_a for each of 35 configurations



Pitch contribution to ω_a for each of 35 configurations



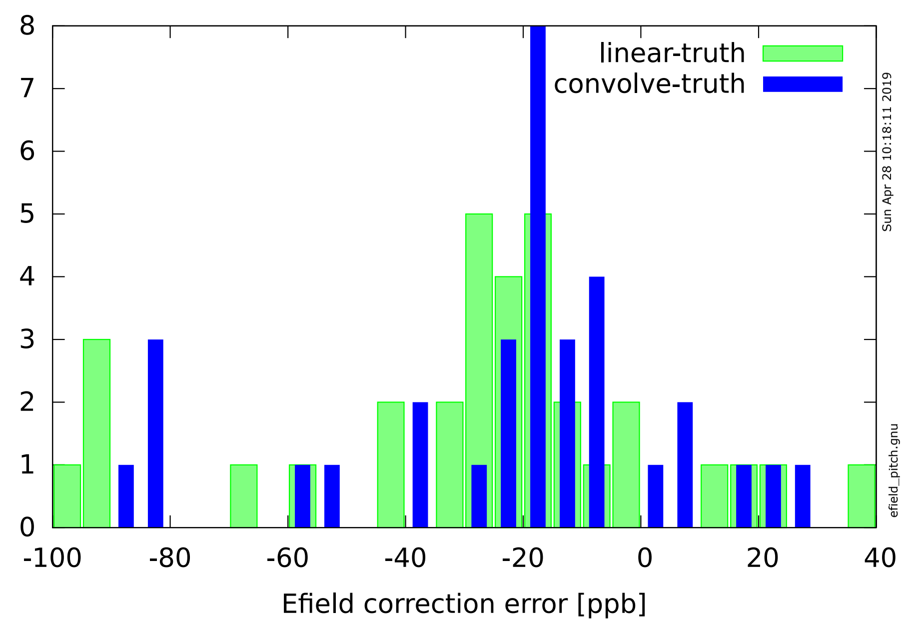
Discrepancy between real C_e(truth) and result from convolution C_e(convolve)



C_e (linear) computed by 'standard' method

$$C_E = -2\beta^2 n(1-n)\frac{\langle x_e^2 \rangle}{r_0^2}$$

Assuming quad index is measured

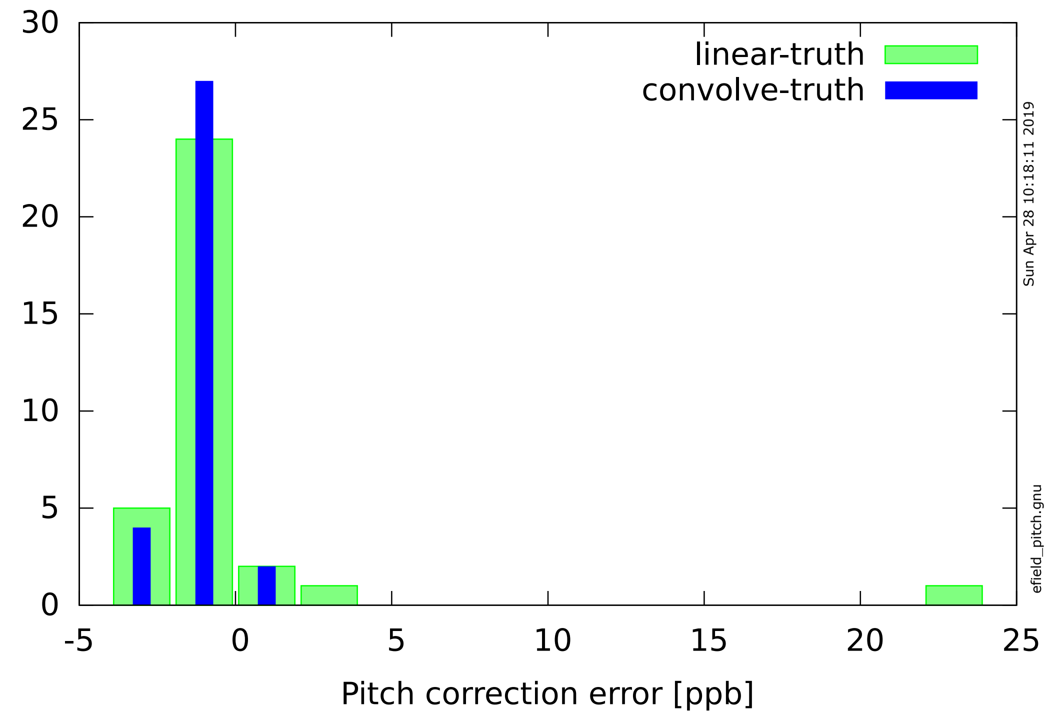


ΔC_e(convolve) < ΔC_e(linear)

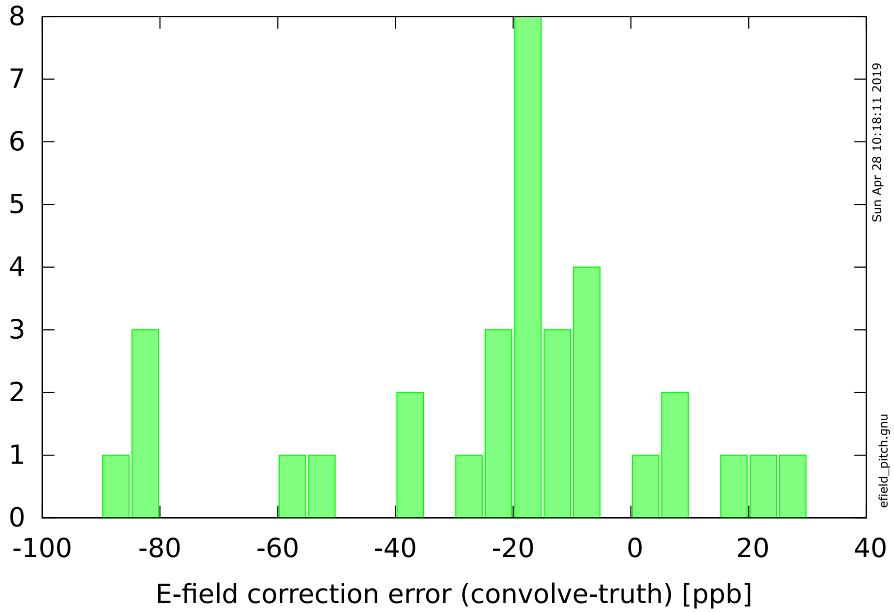
Discrepancy in evaluation of pitch correction using convolution vs linear (standard method)

$$C_p = -\frac{n\langle y^2\rangle}{2R_0^2}$$

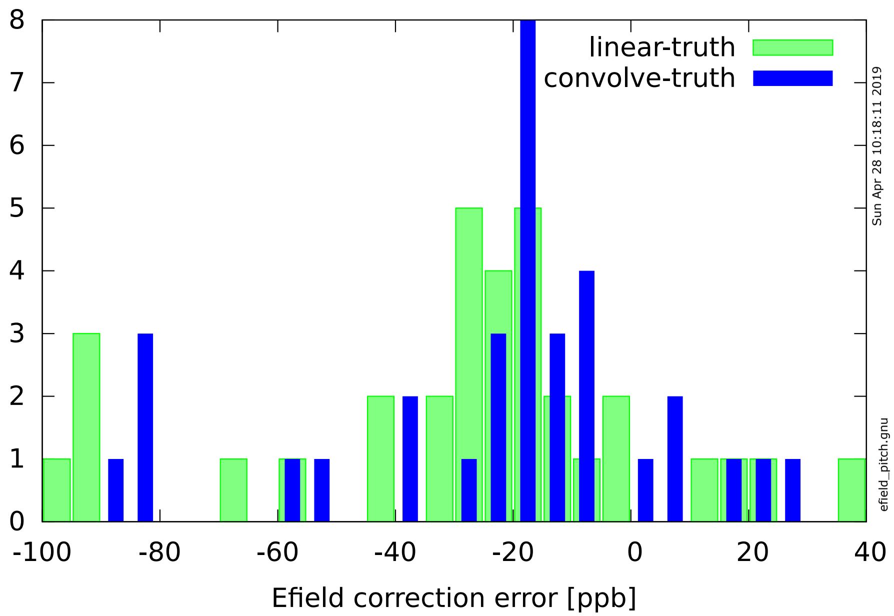
Assuming quad index n is measured



ΔC_p(convolve) < ΔC_p(linear)



