Corrections for Electric Field and Pitch

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Electric field and vertical pitch systematically shift ω_a

- The electric field correction C_e is based on measurement of the equilibrium radial distribution, and the focusing index
- The pitch correction C_p is based on a measurement of the vertical distribution at the decay muons, and the focusing index

Assuming perfect measurement of the distributions and index *n*

What is the uncertainty in C_e and C_p due to

- *Quadrupole field nonlinearity*
- *Misalignment of quadrupole plates*
- *Quad voltage errors*
- *Radial magnetic field ?*

Outline

- Electric field contribution to ω_a
  - Analytic description − nonlinearity and misalignment
  - Spin tracking and integrating
- Contribution from pitch
  - Theory
  - Spin Tracking and Integration
- Simulation
  - Evaluation of dependence on nonlinearity, misalignment, voltage errors, B-radial
  - Incorporating details of ring model into measurement

How does quad nonlinearity, field errors and misalignment alter effects of electric field and pitch?

How large are those effects?

Can we correct for those effects?

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{m^2}{p^2} \right) \frac{\vec{\beta}\times\vec{E}}{c} \right]$$

$$C_e \sim -2\frac{\Delta p}{p}\langle \frac{\vec{\beta}\times\vec{E}}{Bc} \rangle$$

Electric field

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{m^2}{p^2} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

$$C_e \sim -2\frac{\Delta p}{p}\langle \frac{\vec{\beta} \times \vec{E}}{Bc} \rangle$$

- Measure $\Delta p/p$ and E-field
- As long as the quadrupole field is linear in displacement

$$\langle E_r \rangle = n\left(\frac{v_s B}{R_0}\right) x_e$$

$$\frac{\Delta p}{p} = \frac{x_e}{\eta}$$

$$C_e \sim -2\beta^2 n(1-n)\frac{x_e^2}{R_0^2}$$

Measurement of radial closed orbit, x_e => E-field correction

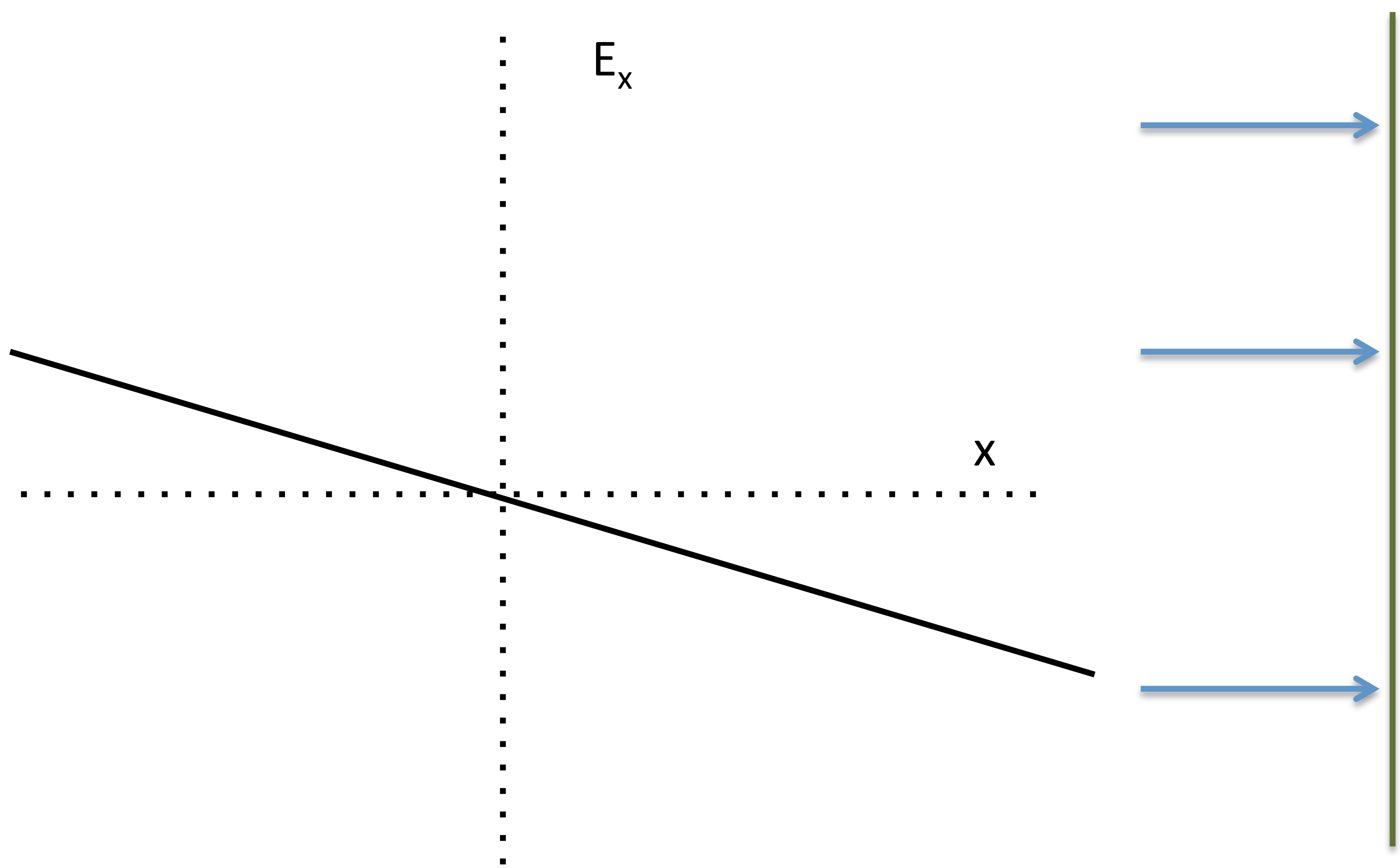
$$C_e \sim -2\beta^2 n(1-n)\frac{x_e^2}{R_0^2}$$

Measurement of radial closed orbit, x_e => E-field correction

We measure distribution of equilibrium radii $\langle x_e^2 \rangle$ with analysis of fast rotation signal

Quad Nonlinearity

- E_r is not simply linear in x
- Index n and dispersion η depend on x
- Quad curvature => quadratic dependence (sextupole-like)
- Sextupole component => amplitude dependent shift of the closed orbit



$$\frac{dE_x}{dx} = 0$$

Quad index n is proportional to $\frac{dE_x}{dx}$

$$-\nabla V = \mathbf{E} \sim k\left((x - \frac{x^2}{2\rho_0} + \ldots)\hat{\rho} - z\hat{\mathbf{z}}\right)$$

$$x = x_e + x_\beta = \eta\delta + x_\beta$$

$$\begin{aligned}
\langle E_r(s)\rangle &= k\langle\left(\eta\delta + x_\beta - \frac{1}{2\rho_0}(\eta\delta + x_\beta)^2\right)\rangle \\
&= \frac{k}{L}\int_0\left(\eta\delta + x_\beta - \frac{1}{2\rho_0}(\eta\delta + x_\beta)^2\right) dl \\
&= \frac{k}{L}\int_0\left(\eta\delta + x_\beta - \frac{1}{2\rho_0}(\eta\delta + x_\beta)^2\right)(1 + x_\beta/\rho_0)ds
\end{aligned}$$

sextupole

Path length

The average E-field for a muon with momentum $p_0 + \Delta p$ and betatron amplitude x_β is

$$\langle E_r \rangle \quad = \quad k\left(\eta\delta + \frac{1}{2\rho_0}((\eta\delta)^2 + \frac{1}{2}x_{\beta 0}^2) + \ldots\right)$$

$$(x_e = \eta\delta)$$

$$C_e \sim -2\delta\langle E_r \rangle$$

If $\langle \delta \rangle = \langle \frac{\Delta p}{p} \rangle = 0$ then sextupole contribution vanishes.

Otherwise for a positive momentum offset
E-field correction *increases* with betatron amplitude

If $\eta\delta \sim x_{\beta 0} \sim 2\mathrm{cm}$ then $E_2/E_1 \sim 0.07\%$

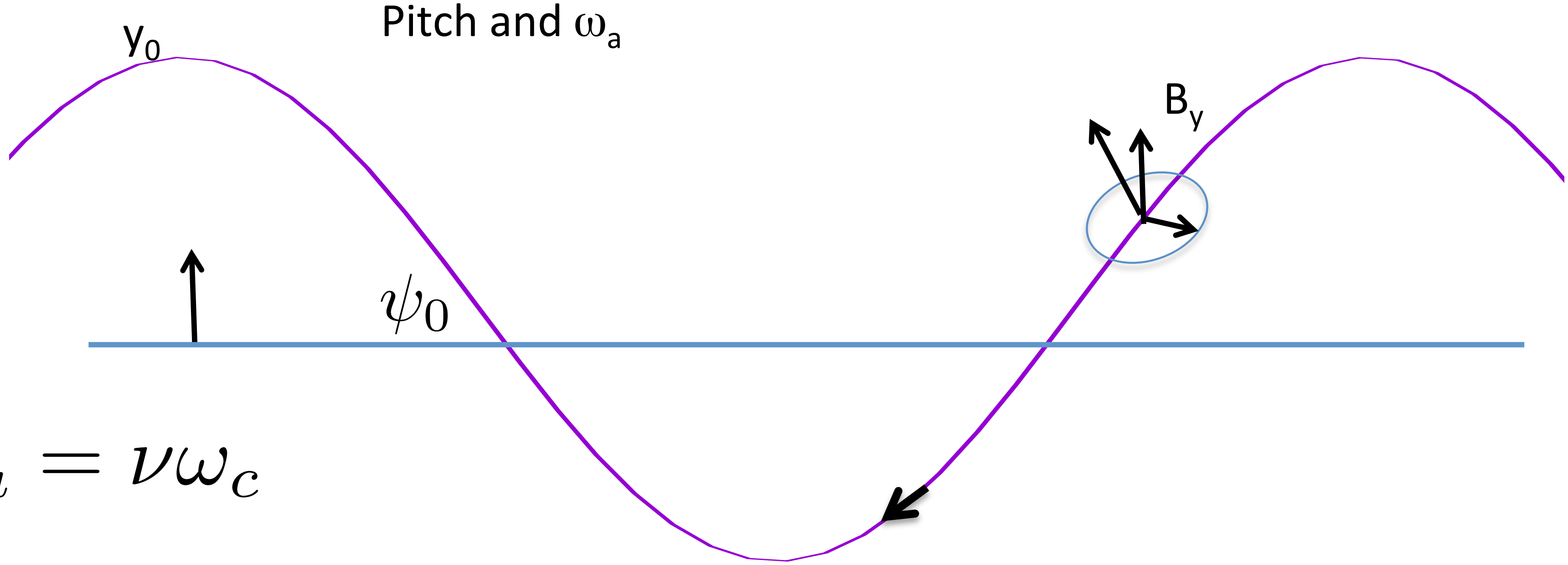
Conclusion so far *re* E-field correction

- In linear regime $C_e \propto \langle x_e^2 \rangle$ and we know how to measure x_e
- Sextupole component does indeed result in dependence of E-field correction on betatron amplitude, but it looks to be a very small effect
- Effect of other multipoles (and especially amplitude dependence of quad index), to be estimated with simulation

Pitch correction

$$\omega_a = -\frac{q}{m}\left(a_\mu \mathbf{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\beta \cdot \mathbf{B})\right)$$

Pitch and ω_a

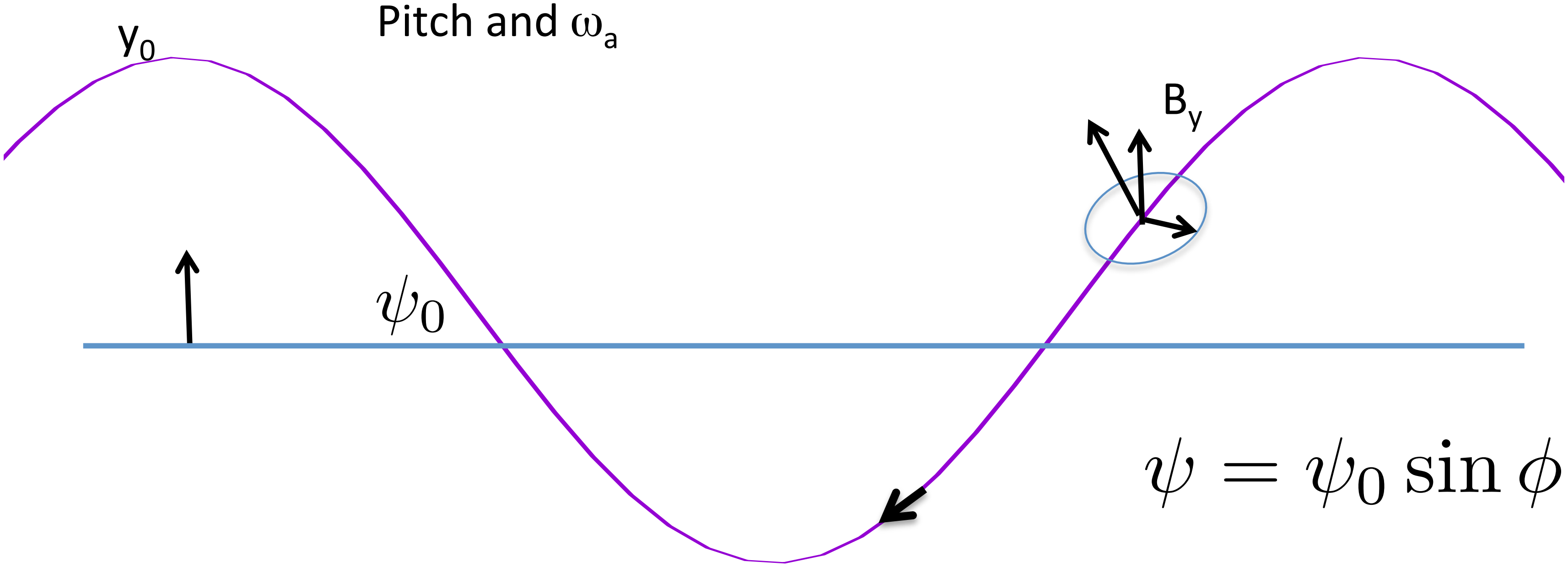


$$\omega_a = \nu \omega_c$$

We measure precession about the axis perpendicular to the direction of motion.

- The component of the magnetic field along that perpendicular axis is B cos ψ.
- The spin tune $\nu \propto \oint B_\perp dl = \oint B \cos\psi dl$
- Path length $\sim L(1 + \frac{1}{4}\psi_0^2)$, cyclotron frequency $\omega_c(\psi_0) \sim \omega_c(0)(1 - \frac{1}{4}\psi_0^2)$

Pitch and ω_a



$$\psi_0$$

$$\psi = \psi_0 \sin\phi$$

$$\omega_a = \nu\omega_c$$

We measure precession about the axis perpendicular to the direction of motion.

- The component of the magnetic field along that pendicular axis is B cos ψ.
- The spin tune $\nu \propto \oint B_\perp dl = \oint B \cos\psi dl$
- Path length $\sim L(1+\frac{1}{4}\psi_0^2)$
- => spin tune (ν) is independent of pitch
- But $\omega_c(\psi_0) \sim \omega_c(0)(1-\frac{1}{4}\psi_0^2)$ $\rightarrow \omega_a(\psi_0) = \omega_a(0)(1-\frac{1}{4}\psi_0^2)$

How do we measure ψ_0 ?