8
7
6
5
4
3
2
1
0
-100
-80
-60
-40
-20
0
20
40
E-field correction error (convolve-truth) [ppb]
Sun Apr 28 10:18:11 2019
efield_pitch.gnu



linear-truth
convolve-truth
8
7
6
5
4
3
2
1
0
-100
-80
-60
-40
-20
0
20
40
Efield correction error [ppb]
Sun Apr 28 10:18:11 2019
efield_pitch.gnu

Summary

- Error in estimation of contribution to wa from pitch due to field errors and misalignment is < 5 ppb

- Error in estimate from E-field

$$-90 < \Delta C_e[\mathrm{ppb}] < 30$$

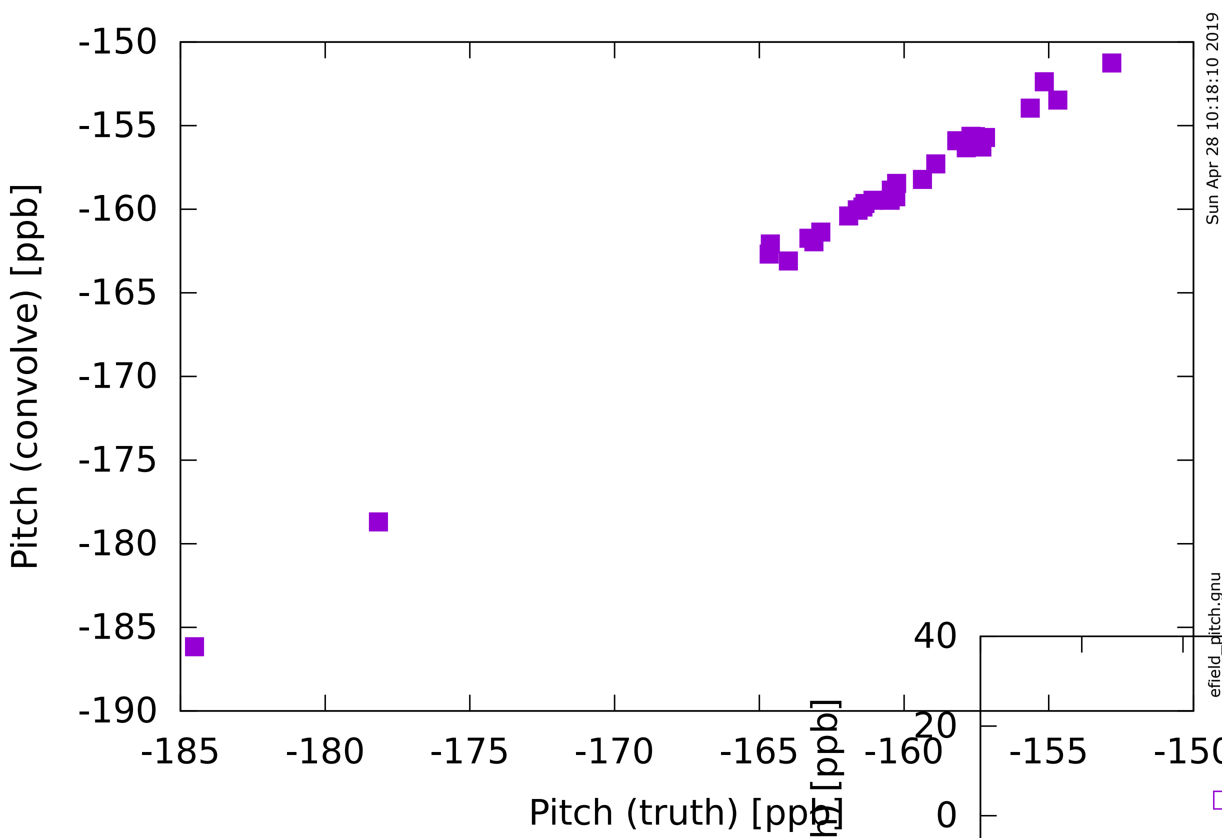
For alignment errors of ±2mm and voltage errors of ± 10%

Alignment uncertainty from survey < ± 1mm
Voltage error expected to be < 5%

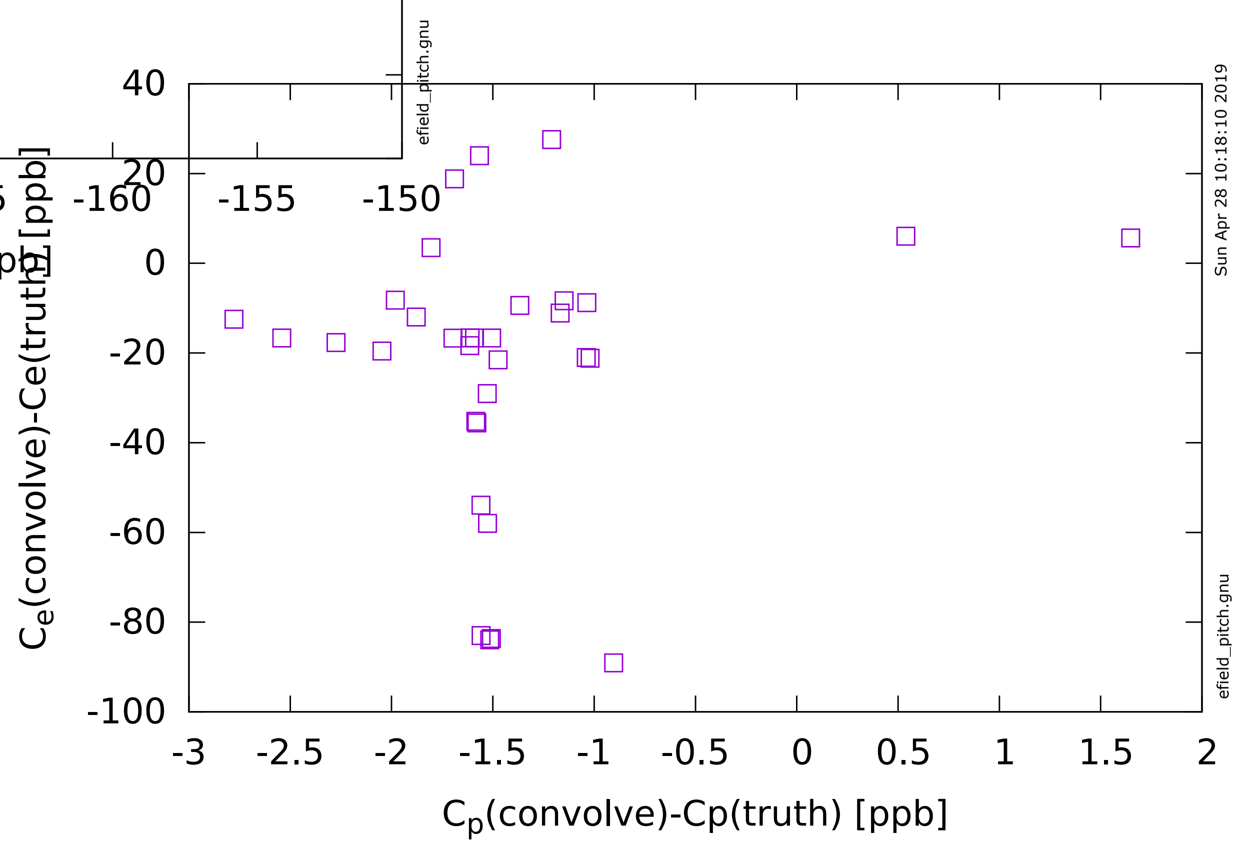
Next step

Generate a ‘complete’ set of configurations based on measured uncertainties and evaluate errors in determining C_e and C_p

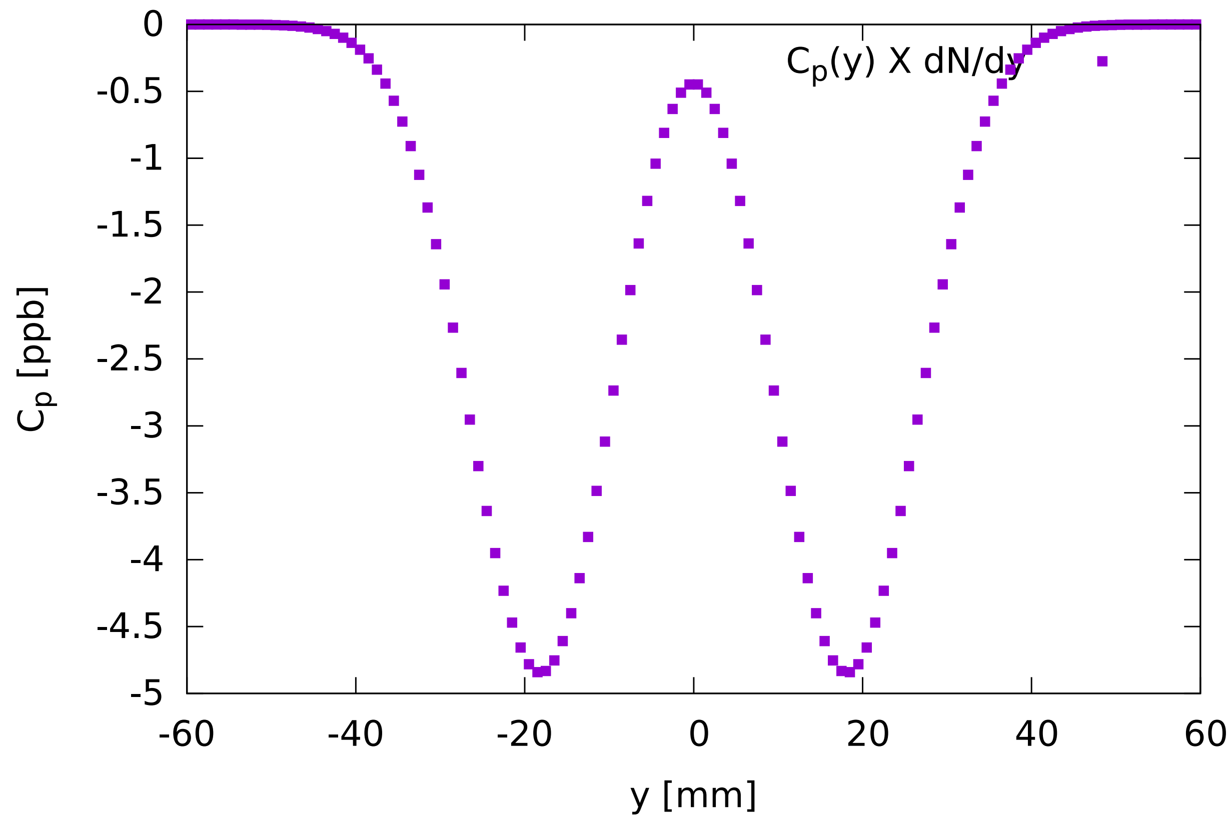
To set conservative bounds on effects of field errors, and misalignment backup



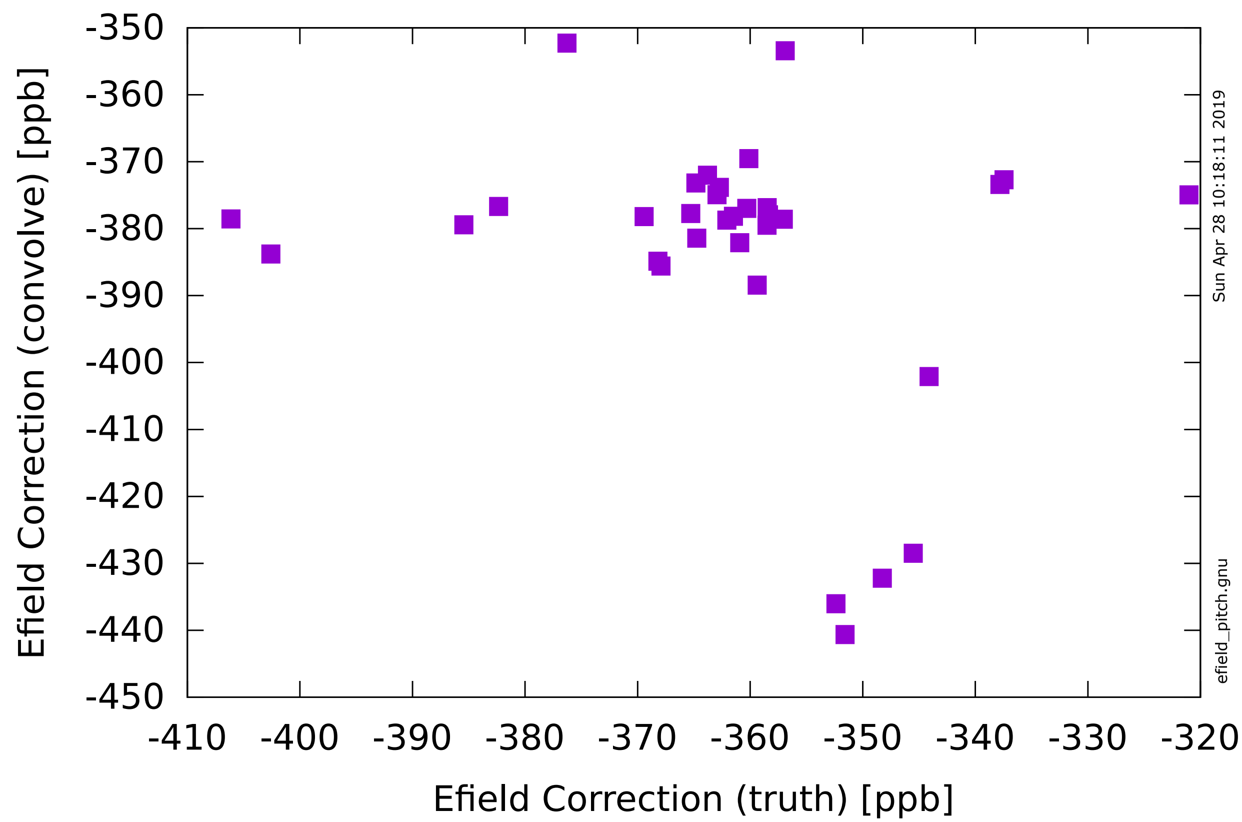
Pitch (convolve) [ppb]
Pitch (truth) [ppb]
-150
-155
-160
-165
-170
-175
-180
-185
-190
-185
-180
-175
-170
-165
-160
-155
-150
efield_pitch.gnu
Sun Apr 28 10:18:10 2019