If motion (quad field) is linear

$$\begin{aligned} y &= \sqrt{a\beta}\cos\phi \\ \psi &= \psi_0 \sin\phi = \sqrt{\frac{a}{\beta}}\sin\phi \\ \langle \psi^2(a)\rangle_\phi &= \frac{1}{2}\psi_0^2(a) = \frac{\langle y^2(a)\rangle_\phi}{\beta^2} \end{aligned}$$

Average over all ϕ for a given amplitude *a*

Average over all amplitudes *a*

$$\begin{aligned} \langle\langle y^2\rangle_\phi\rangle_a &= \frac{1}{2}\beta\frac{\int^{ap} aP(a)da}{\int^{ap} P(a)da} \\ \langle\langle \psi^2\rangle_\phi\rangle_a &= \frac{1}{2}\langle\psi_0^2\rangle_a = \frac{1}{\beta^2}\langle\langle y^2\rangle_\phi\rangle_a \end{aligned}$$

$$C_p = -\frac{n\langle y^2\rangle}{2R_0^2}$$

Average over all ϕ for a given amplitude *a*

$$
\begin{aligned}
y &= \sqrt{a\beta}\cos\phi \\
\psi &= \psi_0 \sin\phi = \sqrt{\frac{a}{\beta}}\sin\phi \\
\langle\psi^2(a)\rangle_\phi &= \frac{1}{2}\psi_0^2(a) = \frac{\langle y^2(a)\rangle_\phi}{\beta^2}
\end{aligned}
$$

Assumes linearity

$$\beta(y) = \frac{R_0}{\sqrt{n(y)}}$$

We know that the effective quad index decreases with amplitude y

$$|C_p| < \frac{n\langle y^2\rangle}{2R_0^2}$$

Conclusion so far *re* Pitch correction

- In linear regime $C_p = -\frac{n\langle y^2\rangle}{2R_0^2}$ and we know how to measure vertical distribution
- The amplitude dependence of the field index will alter the correction
- Effect of amplitude dependence and multipoles to be evaluated with simulation

For both E-field and Pitch corrections,

In addition to nonlinearity

- Voltage errors on individual quad plates can distort closed orbit, and, η and β
- Misalignment of quad plates will distort closed orbit, introduce additional nonlinearity, alter focusing

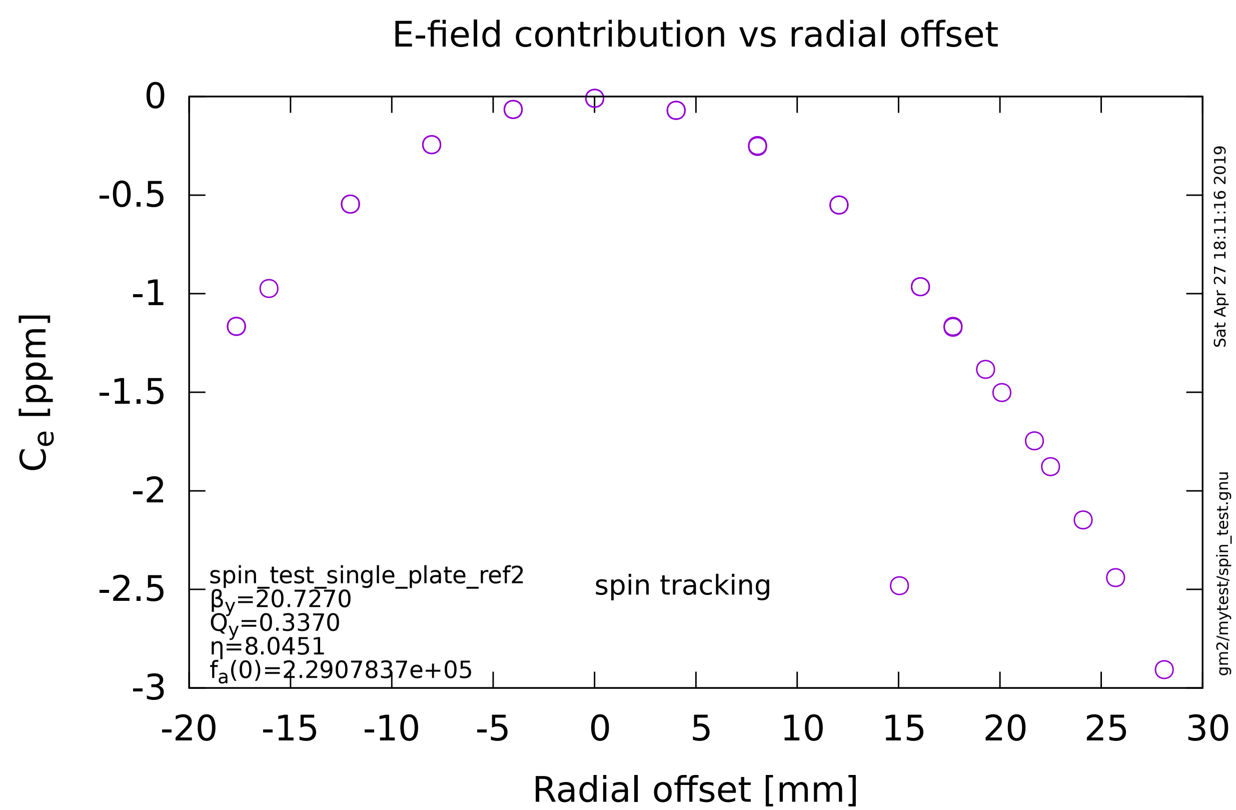
In tracking simulation it is convenient to compute

$$
\begin{aligned}
C_e(T) &= -2\frac{\Delta p}{p}\frac{1}{T}\int^T \frac{\tilde{\beta}\times\mathbf{E}}{Bc}dt \\
C_p(T) &= \frac{1}{T}\int^T (\tilde{\beta}\cdot\mathbf{B})\tilde{\beta}dt
\end{aligned}
$$

to give E-field and pitch correction along the trajectory of the muon as a proxy for spin tracking (slow)

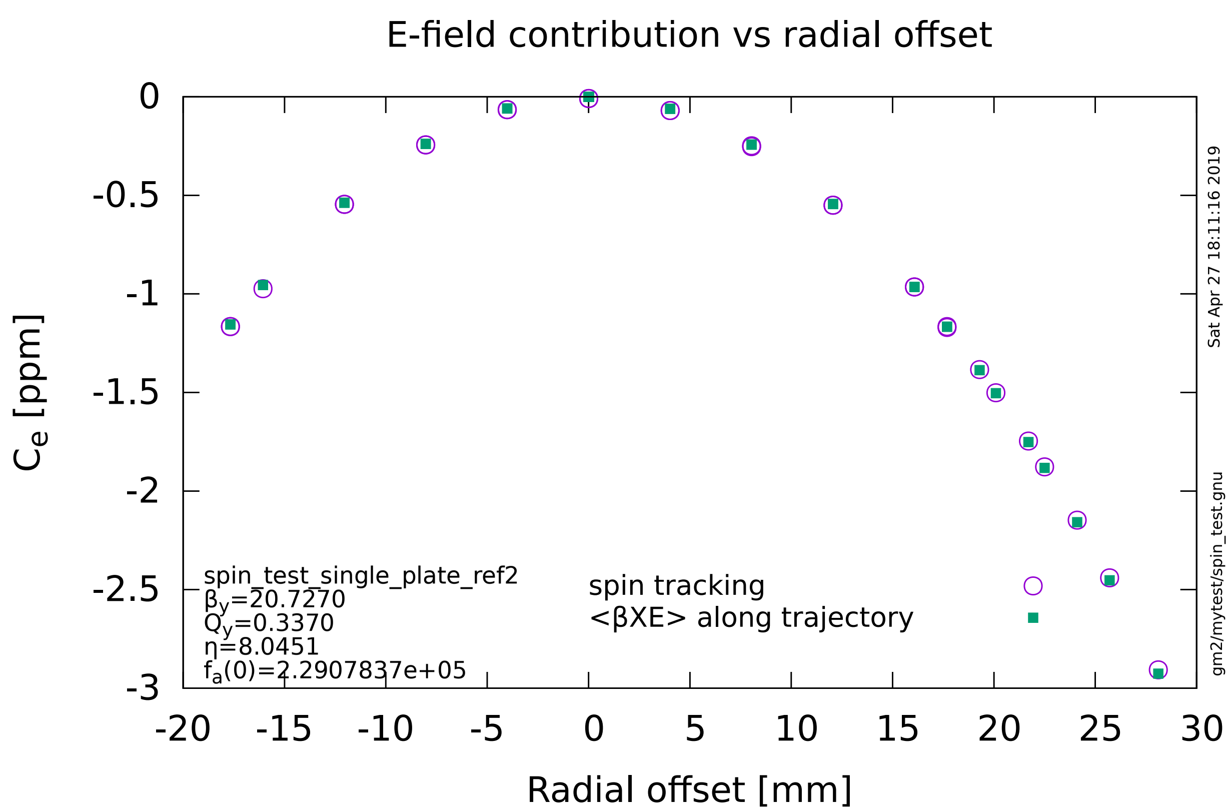
First we need to establish equivalence of spin tracking and integration

E-field
Spin tracking



Trajectories initiated with $x = \eta\delta, x' = y = y' = 0$

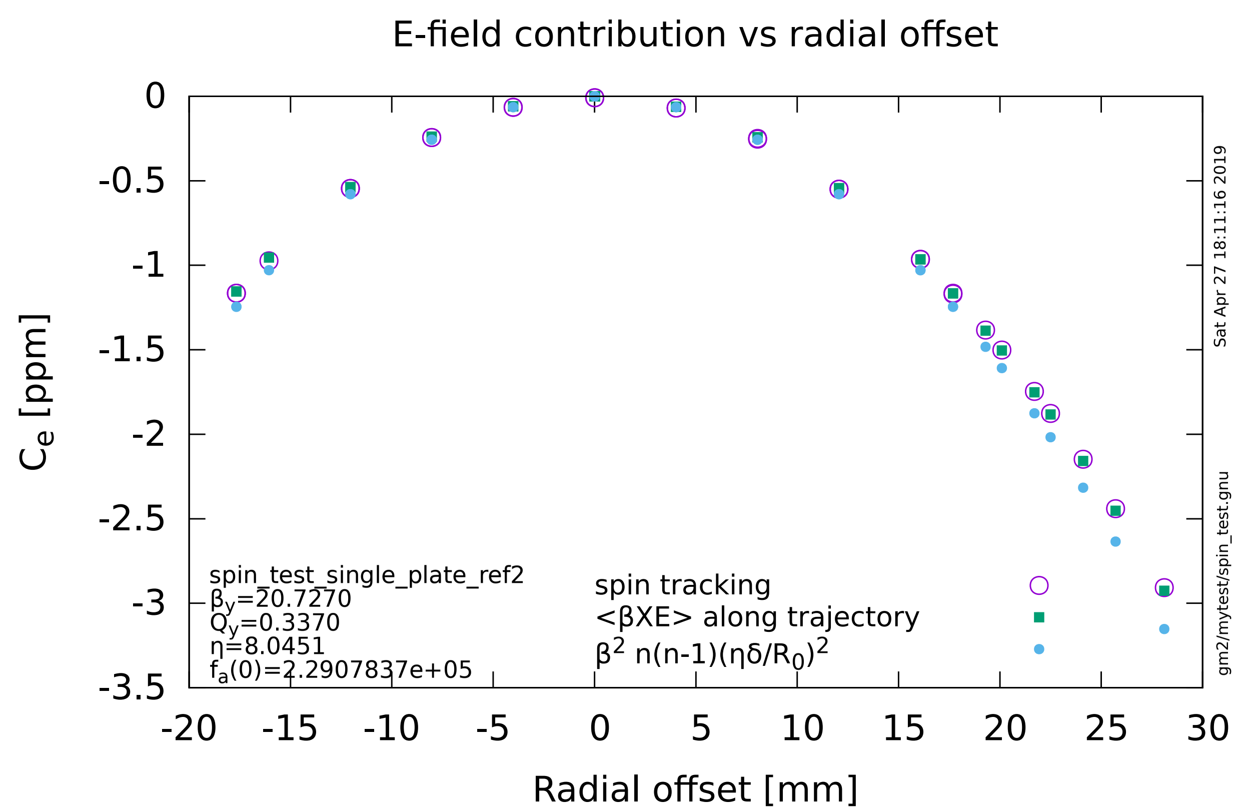
Spin tracking
Integration $\vec{\beta} \times \vec{E}$



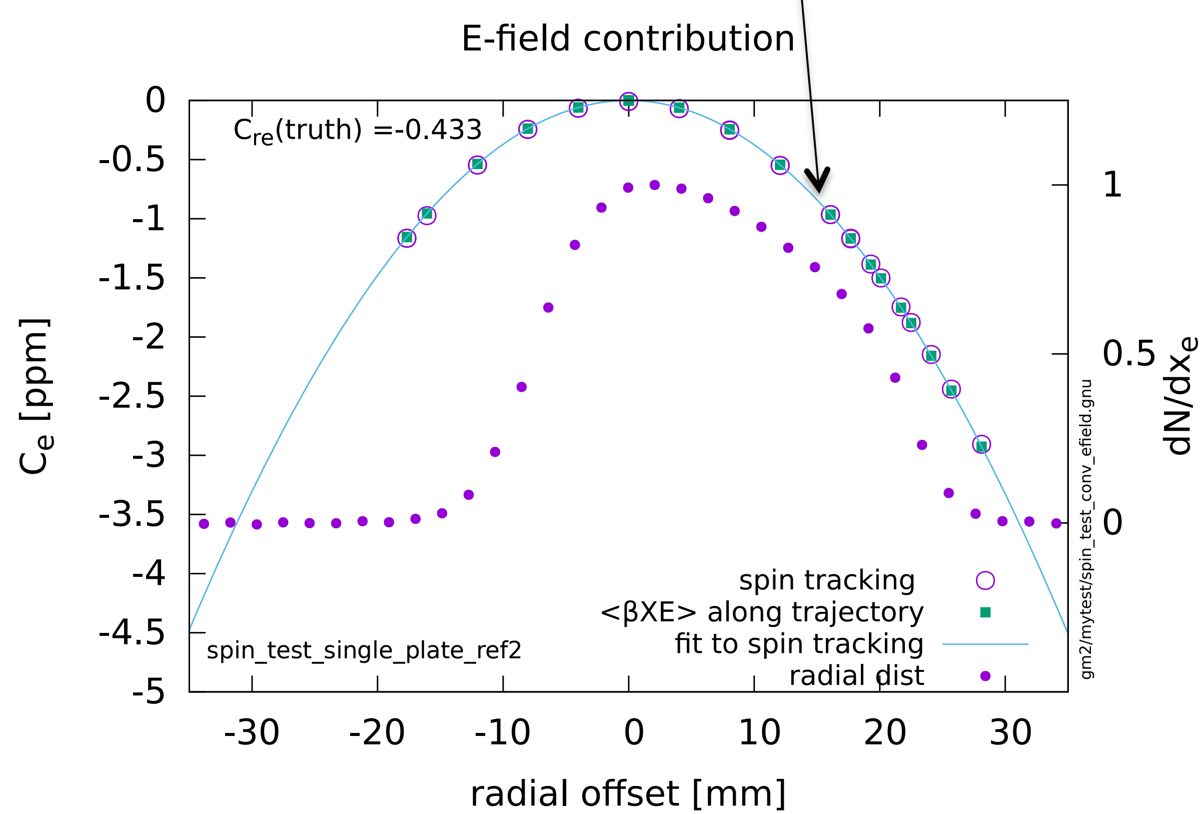
Spin tracking

Integration $\vec{\beta} \times \vec{E}$

Linear method $C_e = -2\beta^2 n(1-n)\frac{x_e^2}{R_0^2}$



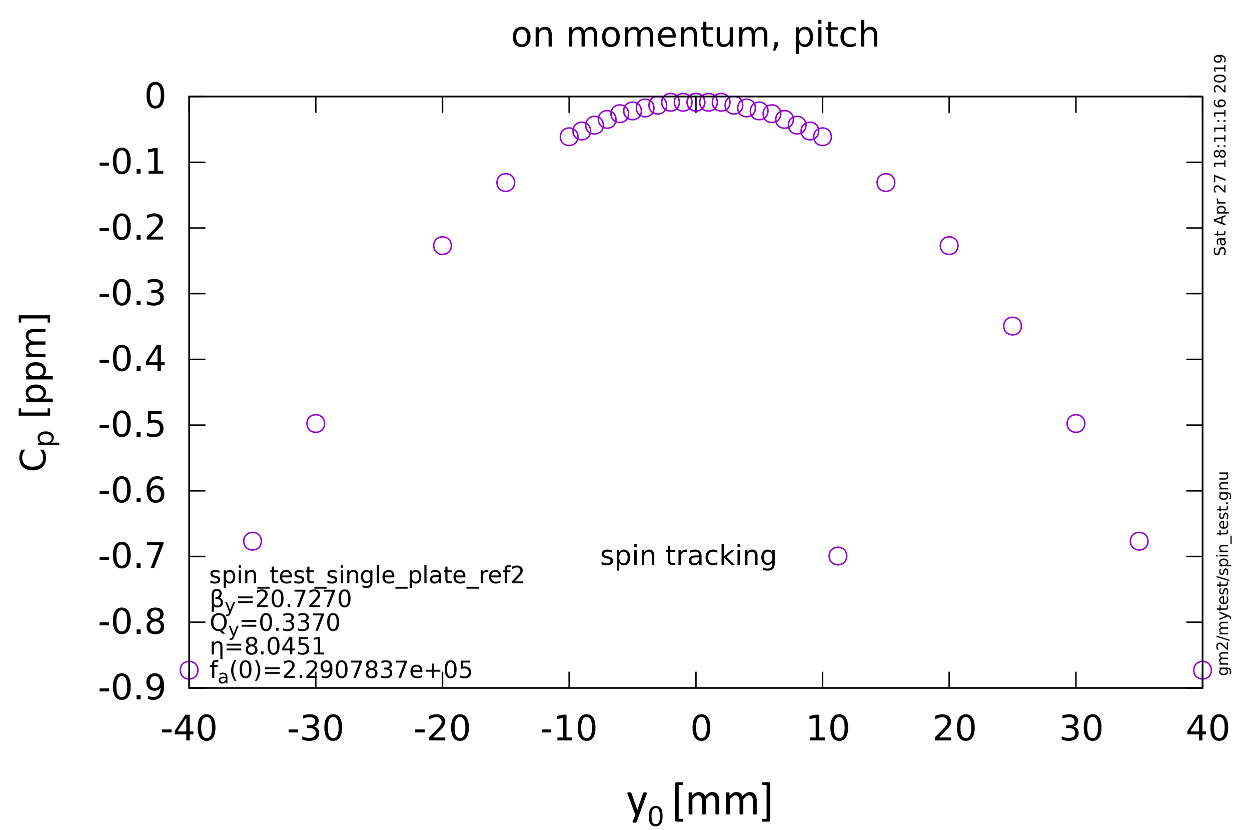
Given the measured distribution of equilibrium radii
We determine C_e by convolution of with $C_e(x_e)$
(Note the x_e is uniquely defined for each muon trajectory)



$C_e(x)$ is computed by tracking through model that includes our best estimate of quad alignment, nonlinearity, B-field errors, etc.