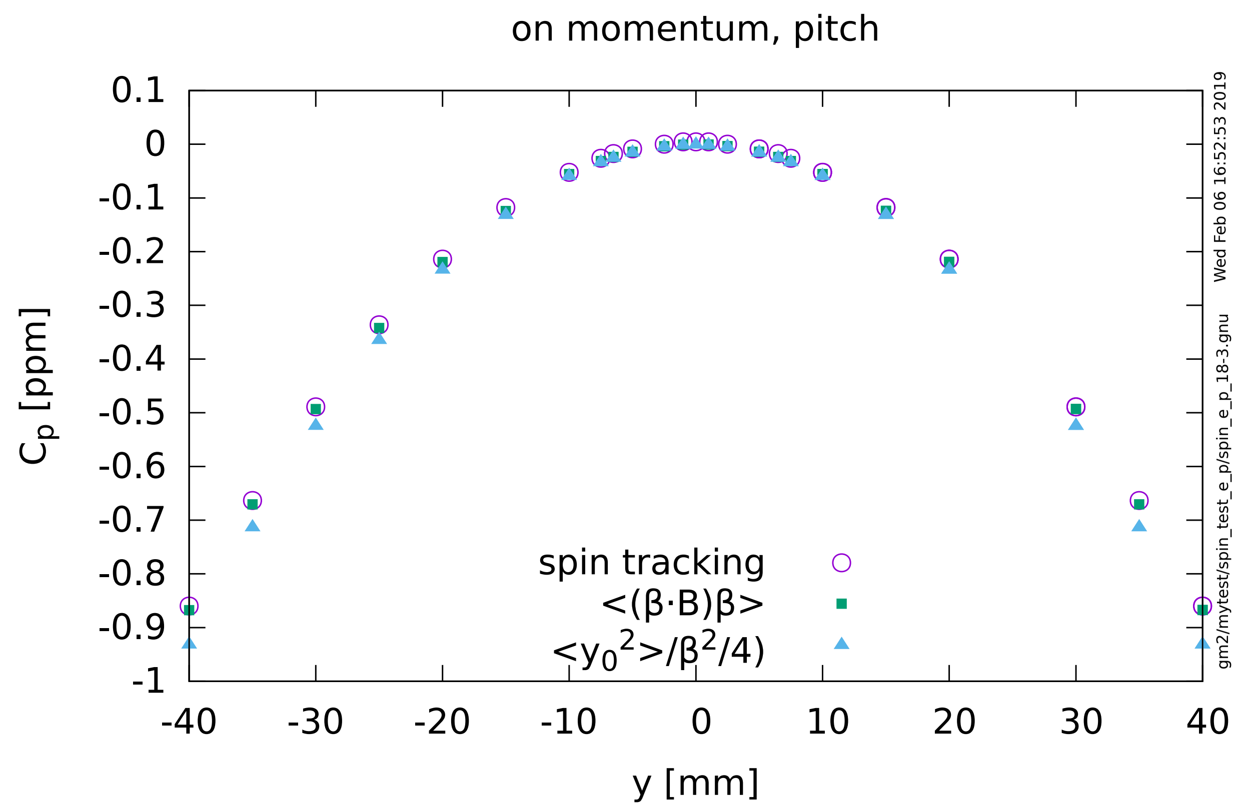


Ce(convolve)-Ce(truth) [ppb]
Cp(convolve)-Cp(truth) [ppb]
40
20
0
-20
-40
-60
-80
-100
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
efield_pitch.gnu
Sun Apr 28 10:18:10 2019



0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
-60
-40
-20
0
20
40
60
C_p [ppb]
y [mm]
C_p(y) X dN/dy


Efield Correction (convolve) [ppb]
-350
-360
-370
-380
-390
-400
-410
-420
-430
-440
-450
-410
-400
-390
-380
-370
-360
-350
-340
-330
-320
Efield Correction (truth) [ppb]
Sun Apr 28 10:18:11 2019
efield_pitch.gnu



Vertical amplitude $(y_0)/\beta_v$ is not a good measure of angle ψ at large amplitude

Quantify impact of nonlinearity, field and alignment errors with simulation

- Establish integration along trajectory as proxy for spin phase advance

$$\vec{C}_e(T) \sim 2\frac{\Delta p}{p}\frac{1}{T}\int_0^T \frac{\vec{\beta}\times\vec{E}}{Bc}dt$$

$$C_p = \frac{1}{T}\int_0^T (1-\hat{\beta}\times\hat{\mathbf{B}})dt$$

- Introduce field and alignment errors into model
- Track ‘realistic’ distribution and compute pitch and E-field corrections for every muon.

  Note that the field and alignment errors effect the distribution as well as the correction for each trajectory.

Pitch correction

3 ways to compute pitch correction in simulation

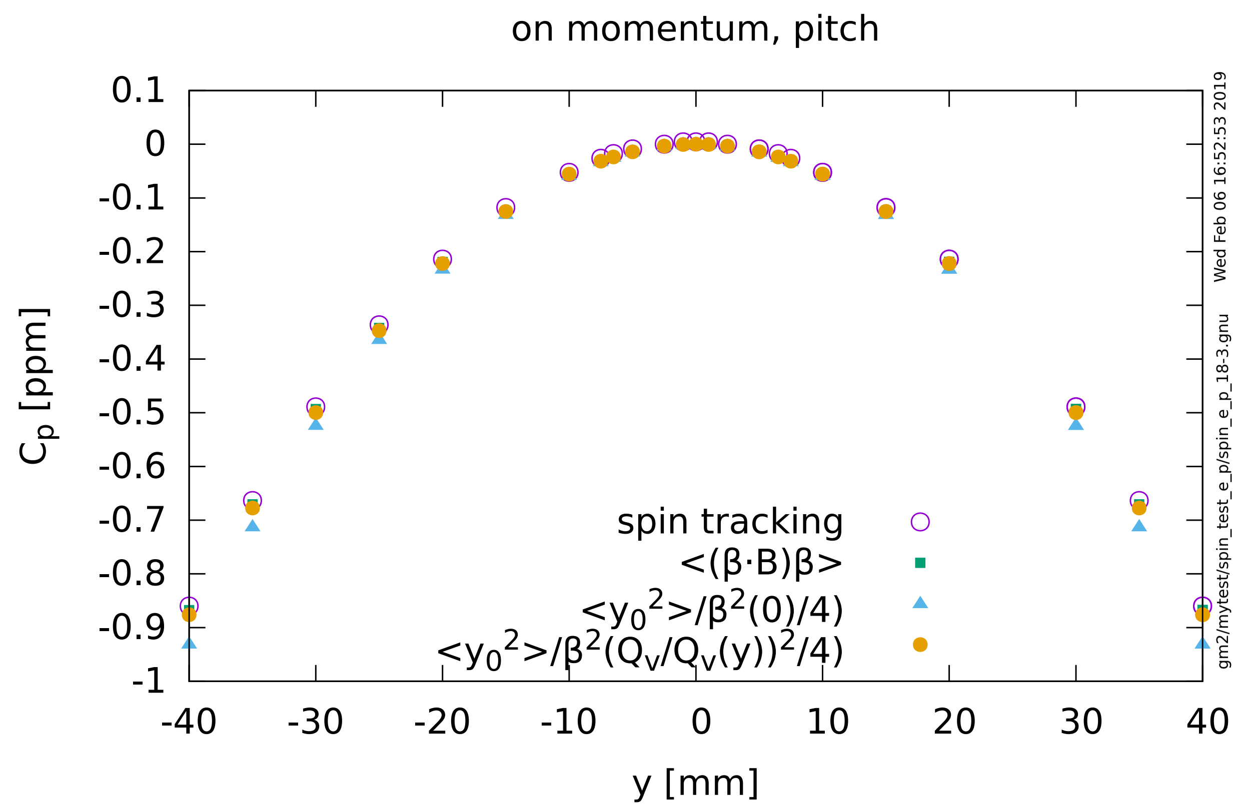
- Spin tracking − *Includes everything, but ppb precision requires many turns*
- Integration along trajectory − *very good approximation far from resonances*

$$C_p = \frac{1}{T}\int_0^T (1 - \hat{\beta} \times \hat{\mathbf{B}})dt$$

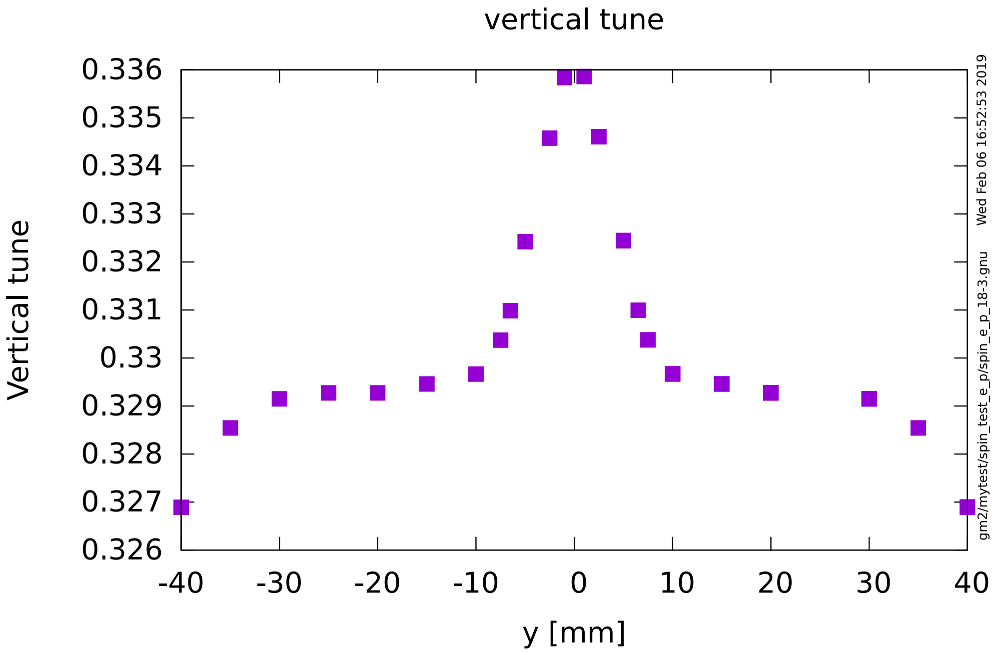
- Measurement of vertical amplitude − *assumes quad linearity*

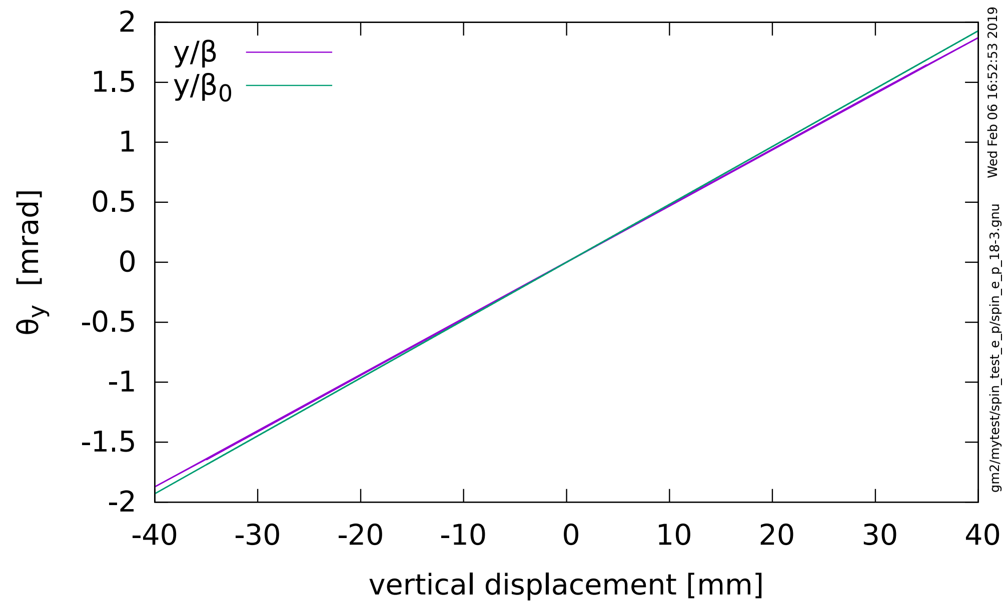
$$C_p = -\frac{n\langle y^2\rangle}{2R_0^2} = -\frac{\langle y^2\rangle}{2\beta_y^2} = -\frac{\langle \psi^2\rangle}{2}$$

$$\beta(y) = \left(\frac{Q_v(0)}{Q_v(y)^2}\right)^2 \beta_0$$

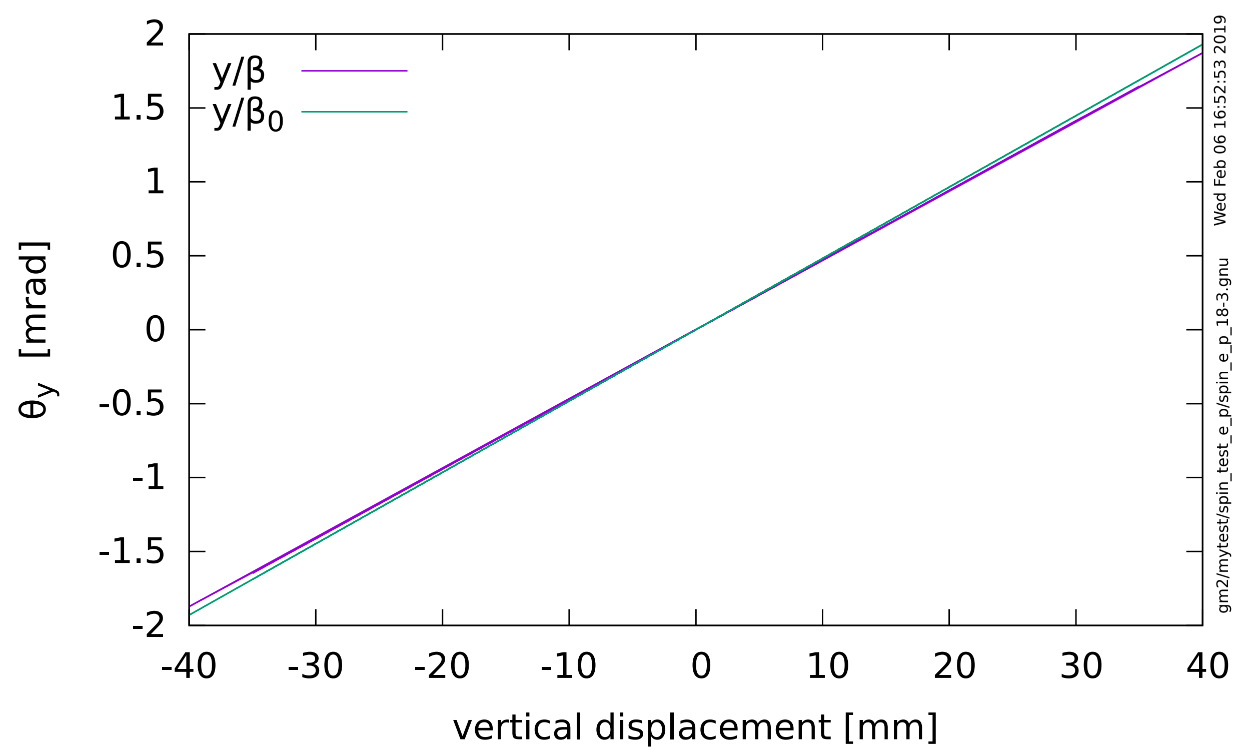


- We can correct for the amplitude dependence by measuring the vertical tune
- Alternatively, measure the angular $\langle \psi^2 \rangle$ distribution directly





Quad nonlinearity => amplitude dependence of tune and β
And nonlinear dependence of ψ on y_0


θy [mrad]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
y/β
y/β0
-40
-30
-20
-10
0
10
20
30
40
vertical displacement [mm]
gm2/mytest/spin_test_e_p/spin_e_p_18-3.gnu
Wed Feb 06 16:52:53 2019

E field correction

3 ways to compute E-field contribution to ω_a

1. Spin tracking (BMT equation)
2. Integration

$$\vec{C}_e(T) \sim 2\frac{\Delta p}{p}\frac{1}{T}\int_0^T \frac{\vec{\beta}\times\vec{E}}{Bc}dt$$