Pitch

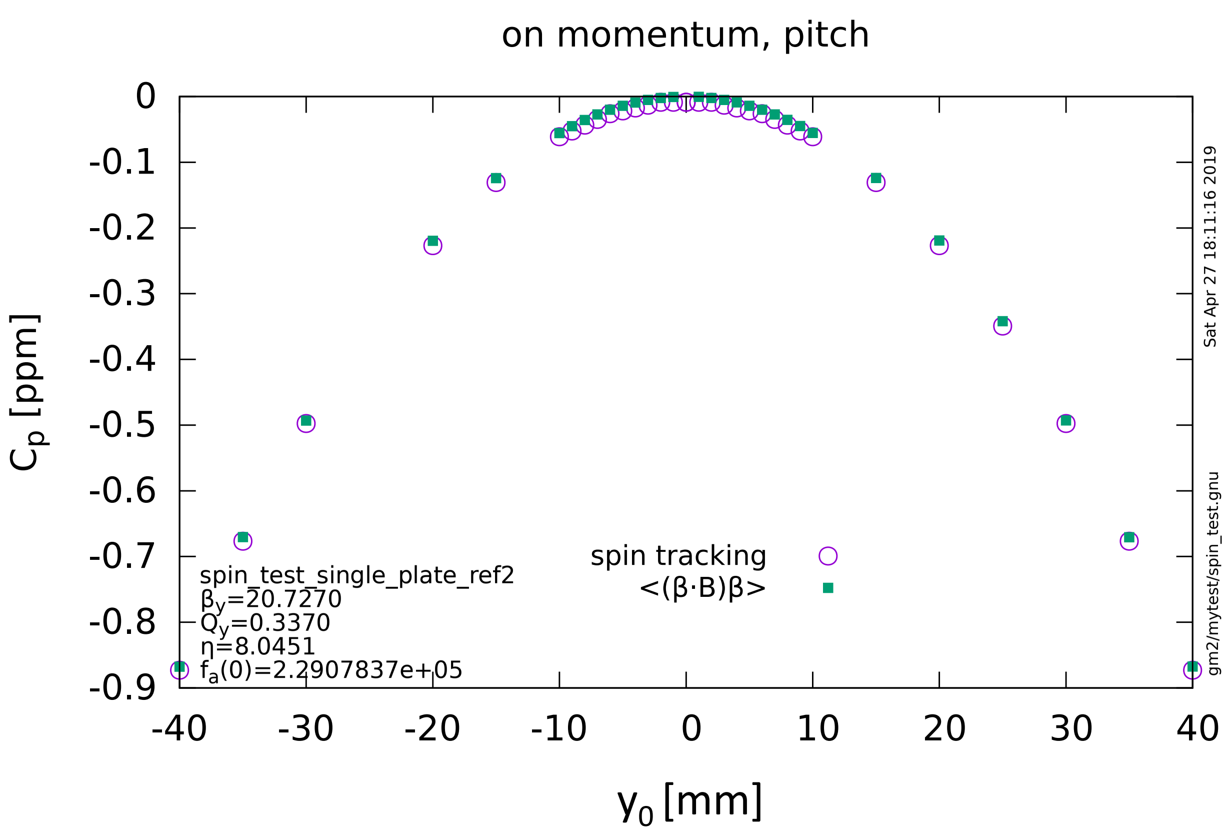
Spin tracking using BMT



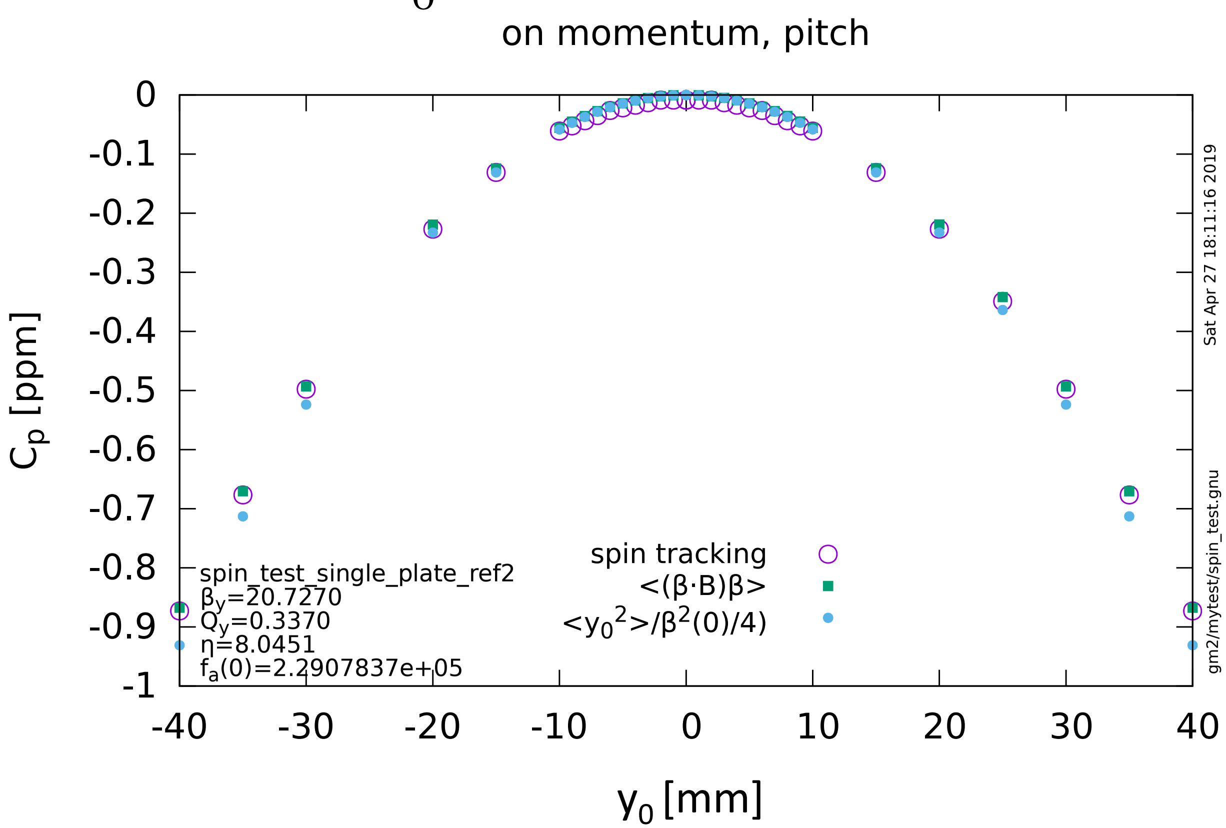
Pitch correction vs vertical amplitude

Trajectory initiated with finite $y, \quad y' = x = x' = \delta = 0$

Spin tracking
Integration $\vec{\beta} \cdot \vec{B}$



- Spin tracking
- Integration $\vec{\beta} \cdot \vec{B}$
- Linear method $C_p = -n \frac{\langle y^2 \rangle}{2R_0^2}$



Convolution?

We measure y at the decay point
The pitch correction depends on ψ_0^2

But $y = \beta\psi_0 \cos\phi$

Muons that decay with y=0 (for example) will have all possible values of ψ_0

(For muons that decay at $y=y_{max}$, all $\psi_0 = \psi_0^{max}$)

The average $\psi_0(y)$ will depend on the muon distribution

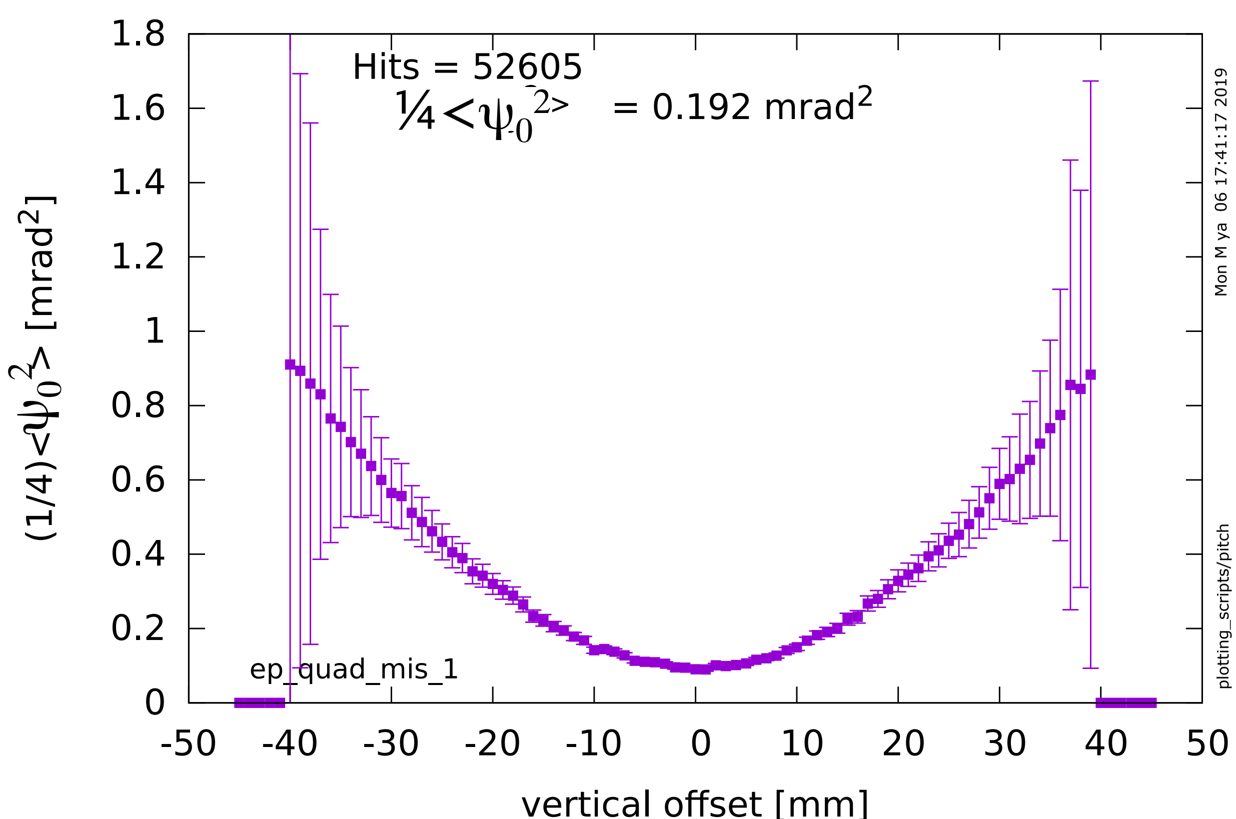
Track a distribution

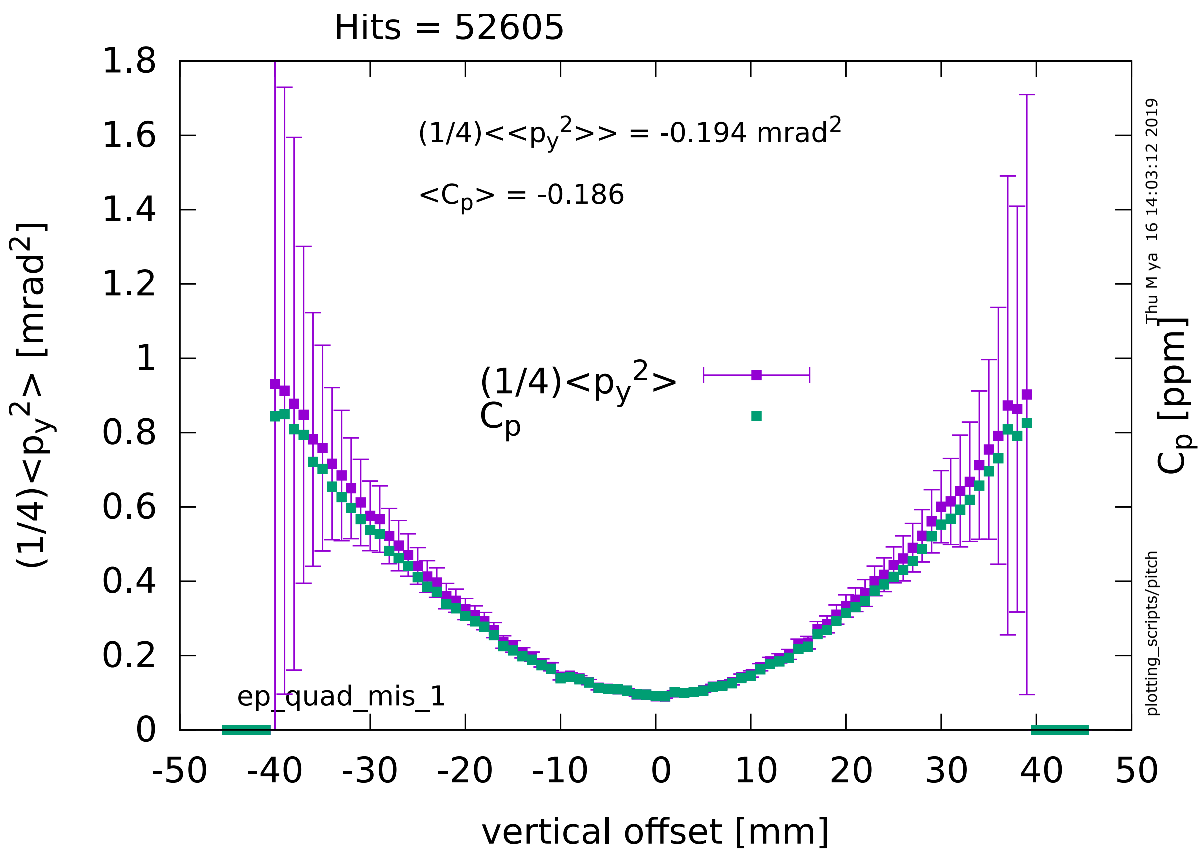
Determine a,ψ_0 from y,ψ at decay point

$$a = \frac{y^2}{\beta} + \psi^2\beta$$

$$\psi_0 = \sqrt{\frac{a}{\beta}}$$

Plot ψ_0 vs y





$p_y = \psi_0$

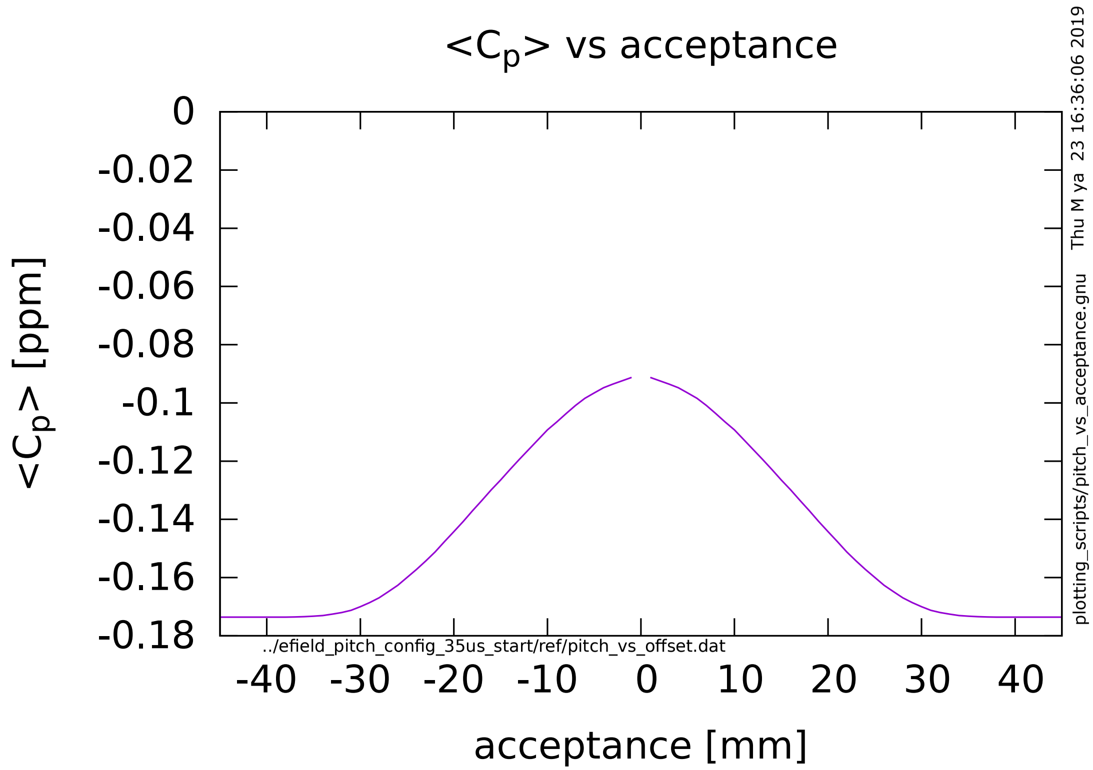
Better to convolute the measured dN/dy vs y with a curve fit to the green points?