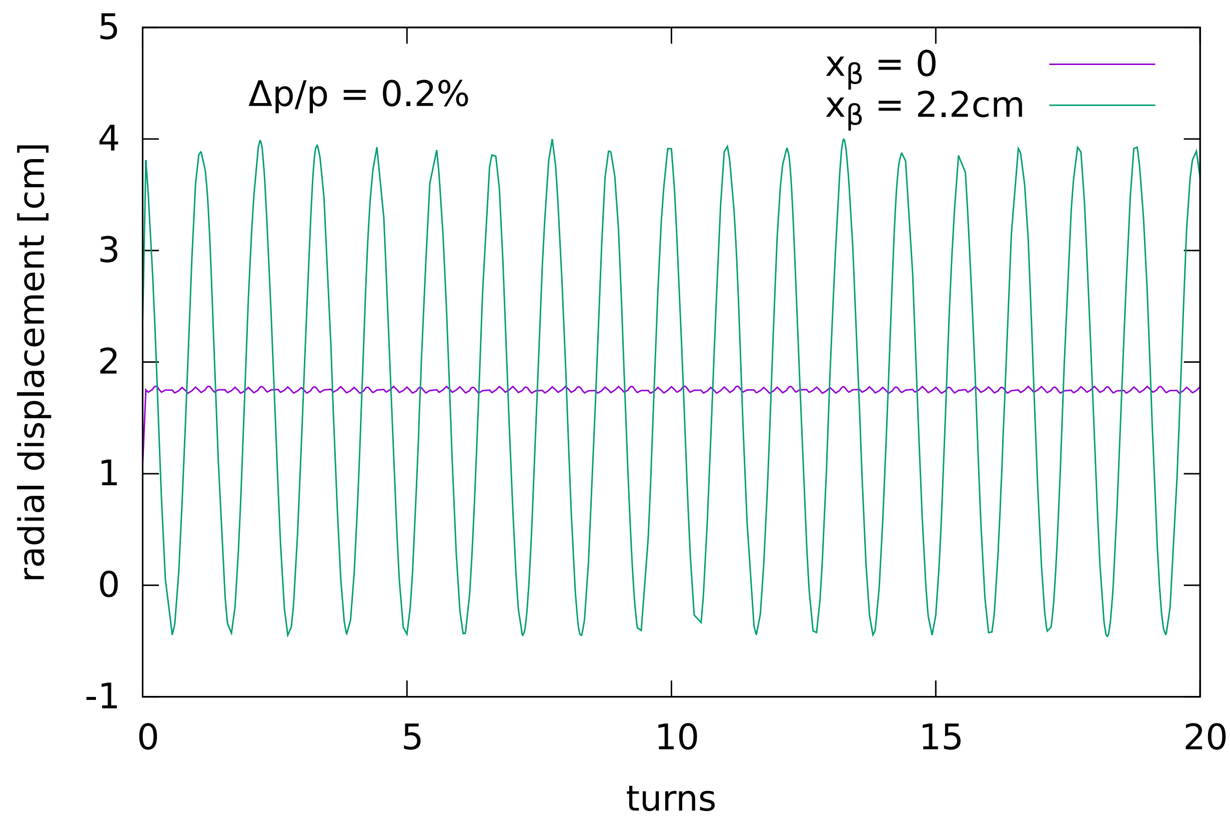
   a) Integration along trajectory (includes betatron oscillations)
   b) Integration along closed orbit ($x = \eta\delta$)

Note that method 2b) is most nearly equivalent to the 'classic' method, namely

$$C_E = -2\beta^2 n(1-n)\frac{\langle x_e^2 \rangle}{r_0^2}$$

Compare the 3 methods in simulation to determine

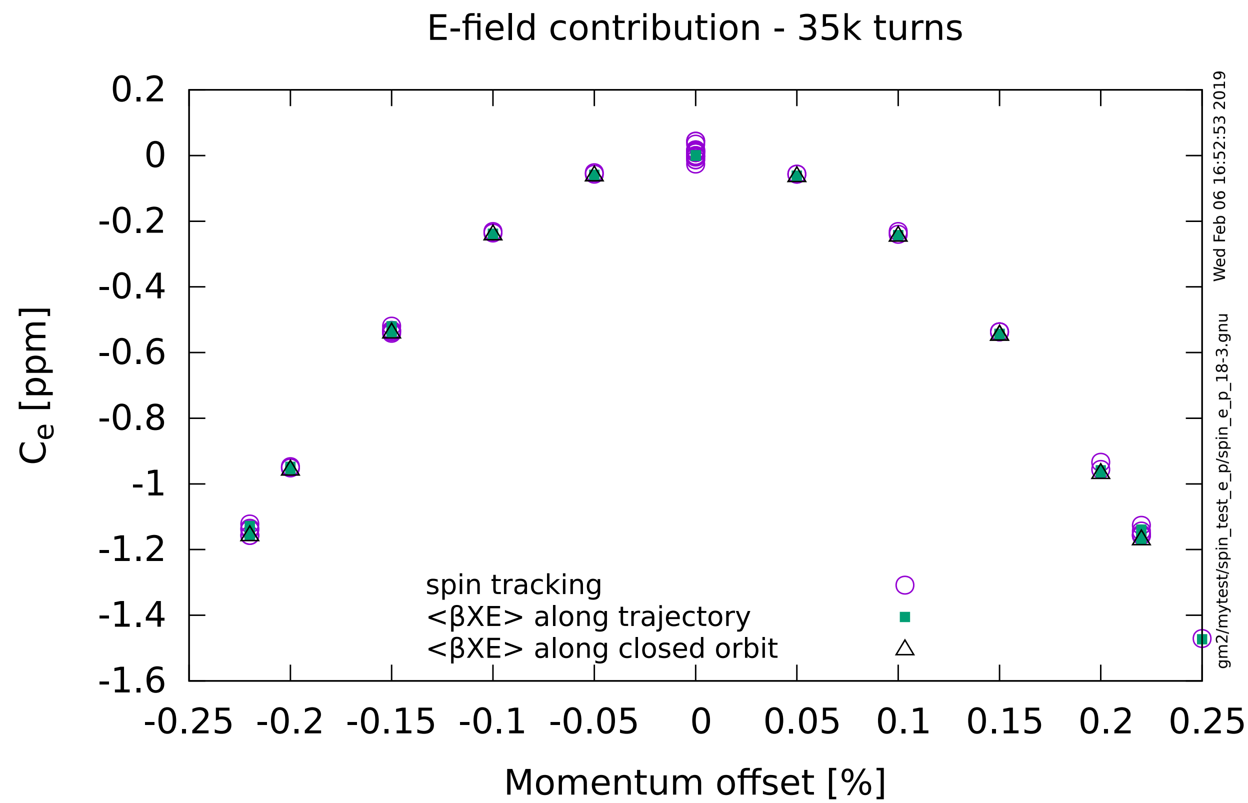
1. If integration is a reliable proxy for spin tracking
2. The size of the contribution from finite betatron oscillation amplitude
3. Effect of quad nonlinearity



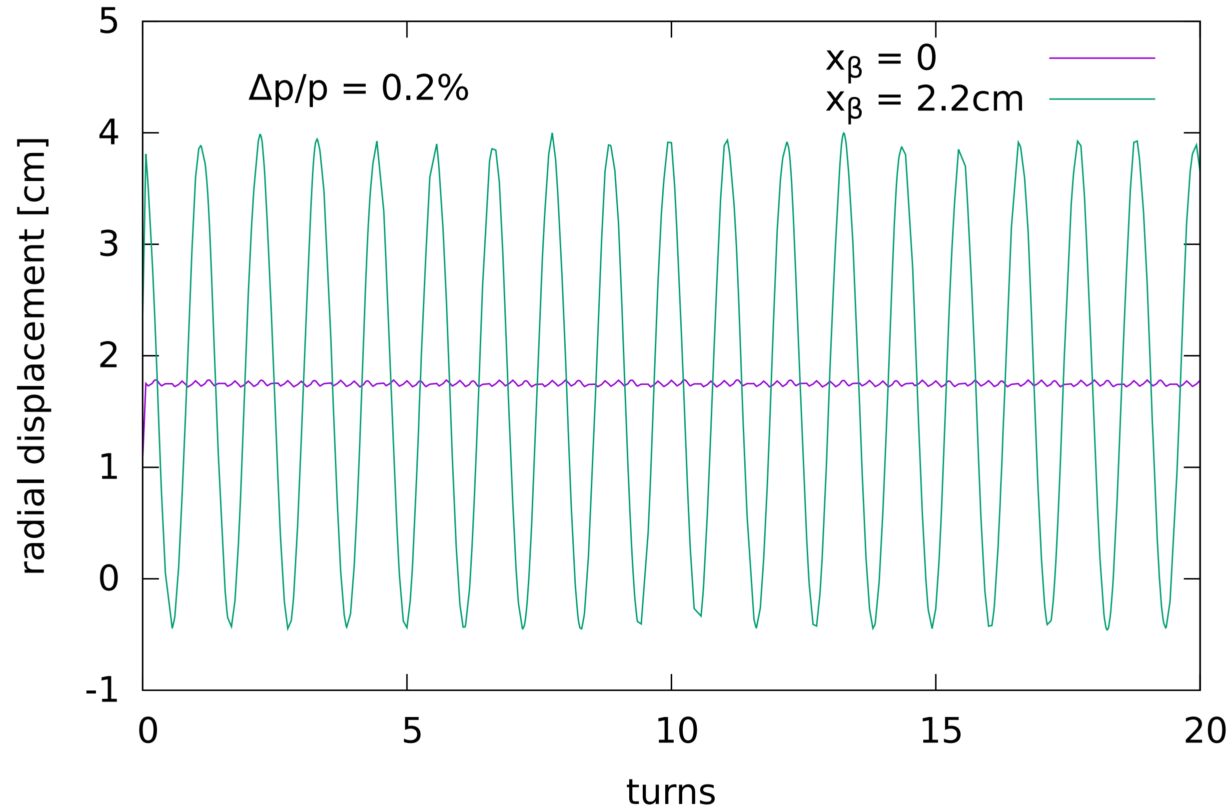
Distinct trajectories with common momentum offset