(C_p by integration along trajectory (green points))

Pitch and acceptance

The maximum pitch angle is determined by the physical aperture (collimators)
The average pitch correction for the distribution depends on the vertical acceptance of the detector

If aperture is 45mm then

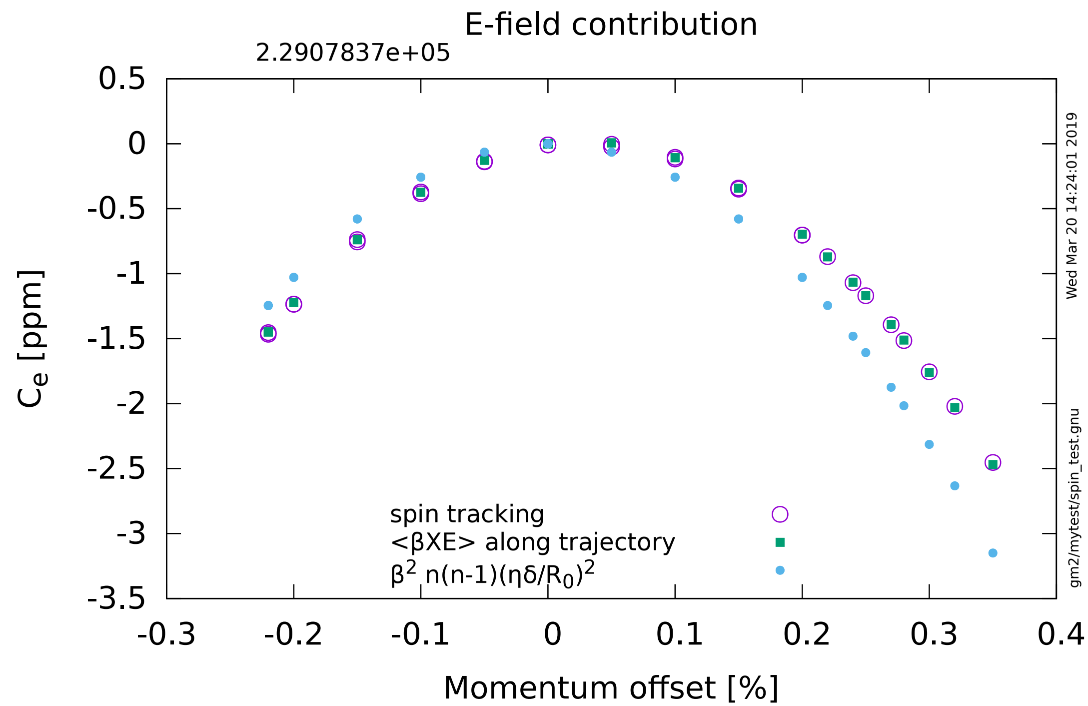


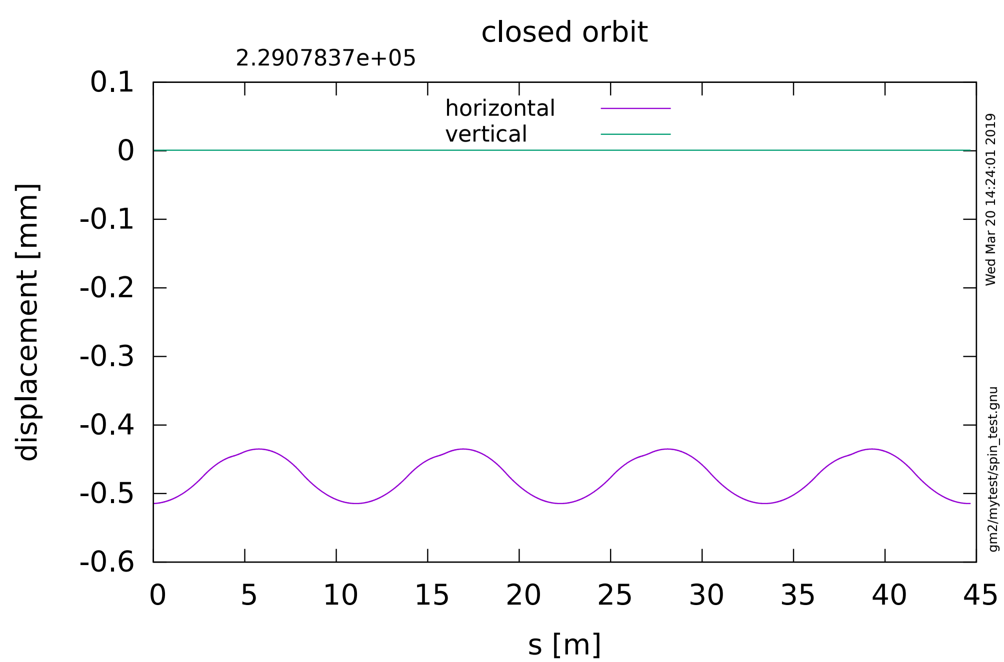
Acceptance	C_p [ppm]
± 1 mm	-0.09
± 10 mm	-0.11
± 40 mm	-0.175

Effect of misalignment, voltage errors, radial B-field

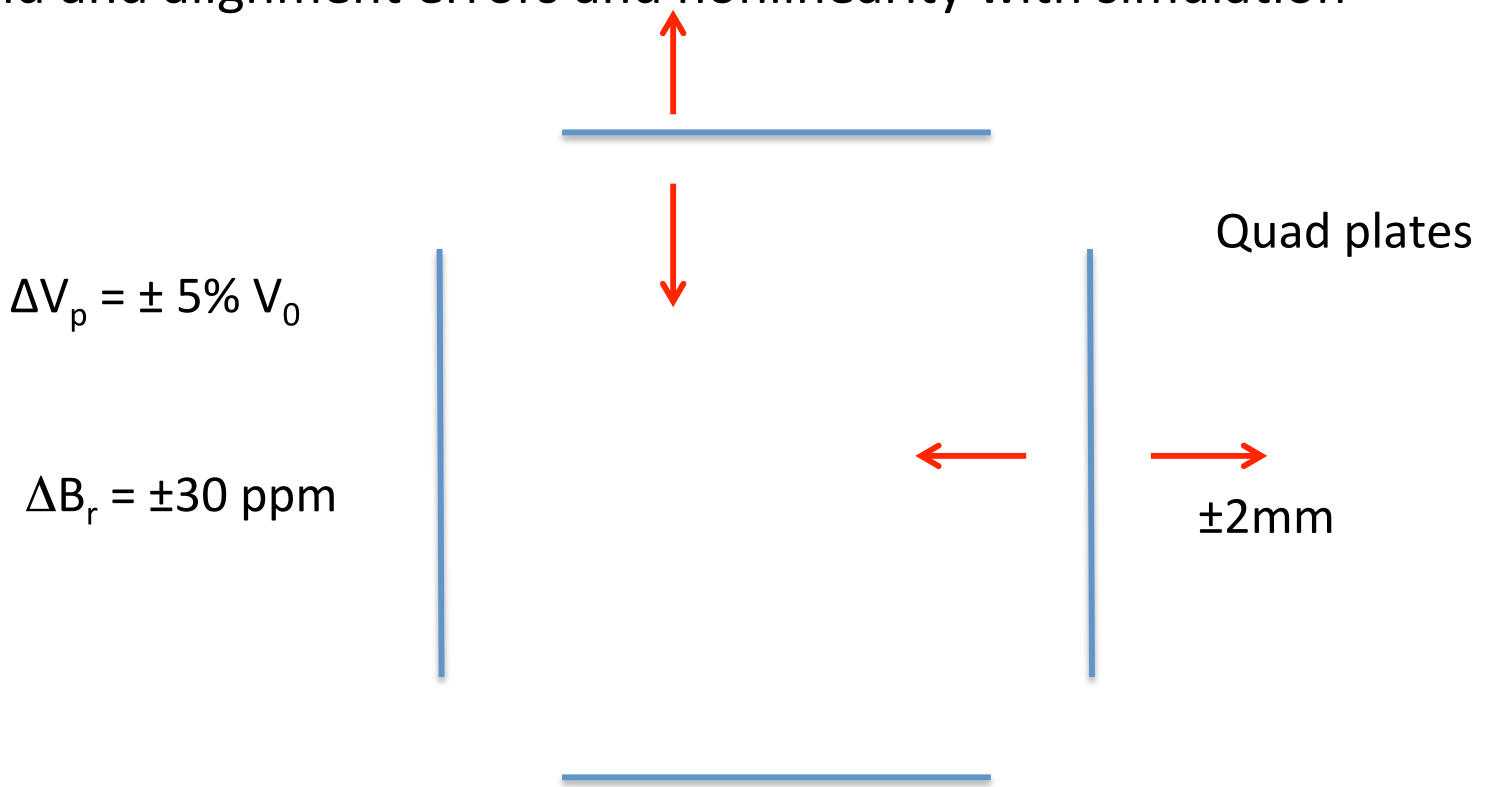
An example misalignment

Suppose all quads are displaced 4mm radially outward





Systematically explore dependence of E-field and Pitch correction on field and alignment errors and nonlinearity with simulation



Create *configurations* that span the space of possibilities

- Vary alignment of each of 2X4X4 quad plates $\Delta x = \pm 2$mm, $\Delta y = \pm 2$mm
- Vary voltage on each of 32 quad plates $\Delta V_p = \pm 5\% V_0$
  Note that both misalignment and voltage errors enhance nonlinearities
- Vary radial magnetic field $\Delta B_r = \pm 30$ ppm (uniformly)

The corrections depend on

- P(y) and $C_p(y)$ for pitch
- $P(x_e)$ and $C_e(x_e)$ for E-field

And all four quantities depend on the *configuration*.

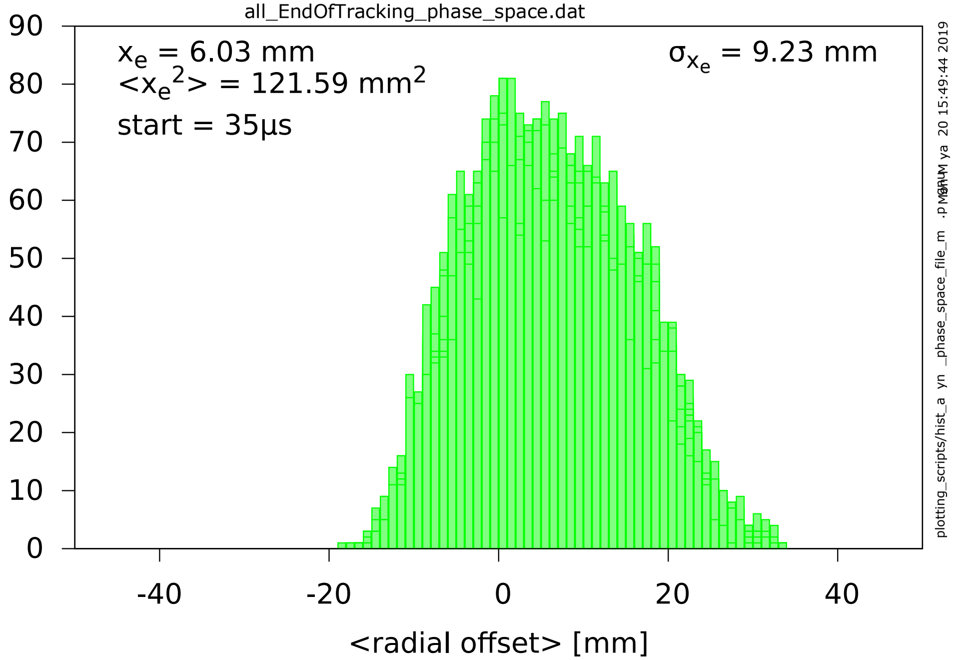
For each *configuration*

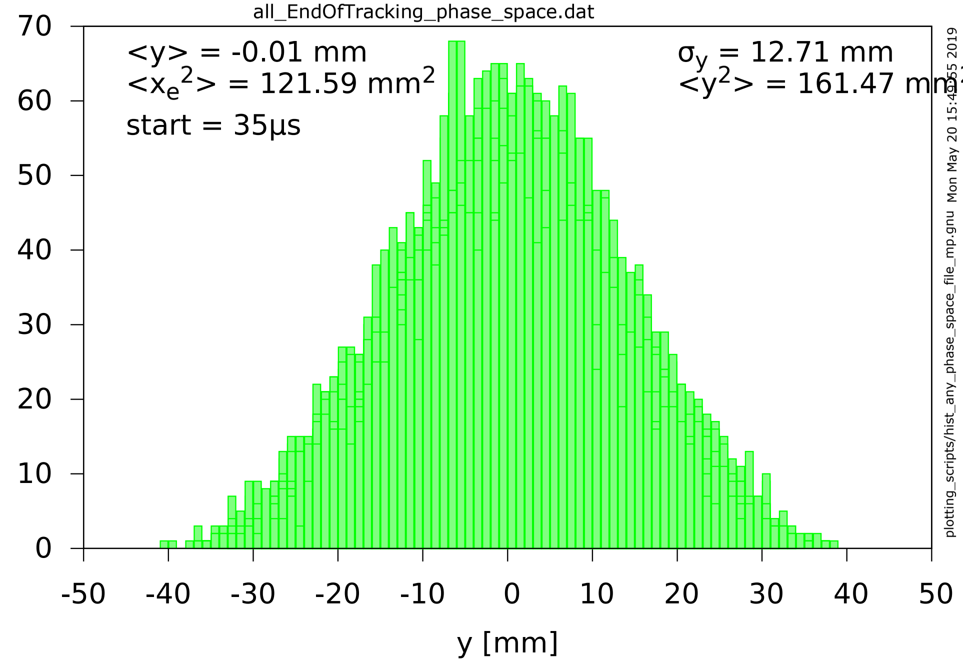
- Track through injection channel into ring to generate ‘realistic’ distribution
- Kicker B=200 G
- Quads at 18.3 kV
- Quad scrape 13.1kV -> 18.3kV
- Muon decay is turned on
- Compute C_p and C_e (by integration along trajectory) for each muon
- Include all muons that decay at t > 35 us

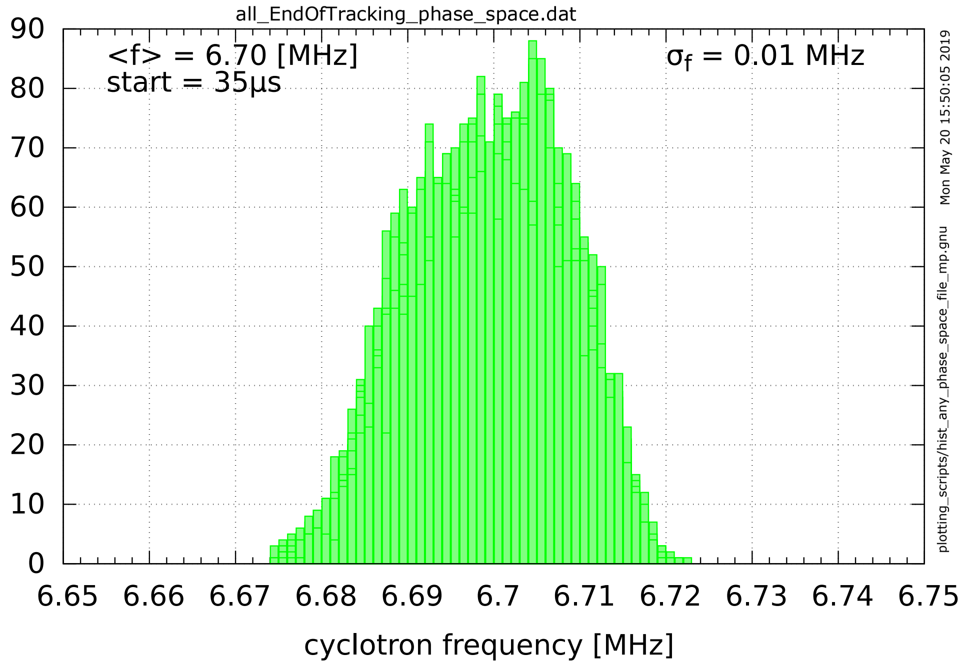
Reference

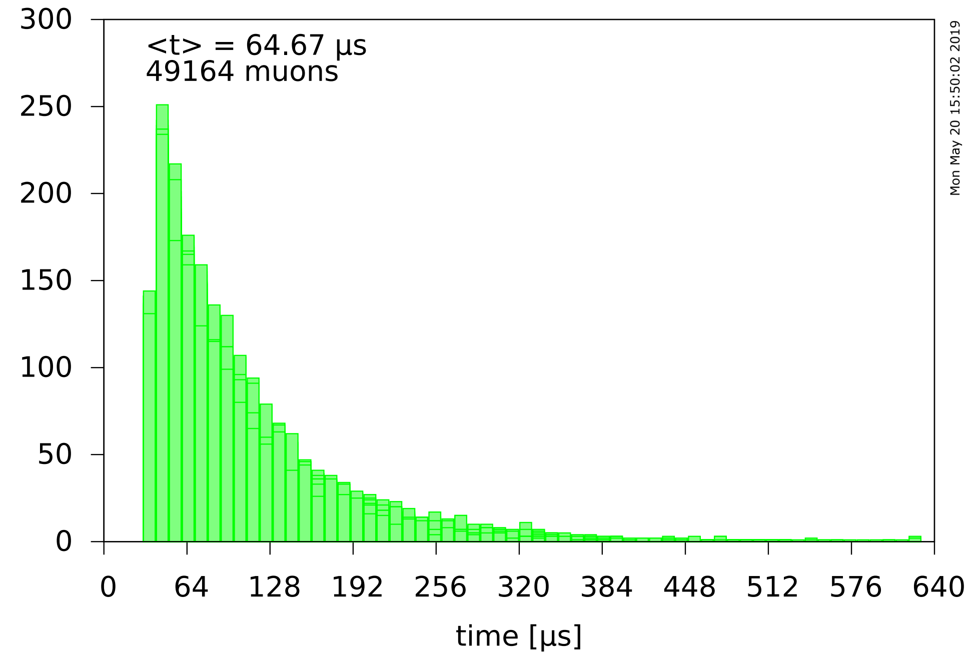
- Nominal quad voltage
- $B_r = 0$
- Quad plates aligned according to survey