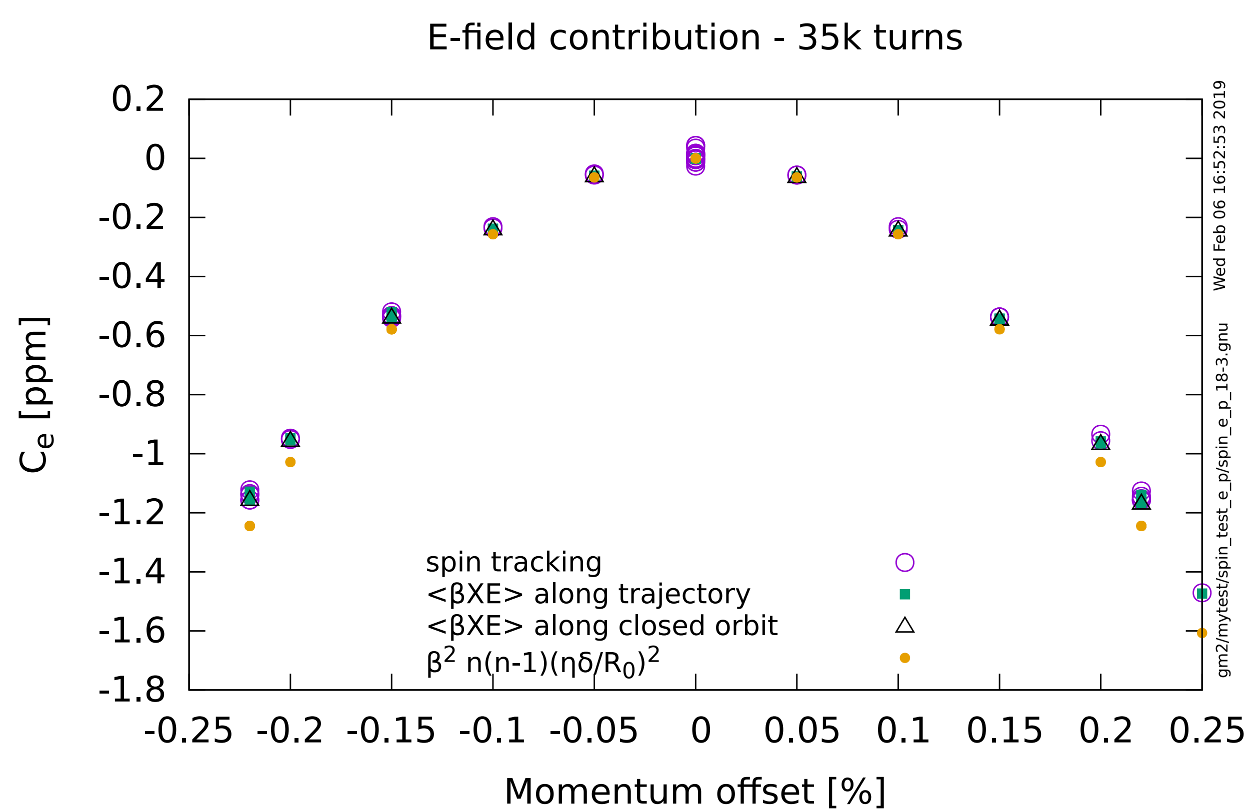
- For trajectory compute ω_a by spin tracking and by integration

Is the Efield correction independent of the betatron amplitude?

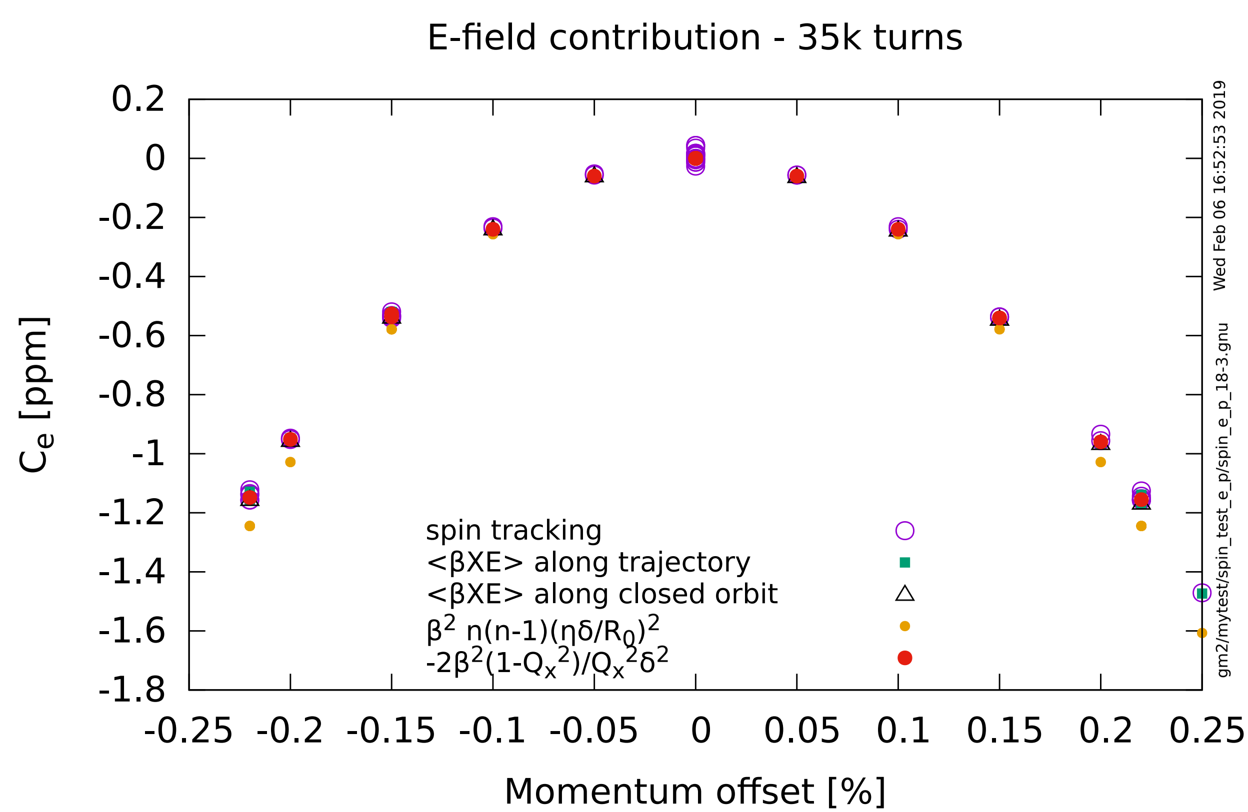


$\langle \beta \times \mathbf{E} \rangle$ along the trajectory is very nearly the same as $\langle \beta \times \mathbf{E} \rangle$ along the closed orbit. (There is little dependence on betatron amplitude)


radial displacement [cm]
turns
Δp/p = 0.2%
xβ = 0
xβ = 2.2cm
5
4
3
2
1
0
-1
0
5
10
15
20