Reference configuration phase space distribution



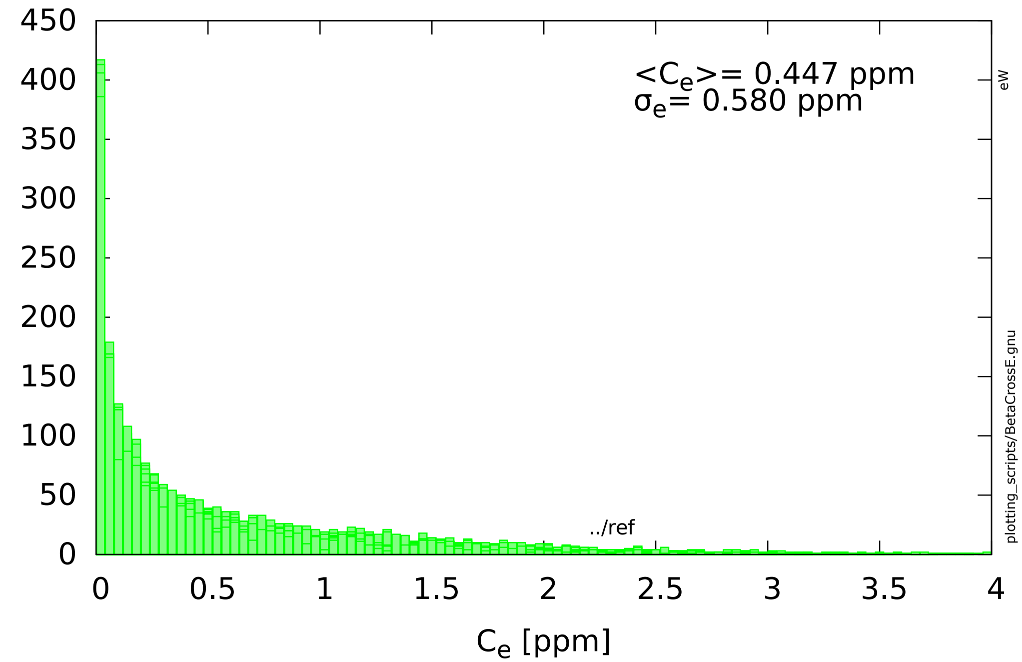






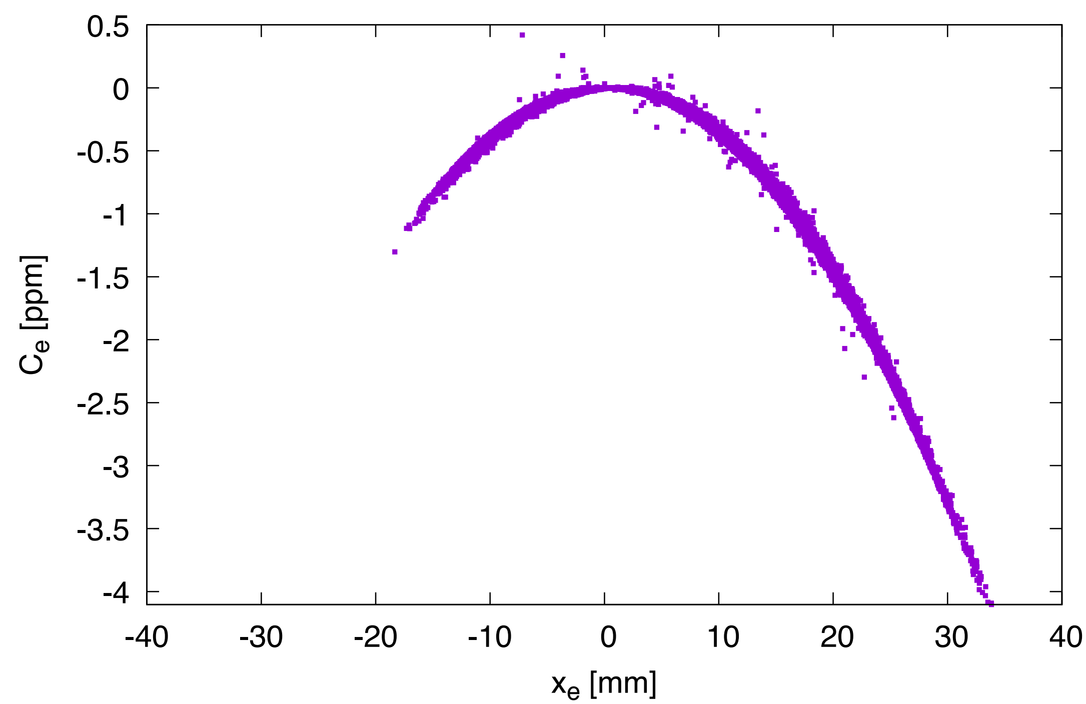
Reference configuration distribution of

E-field contributions



Reference configuration

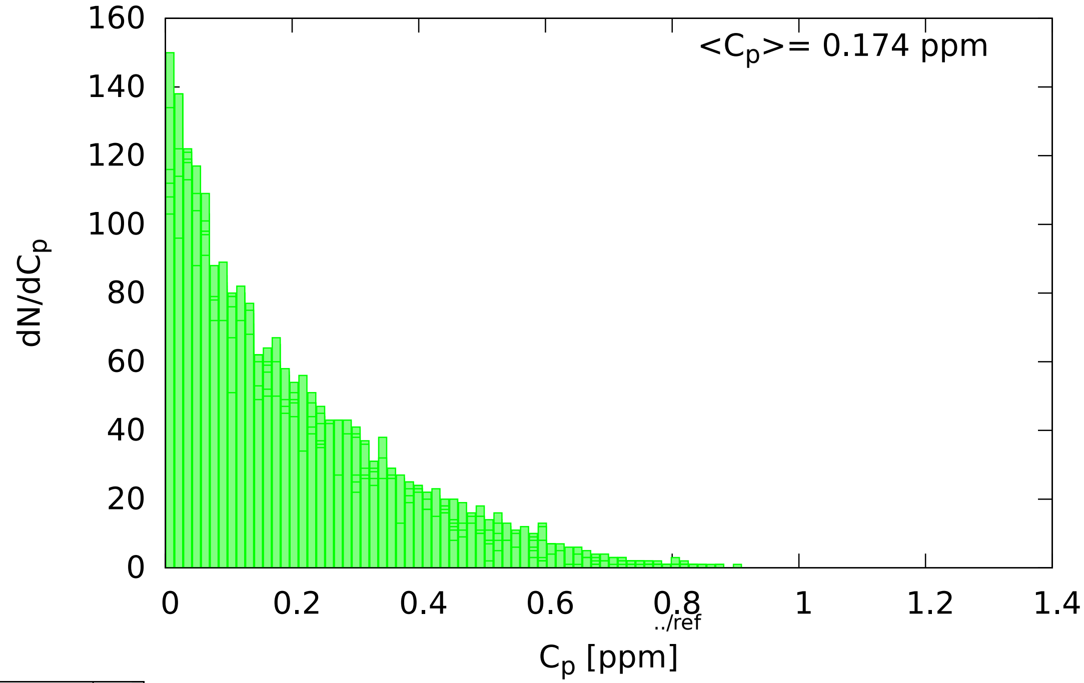
C_e vs equilibrium radius

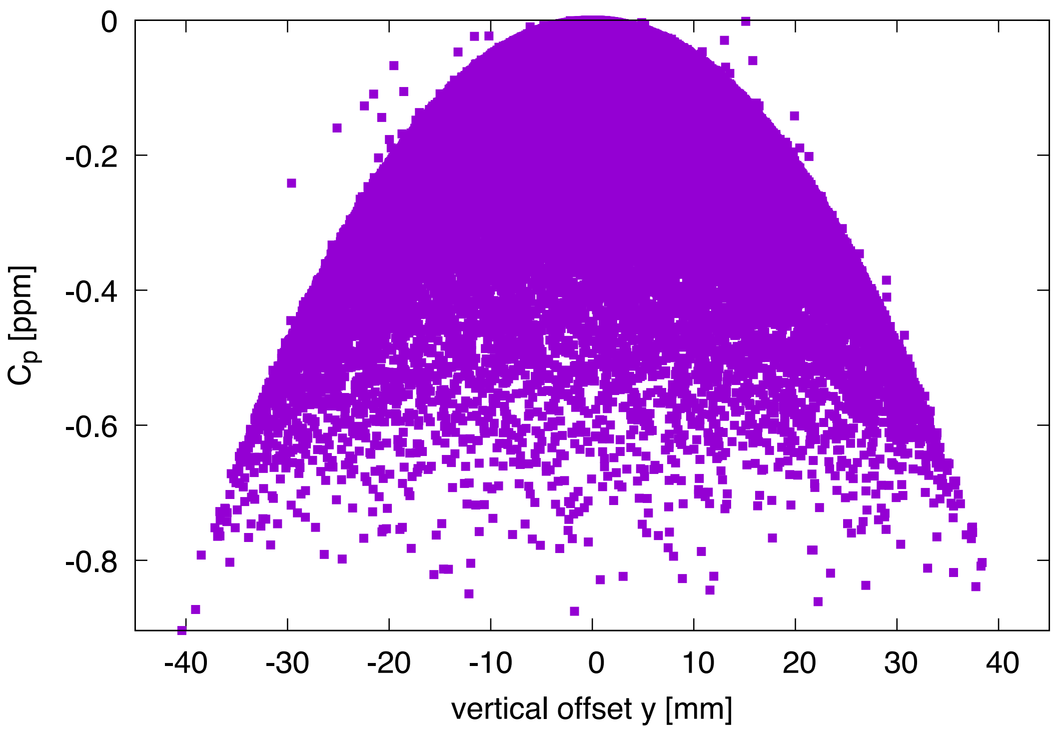


Dependence on betatron amplitude is evidently very small

Reference configuration

Distribution of C_p





C_p vs vertical offset at decay point for reference configuration

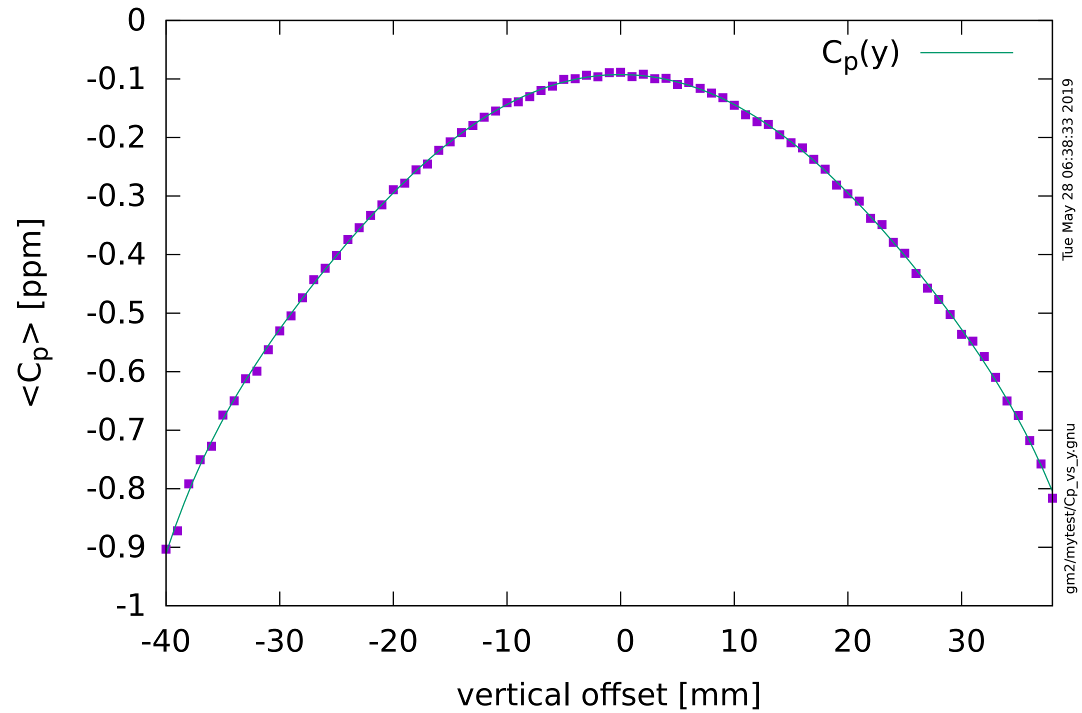
Convolution functions

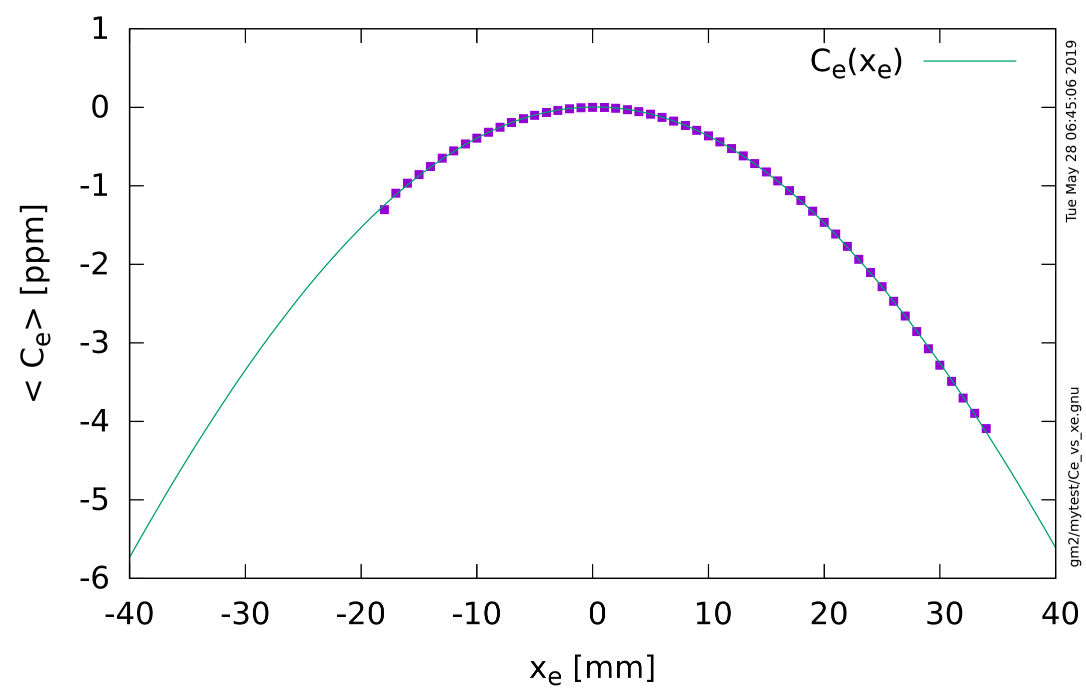
$\langle C_p(y)\rangle$ and $\langle C_e(x_e)\rangle$

generated for reference configuration

$$C_p(\text{convolve}) = \int C_p(y)P(y)dy$$

$$C_e(\text{convolve}) = \int C_e(x_e)P(x_e)dx_e$$





Characterization of a simulated distribution

- Propagate a 'realistic' distribution through the injection channel (through backleg iron and inflector and into storage ring)
  - Assume longitudinal distribution is as measured in Spring 18
  - Kicker pulse shape – as measured
- Track around the ring – (quadrupole field as per Opera 3D map) until muon decays
- For each muon that decays at t > 35μs record:
  - Momentum
  - End phase space coordinates, decay time,
    - closed orbit

$$x_e = \frac{1}{N}\sum_{i=1}^{N} x_i$$

- Fast rotation frequency

$$f_{FR} = \frac{N}{T}$$

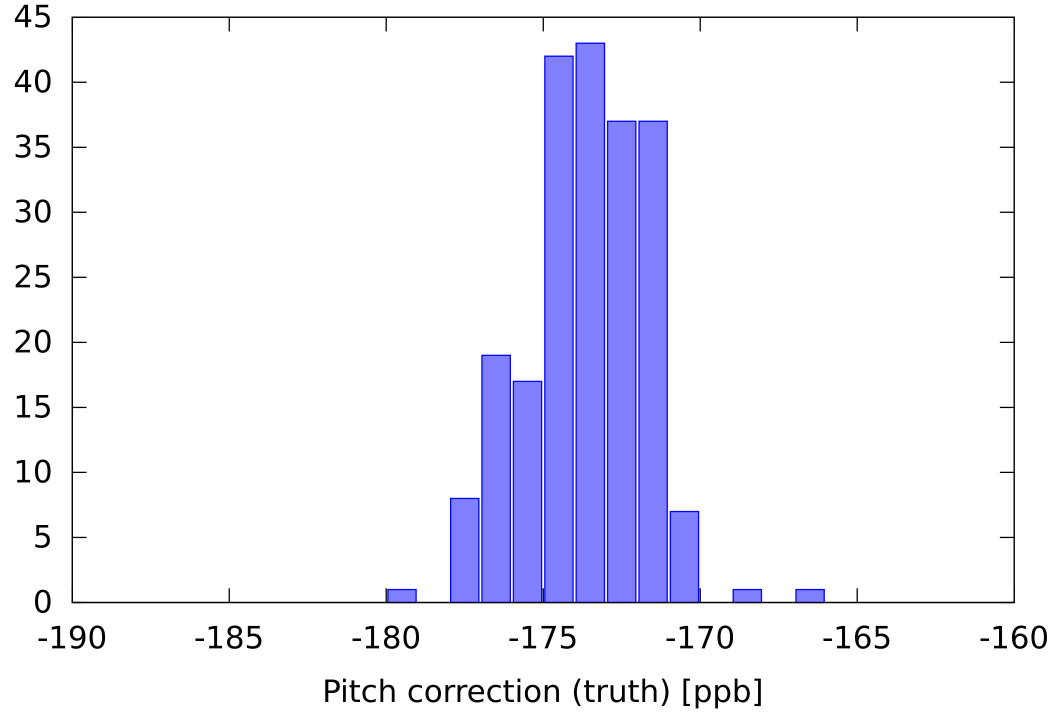
- E-field contribution

$$\vec{C}_e(T) \sim 2\frac{\Delta p}{p}\frac{1}{T}\int_0^T \frac{\vec{\beta}\times\vec{E}}{Bc}dt$$

- Pitch contribution

$$C_p(T) = \frac{1}{T}\int^T (\tilde{\beta}\cdot\mathbf{B})\tilde{\beta}dt$$

Pitch contribution to ω_a for each of 220 configurations



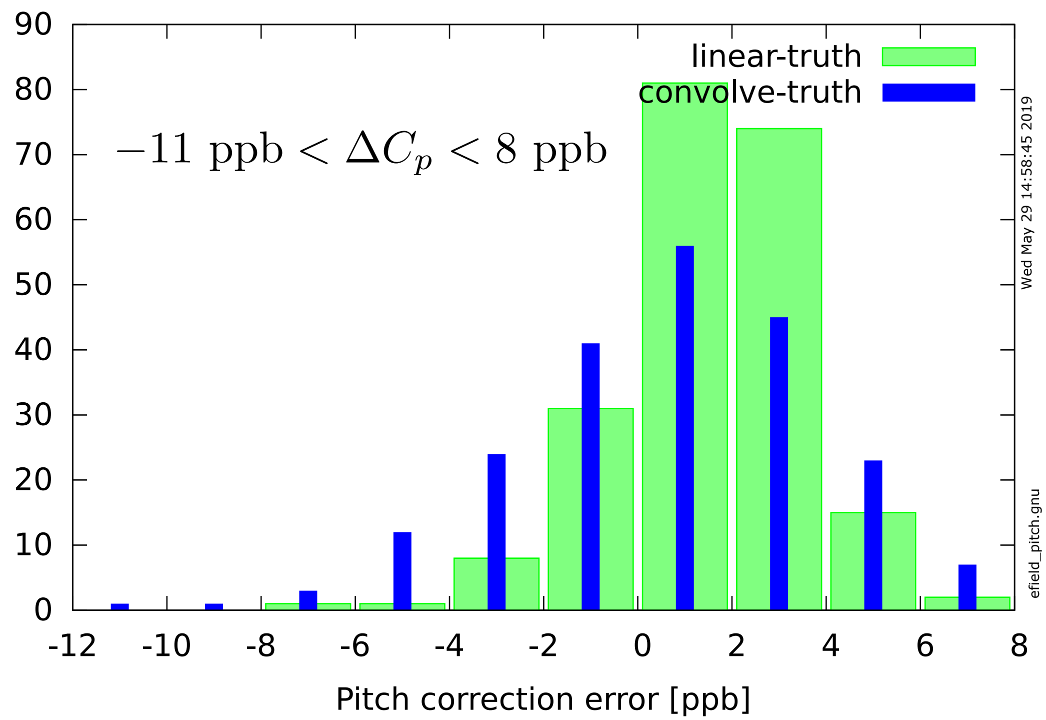
How well do we measure ?

$$C_p(\text{convolve}) - C_p(\text{truth})$$

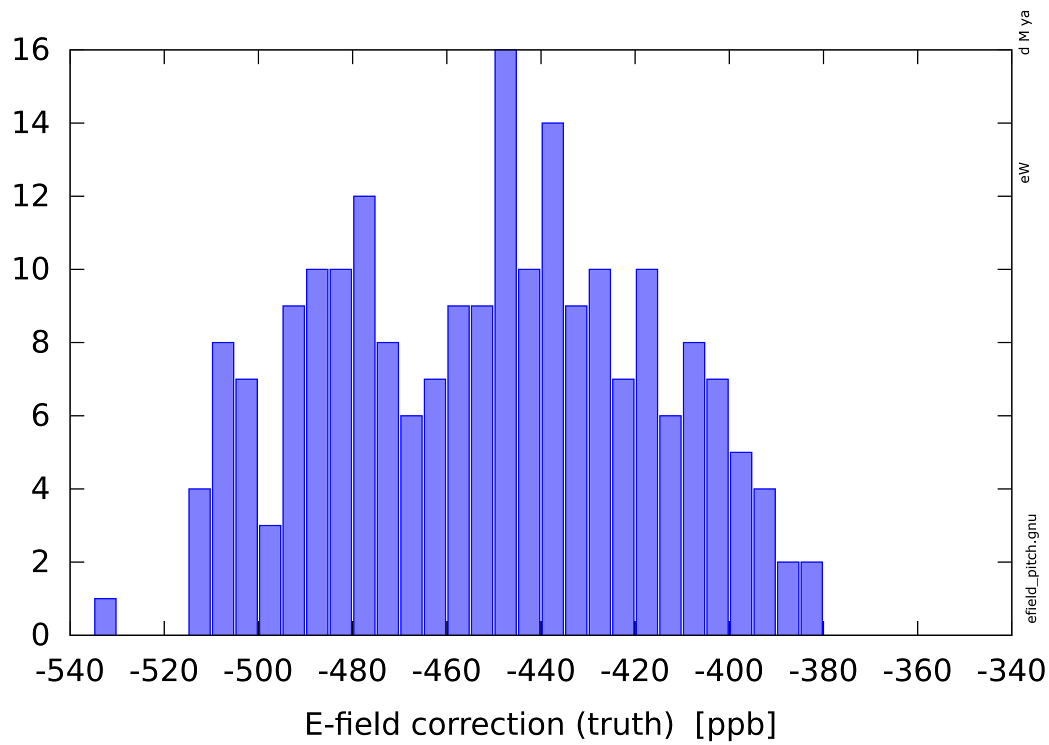
$$C_p(\text{linear}) - C_p(\text{truth})$$

$$C_p(\text{linear}) = -\frac{n\langle y^2\rangle}{2R_0^2}$$

Assuming field index n is measured

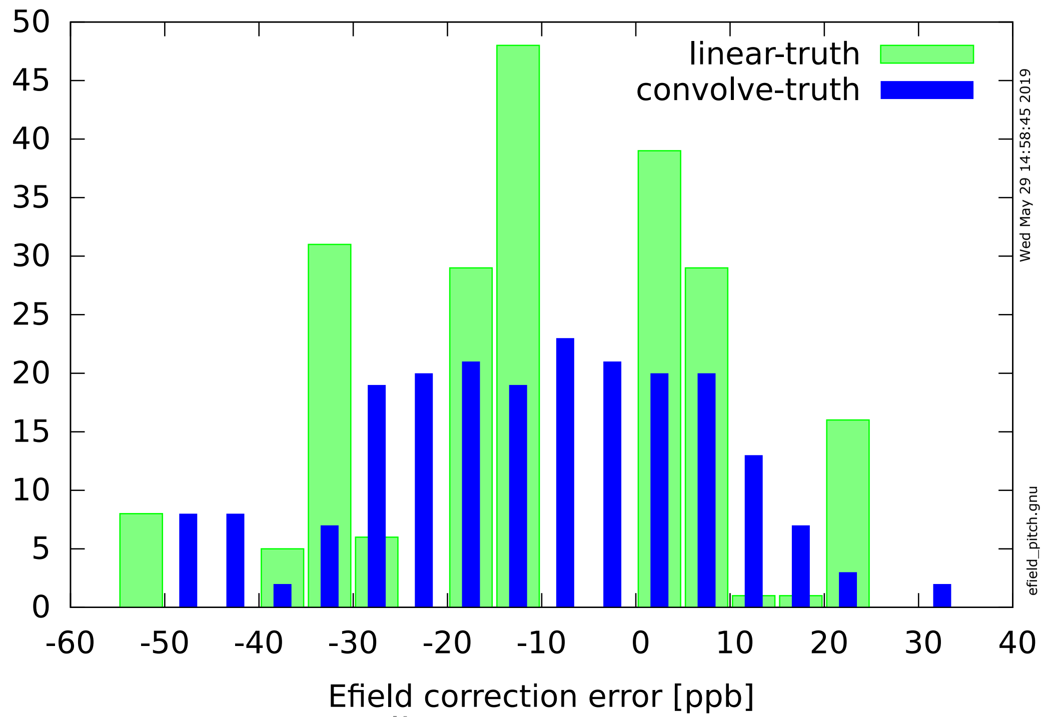


E-field contribution to ω_a for each of 220 configurations



C_e(convolve) – C_e(truth)

C_e (linear) - C_e(truth)



$$C_e(\text{linear}) = -2\beta^2 n(1-n)\frac{\langle x_e^2\rangle}{R_0^2}$$

Assuming field index is measured

$$-50 \text{ ppb} < \Delta C_e < 35 \text{ ppb}$$

Caveats

- We assume ±2 mm misalignment of quad plates
  - Survey and analysis indicates better than ±1 mm (*Manolis, GM2 Doc 16970-v2*)
- And ±5% (1.8 kV) voltage error − *Jason's very conservative estimate*
- Of the $\frac{(2^9)^4}{4!}$ possible configurations, we have evaluated 220.

The 220 configurations considered to date are those with
Δ plate(i) same for all quads - worst case?

END

Next step

Generate a ‘complete’ set of configurations based on measured uncertainties and evaluate errors in determining C_e and C_p

To set conservative bounds on effects of field errors, and misalignment

E-field contribution along any trajectory:

$$\vec{C}_e(T) \sim 2\frac{\Delta p}{p}\frac{1}{T}\int_0^T \frac{\vec{\beta} \times \vec{E}}{Bc}dt$$

To compute correction in simulation, integrate $\langle\vec{\beta} \times \vec{E}\rangle$ along the trajectory of the muon

=> E-field correction as a function of time, $C_e(t)$

Next step. Since we have C_e and x_e for every muon we can compare

Exact

$$\langle C_e \rangle = 2\langle \frac{\Delta p}{p} \frac{1}{T} \int_0^T \frac{\vec{\beta} \times \vec{E}}{Bc} dt \rangle$$

= 0.298 ppm

to

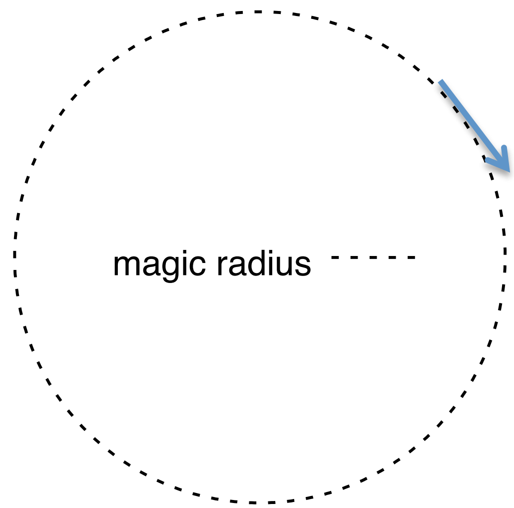
Linear approximation

$$2n(n-1)\beta^2 \langle x_e^2 \rangle / R_0^2$$

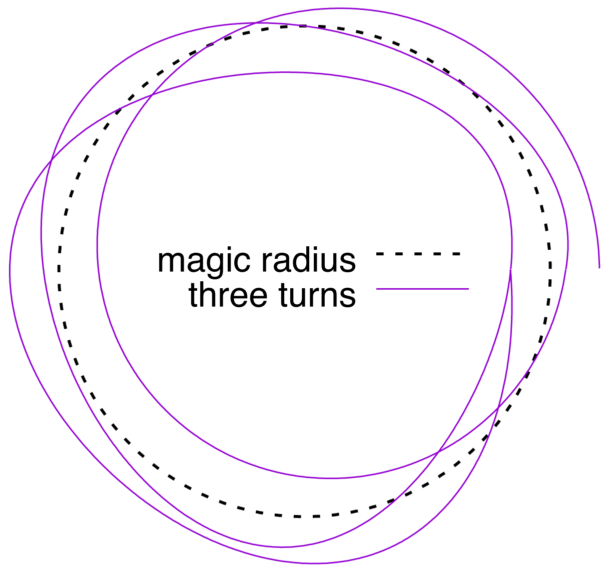
= 0.323 ppm

Conclusion:

If we reconstruct the distribution of equilbrium radii *perfectly*, there is an 8% discrepancy (25 ppb)



The E-field along the trajectory at the magic radius (momentum = p_0) is zero.



But what about the muon with momentum p_0 that oscillates about the magic radius with some betatron amplitude x_β ?

Or the muon with momentum $p_0 + \Delta p$ and betatron amplitude x_β?

$$x = \eta\delta + x_\beta \qquad \delta = \Delta p/p_0$$

E-field correction

$$C_e = \left(1 - \frac{1}{a_\mu}\frac{m^2}{p^2}\right)\frac{\beta E_r}{cB} \sim \left(1 - \frac{m^2}{a_\mu p_0^2}(1 - 2\frac{\Delta p}{p})\right)\frac{\beta E_r}{cB} \tag{1}$$

Magic momentum $\quad m^2/p_0^2 = a_\mu$

$$x_e = \eta\delta$$

$$C_e(\delta, x_{\beta 0}) \approx 2\frac{\beta k}{cB}\left(\frac{x_e^2}{\eta} + \frac{1}{2\rho_0}(\frac{x_e^3}{\eta} + \frac{1}{2}x_{\beta 0}^2\frac{x_e}{\eta})\right)$$

If $\langle x_e\rangle = \langle\delta\rangle\eta = 0$ then correction is independent of x_β

$$\langle C_e(\delta, x_{\beta 0})\rangle = 2\frac{\beta k}{cB}\eta\langle\delta^2\rangle$$

$$\langle C_e(\delta, x_{\beta 0})\rangle = 2\frac{\beta k}{cB}\eta\langle\delta^2\rangle$$

$$k = (22.409\frac{V}{27.2} \times 10^6\mathrm{V/m}^2$$

$$\eta = 8.3\mathrm{m}$$

$$2\frac{(0.999)(22.409 \times 10^6)\frac{V}{V_0}}{(3 \times 10^8)(1.45)}8.3(0.001)^2 = 85.429 \times 10^{-8}$$

$$k_{eff} = k\frac{L_{quad}}{2\pi R_0} = k\frac{156}{360} = (0.43333)k$$

$$\eta_{eff} = \langle\eta\rangle$$

$$k(\mathrm{MV/m}^2) = \frac{22.409}{27.2}V(\mathrm{kV})$$

If $\langle x_e \rangle = \langle \delta \rangle \eta \neq 0$ then according to the Miller/Nguyen rule

To minimize the E-field correction choose p_0 so that

$$2a_\mu \langle \frac{p - p_0}{p_0} \rangle = \frac{m^2}{p_0^2} - a_\mu$$

Then

$$\langle C_e \rangle \sim 2 \left[-\eta(\langle \delta \rangle^2 - \langle \delta^2 \rangle) + \frac{1}{4\rho_0}(\langle \delta \rangle^2 \langle x_{\beta 0}^2 \rangle \right] \frac{\beta k}{cB}$$

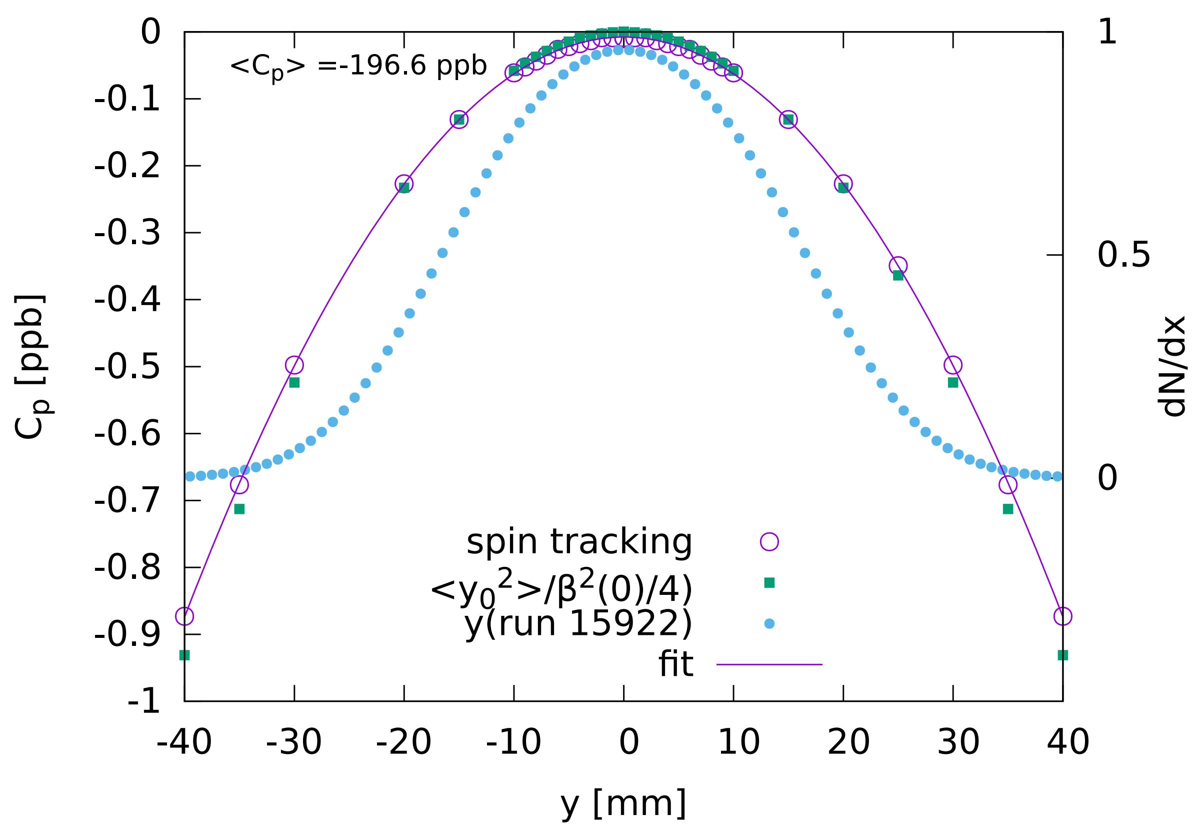
Contribution from betatron amplitude

Correction *increases* with betatron amplitude

Convolution of measured distribution with $C_p(y)$

Each point on the spin vs offset curve is the pitch correction for a trajectory with initial offset y. The pitch angle is y/β