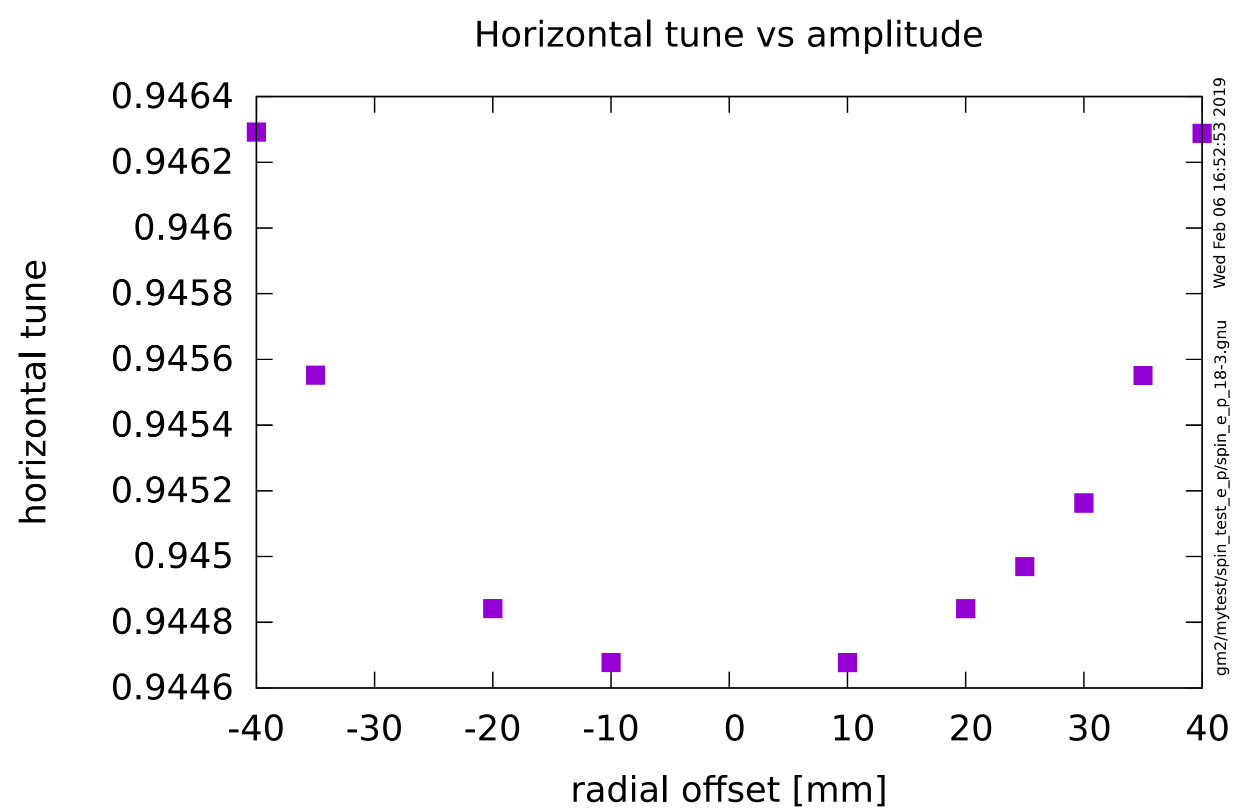


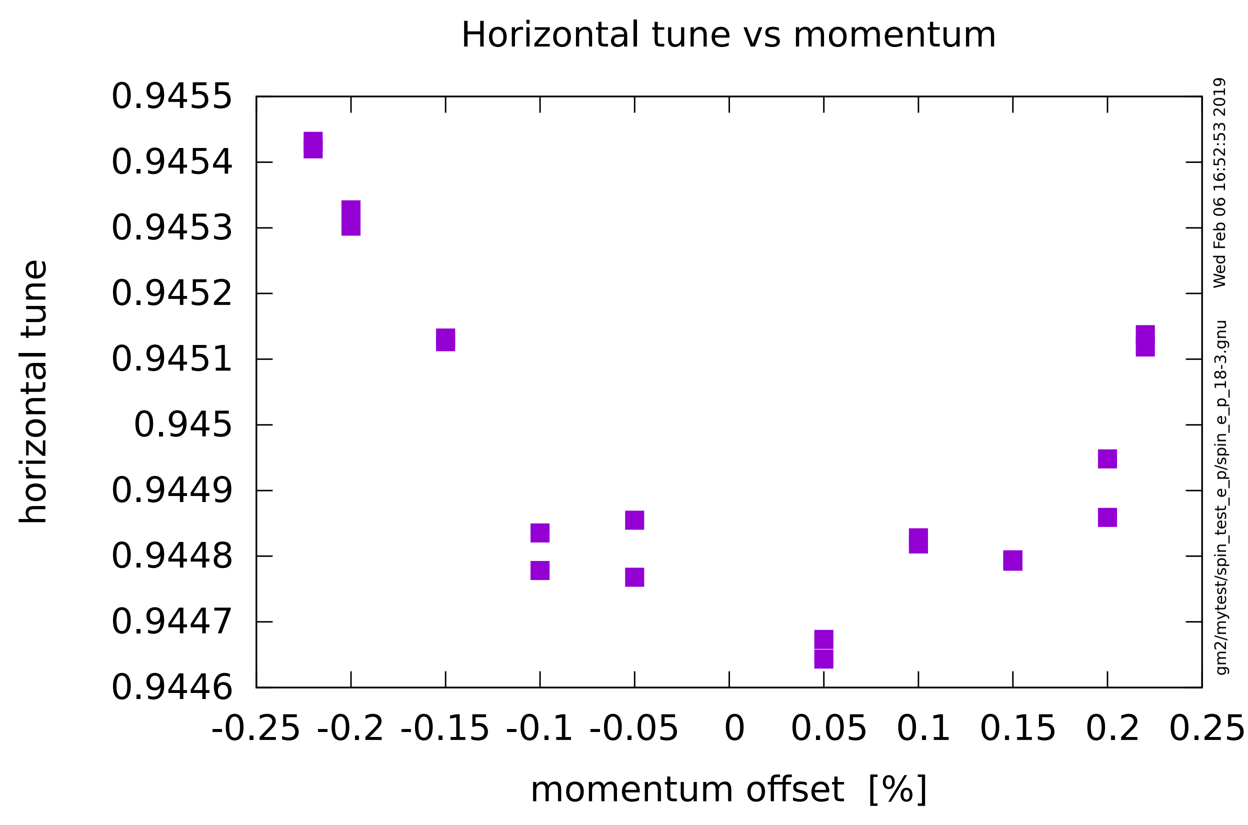
The calculation of the E-field correction that assumes quad linearity, *overestimates* the effect at large momentum offset (where E-field does not increase linearly with displacement)



Replace n, R_0 and η with Q_x and measure Q_x for each momentum => $C_e = -2\beta^2 \frac{(1-Q_x)^2}{Q_x^2}\delta^2$

The effect of quad nonlinearity can be corrected by measuring momentum dependence of horizontal tune


Horizontal tune vs amplitude
horizontal tune
0.9464
0.9462
0.946
0.9458
0.9456
0.9454
0.9452
0.945
0.9448
0.9446
-40
-30
-20
-10
0
10
20
30
40
radial offset [mm]
gm2/mytest/spin_test_e_p/spin_e_p_18-3.gnu
Wed Feb 06 16:52:53 2019


Horizontal tune vs momentum
horizontal tune
0.9455
0.9454
0.9453
0.9452
0.9451
0.945
0.9449
0.9448
0.9447
0.9446
-0.25 -0.2 -0.15 -0.1 -0.05 0 0.05 0.1 0.15 0.2 0.25
momentum offset [%]
gm2/mytest/spin_test_e_p/spin_e_p_18-3.gnu
Wed Feb 06 16:52:53 2019

Comments

- Quad fields are based on an azimuthal slice of 3-D field map with no end effect details
- And perfect relative alignment of plates and absolute alignment about magic radius
- Perfect B-field

Measurement of amplitude/momentum dependence of tunes would be very useful to diagnose quad fields and compensate nonlinearities.