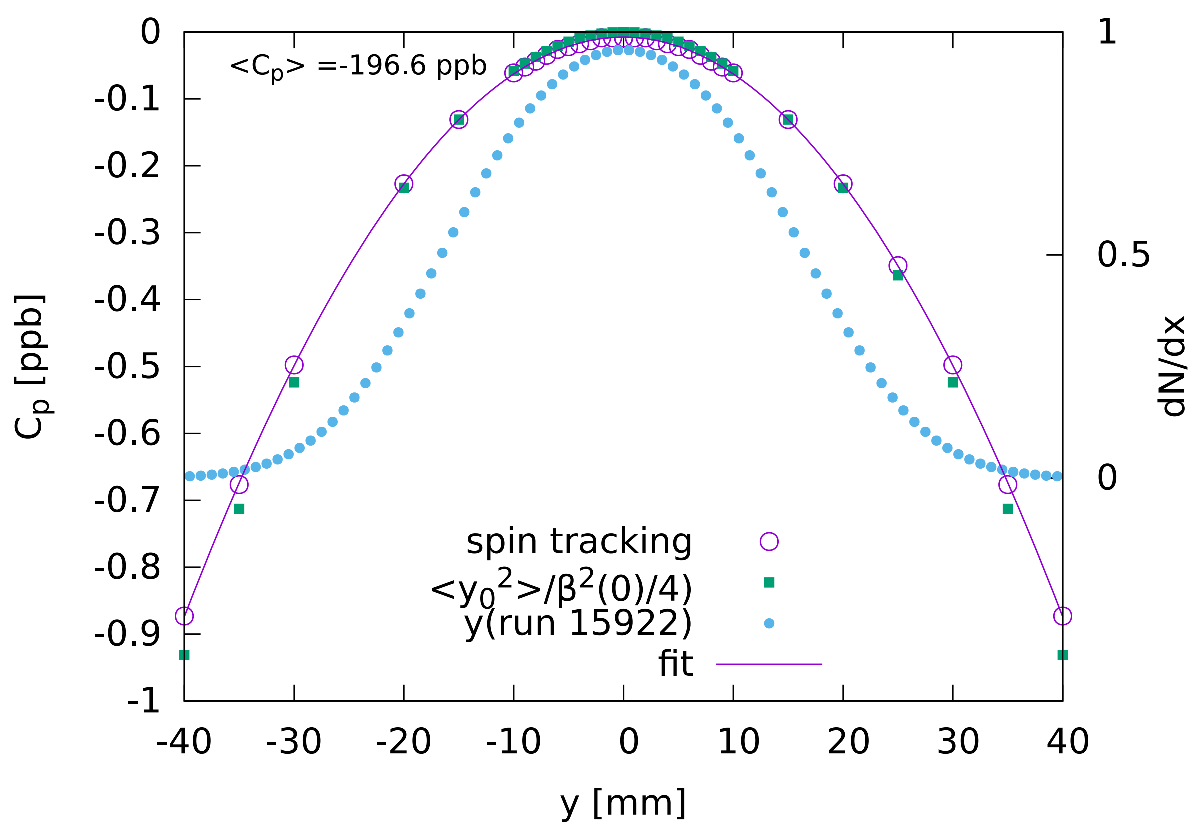
But the number of hits at offset y on the N vs y curve includes a range of pitch angles



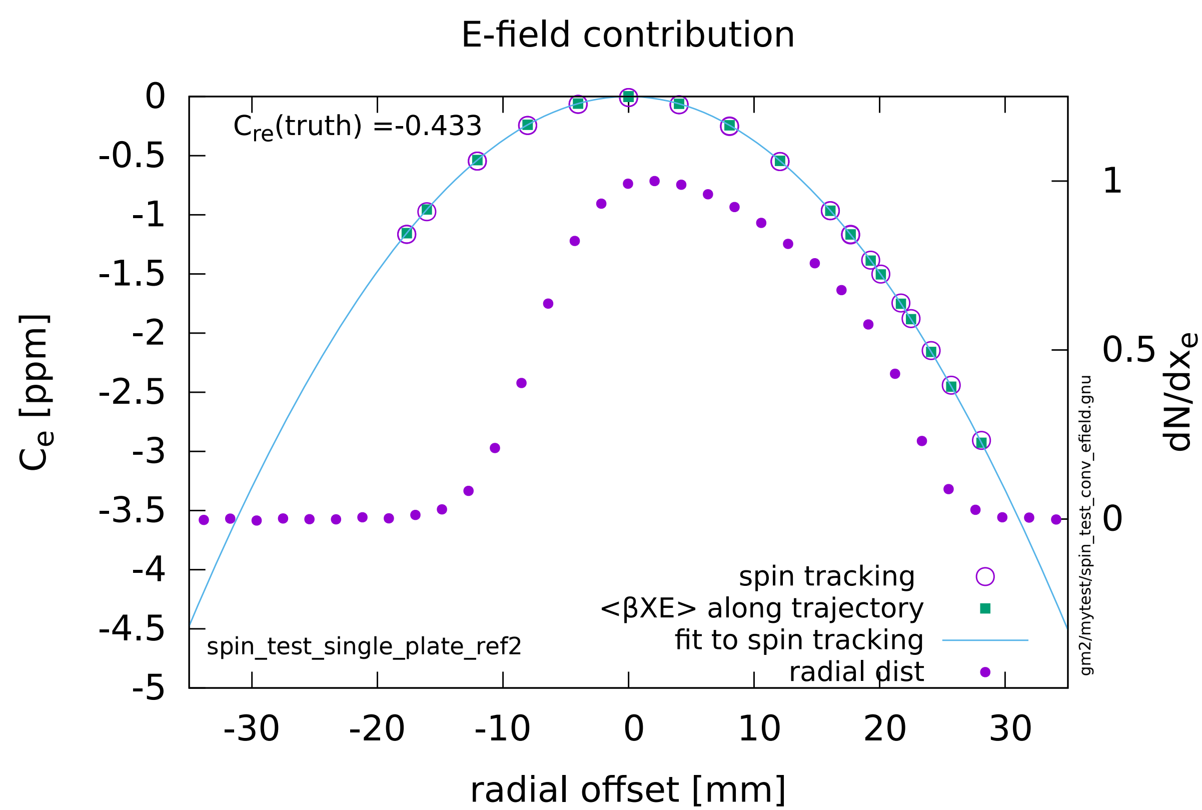
$$y' = y/\beta$$

Quad plate	Δc_e /ΔV [ppb/kV]	Δc_e /Δx/y [ppb/mm]	
Inner	8.6	0.9	
Bottom	9.4	2.9	
Outer	8.8	5.3	
Top	5.7	0.25	

Convolution of measured distribution with $C_p(y)$



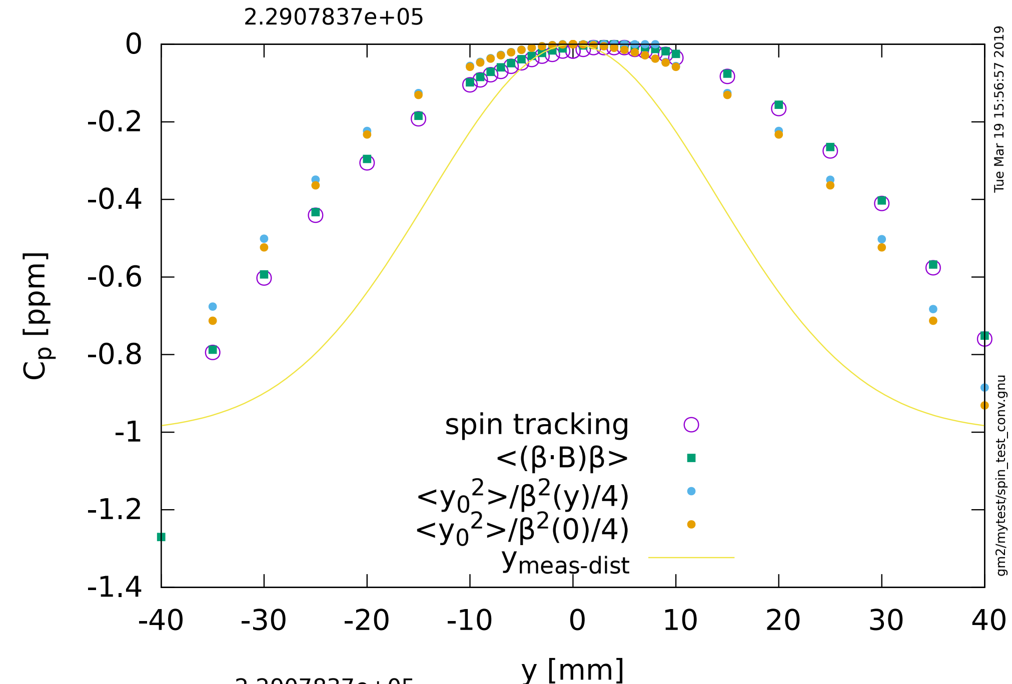
Convolution of measured distribution with $C_e(x_e)$

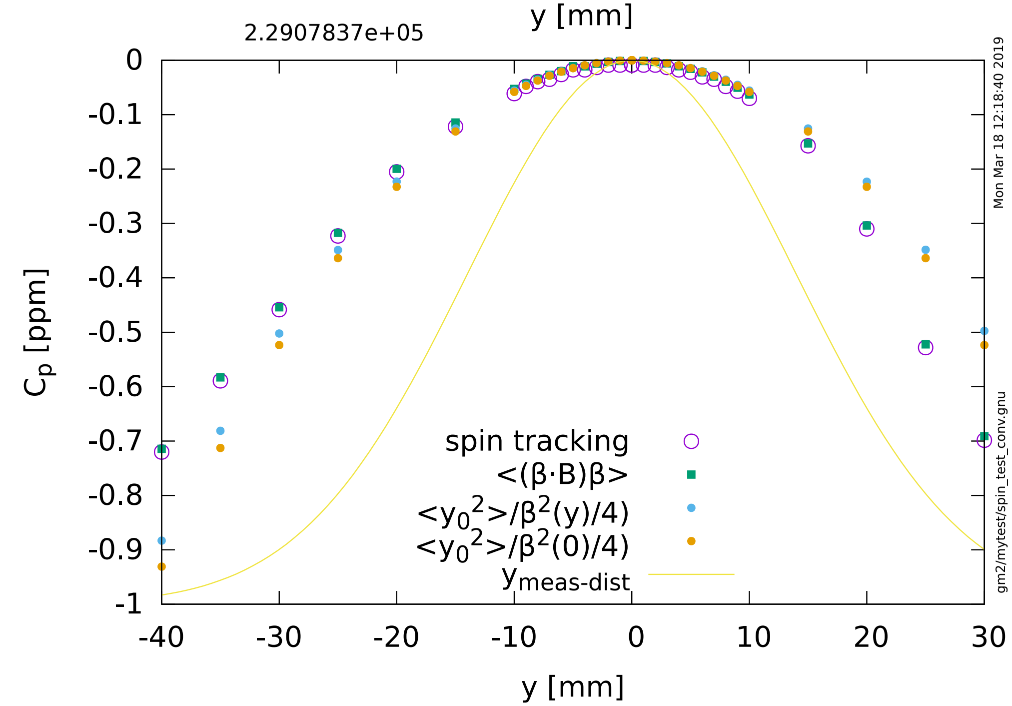


Examples of errors that impact pitch correction

B_{radial} =50 ppm

V [kV]	Inner	Bottom	Outer	Top
Q1	18.3	21.8	18.3	14.8
Q2	18.3	18.3	18.3	18.3
Q3	18.3	14.8	18.3	21.8
Q4	18.3	18.3	18.3	18.3



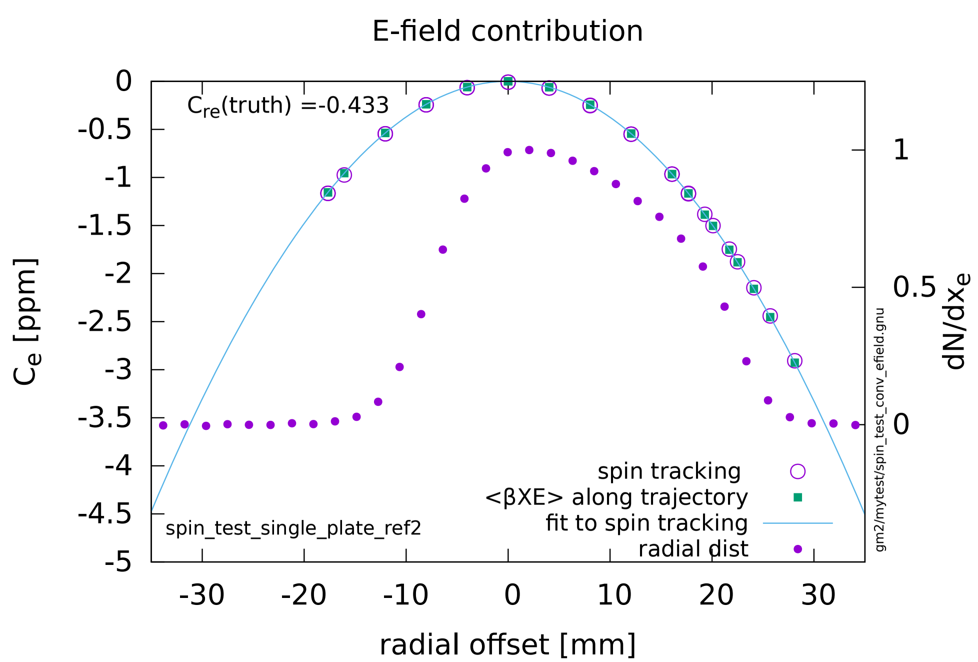


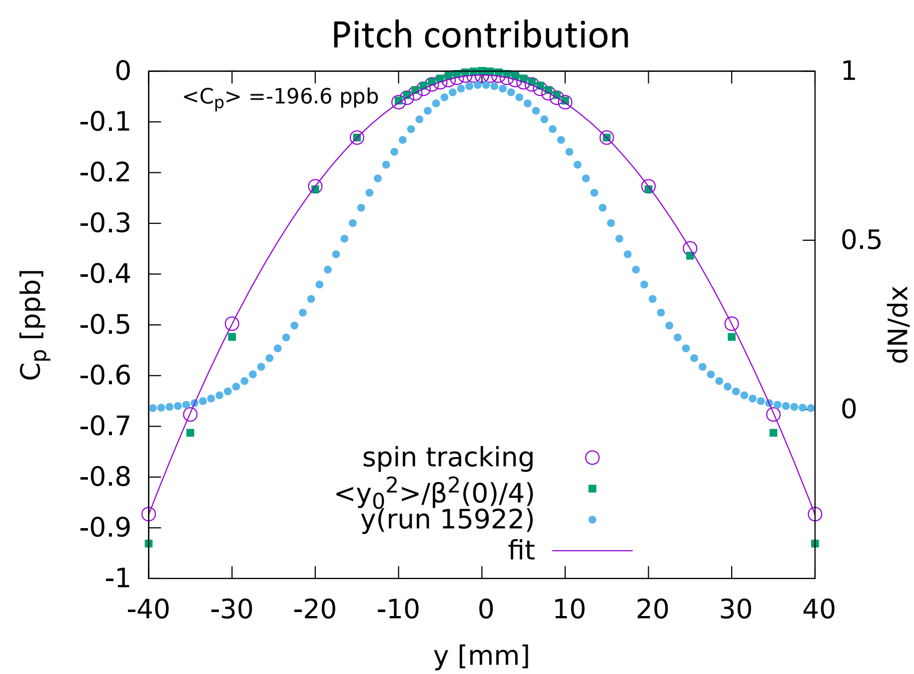
For each configuration

- Compute ‘real’ average C_p and C_e of all trajectories
- Convolute simulated distributions of x_e and y with $C_p(y)$ and $C_e(x_e)$ of the reference configuration

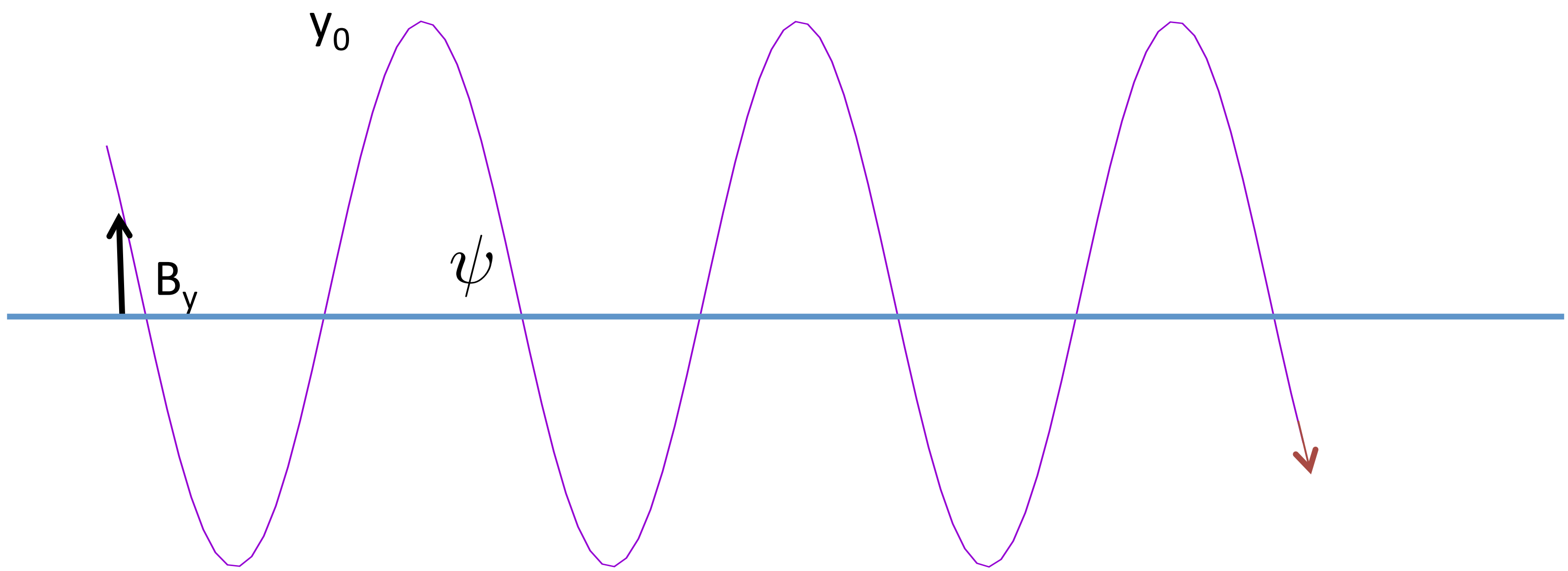
The convolution represents our ‘measurement’

Discrepancy between the ‘measurement’ and the ‘real’ C_p and C_e is the uncertainty due to alignment and field errors and nonlinearity.





Contribution of pitch to ω_a



Our measurement is sensitive to precession about the axis perpendicular to the direction of motion. The component of the magnetic field along that axis is B cos ψ. Path length

Examples of errors that impact pitch correction

B_{radial} =50 ppm

V [kV]	Inner	Bottom	Outer	Top
Q1	18.3	21.8	18.3	14.8
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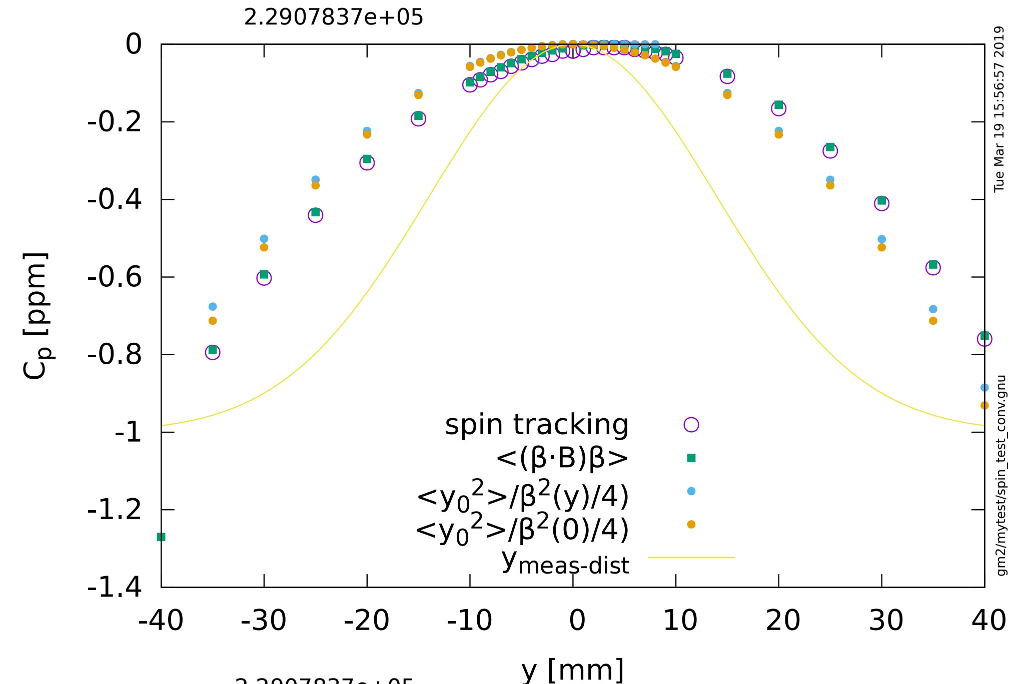
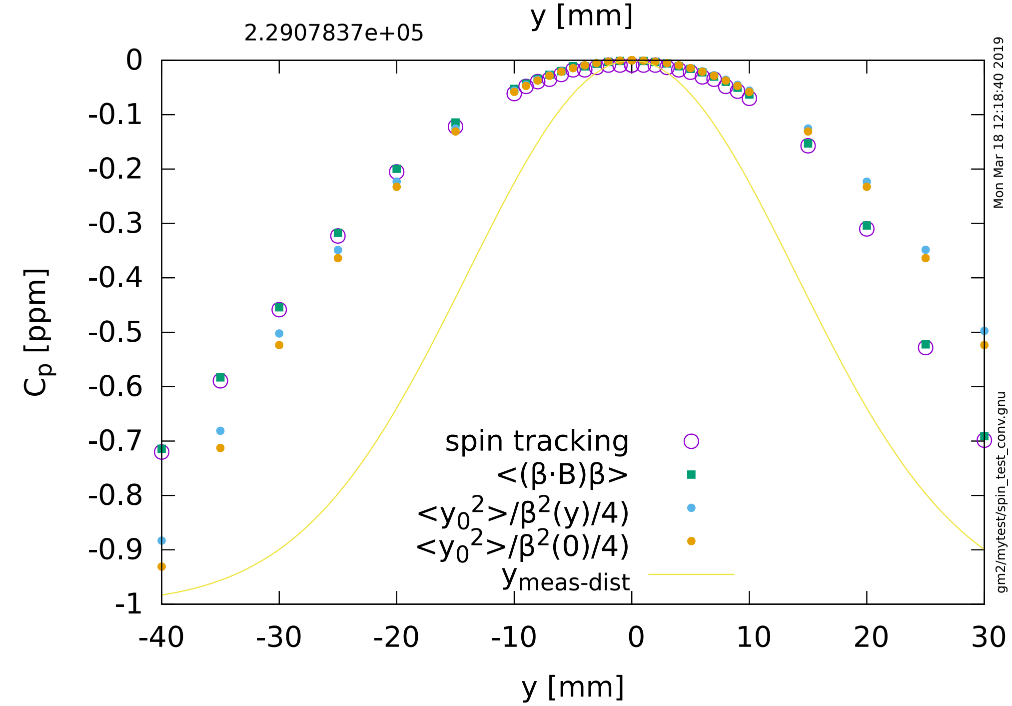
Q3	18.3	14.8	18.3	21.8
Q4	18.3	18.3	18.3	18.3





Our measurement is sensitive to precession about the axis perpendicular to the direction of motion. The component of the magnetic field along that axis is B cos ψ. Path length

$$\int B\cos\psi(s)ds \sim \int_0^\lambda B\cos(\psi_0\cos 2\pi\frac{x}{\lambda})\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx$$

$$y = \sqrt{a\beta}\sin 2\pi\frac{x}{\lambda}$$

$$\psi = \sqrt{a\beta}\frac{2\pi}{\lambda}\cos 2\pi\frac{x}{\lambda} = \sqrt{\frac{a}{\beta}}\cos 2\pi\frac{x}{\lambda} = \psi_0\cos 2\pi\frac{x}{\lambda}$$

$$= \int_0^\lambda B\cos(\psi_0\cos 2\pi\frac{x}{\lambda})\left(1+\frac{1}{2}\psi_0^2\cos^2 2\pi\frac{x}{\lambda}\right)dx$$

$$= \int_0^\lambda B(1-\frac{1}{2}\psi_0^2\cos^2 2\pi\frac{x}{\lambda})\left(1+\frac{1}{2}\psi_0^2\cos^2 2\pi\frac{x}{\lambda}\right)dx$$

$$\sim \int_0^\lambda B\left(1-\left\{\frac{1}{2}\psi_0^2\cos^2 2\pi\frac{x}{\lambda})\right\}^2\right)dx \sim B\lambda$$

The decrease in the component of magnetic field perpendicular to the Precession plane is compensated by the increase in the path length.

$\Delta\phi_a \sim 0$

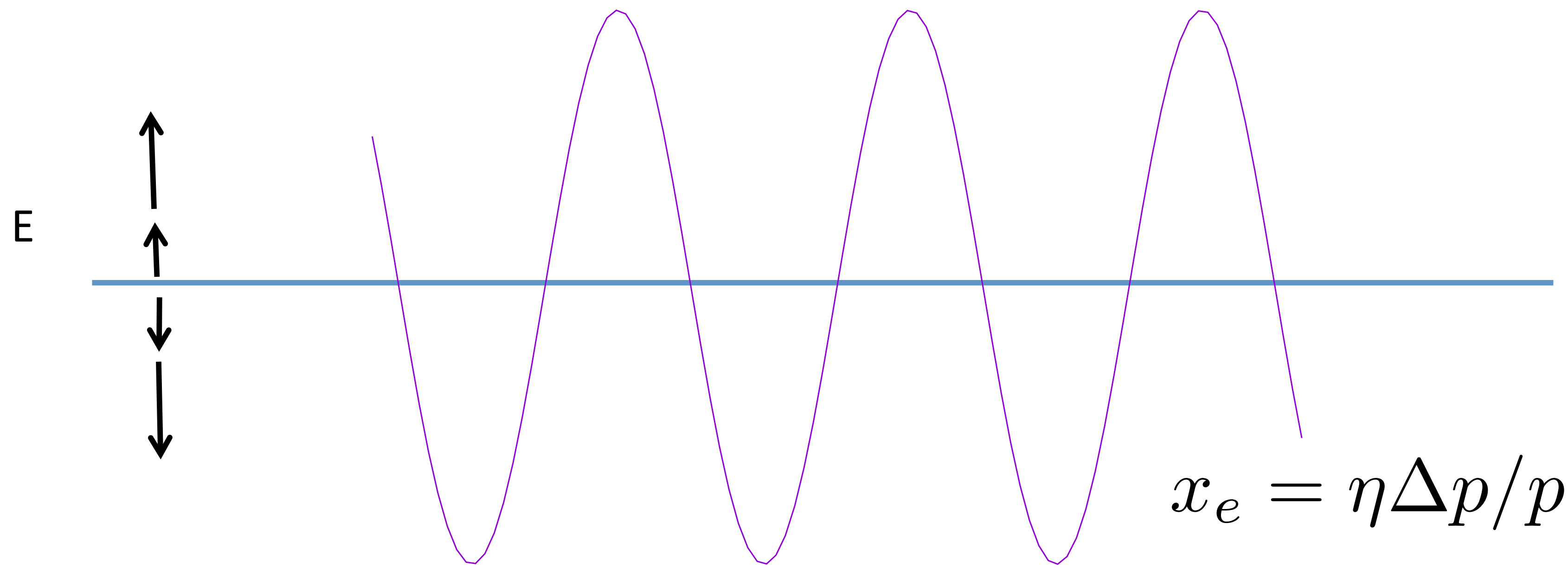
$\Delta\omega_a = \Delta\phi_a \omega_0$ and

$$\frac{\Delta\omega_a}{\omega_a} \sim \frac{\Delta\omega_0}{\omega_0} = -\frac{1}{\lambda}\int_0^\lambda \frac{1}{2}\psi_0^2 \cos^2 2\pi\frac{x}{\lambda}dx = -\frac{1}{4}\psi_0^2$$

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{m^2}{p^2}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

E-field contribution
Quad linearity

In an ideal cartesian geometry and quadrupole where the horizontal field is antisymmetric about the closed orbit, the E-field correction is independent of betatron amplitude

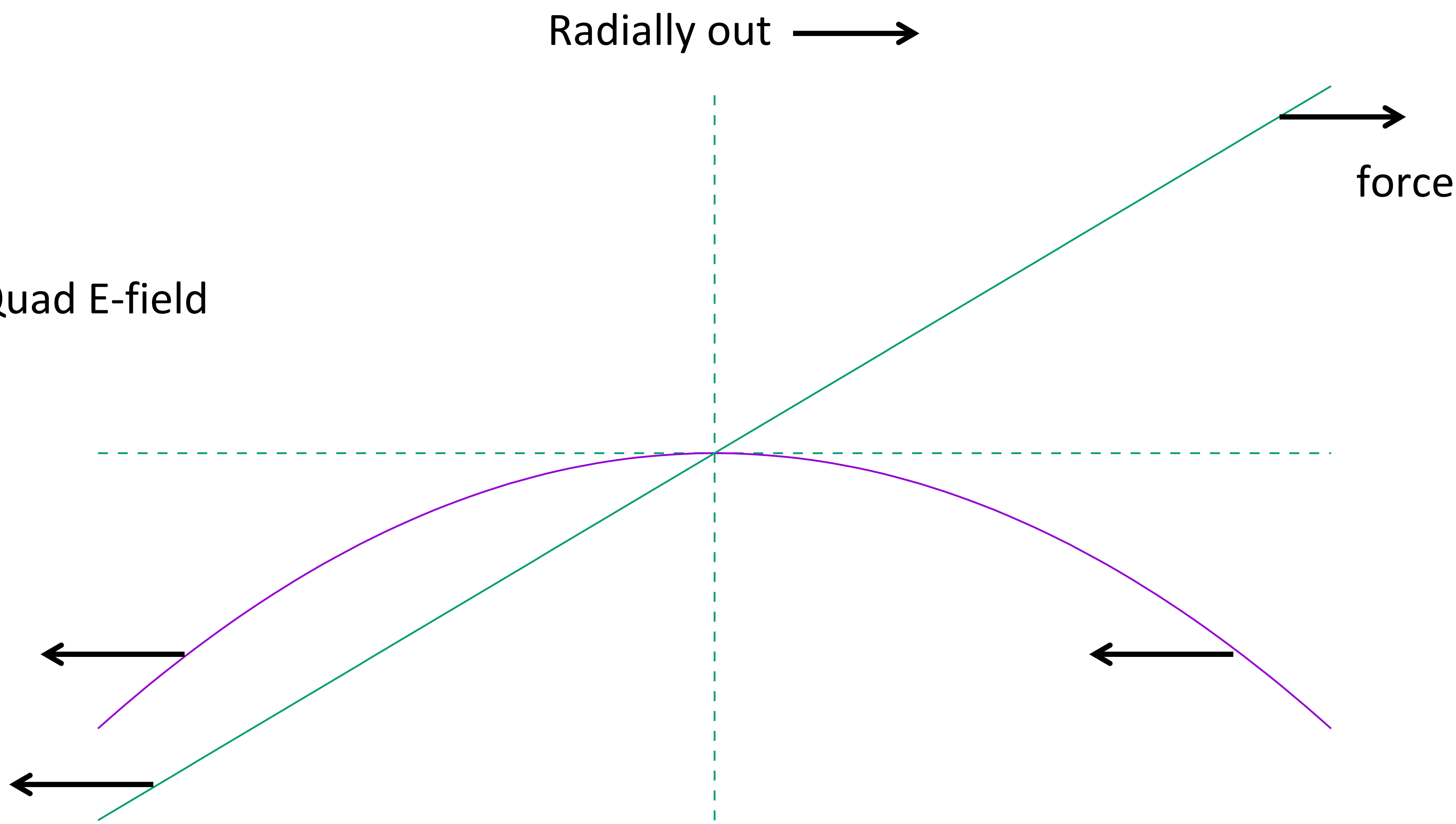


$$x_e = \eta \Delta p/p$$

In a curved geometry, the integrated E-field along the trajectory depends on betatron amplitude in two ways

1. Sextupole (quadratic) component of the quads
   - Sextupole component is symmetric about magic radius
   - Shifts the ‘closed orbit’
2. Path length (asymmetric about magic radius)

Closed orbit depends on betatron amplitude



Sextupole component shifts <x> radially inward by

$$\langle F_x \rangle \sim k \frac{x_\beta^2}{2\rho}$$