Spin tracking vs Integration and effect of quad nonlinearity

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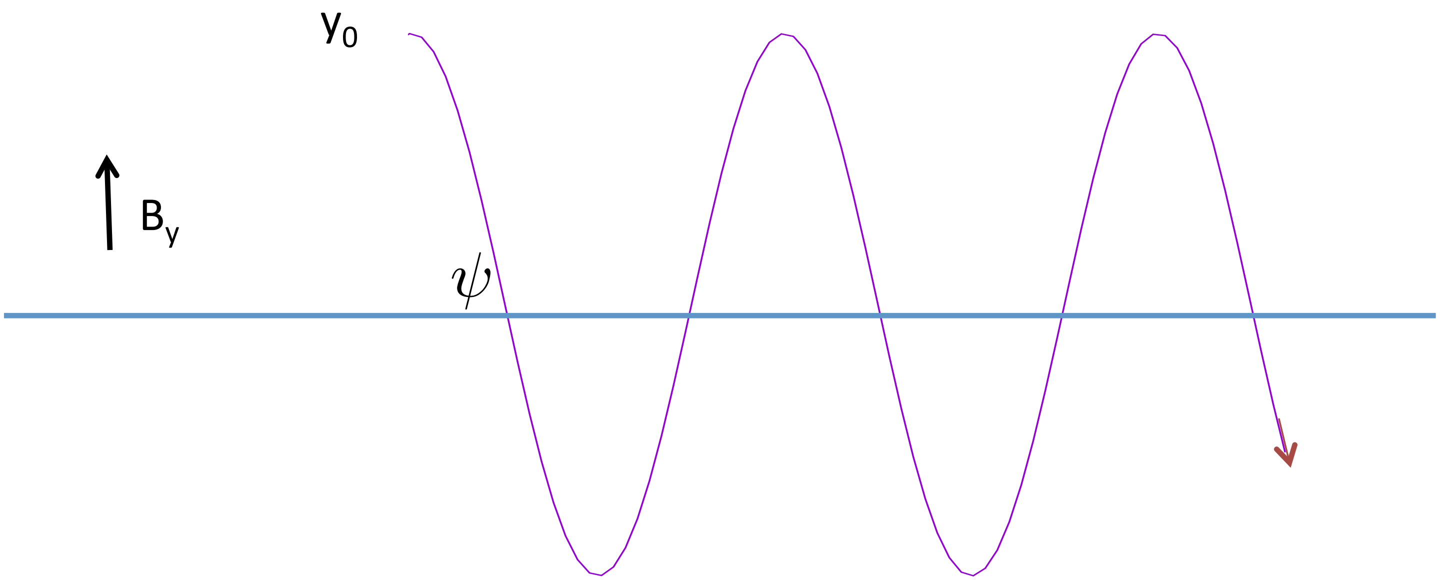
Pitch systematic



$$\frac{\Delta\omega}{\omega} = \langle (1 - \hat{\beta} \times \hat{\mathbf{B}}) \rangle \approx \frac{1}{2}\langle \psi^2 \rangle$$

Pitch correction

3 ways to compute pitch correction in simulation

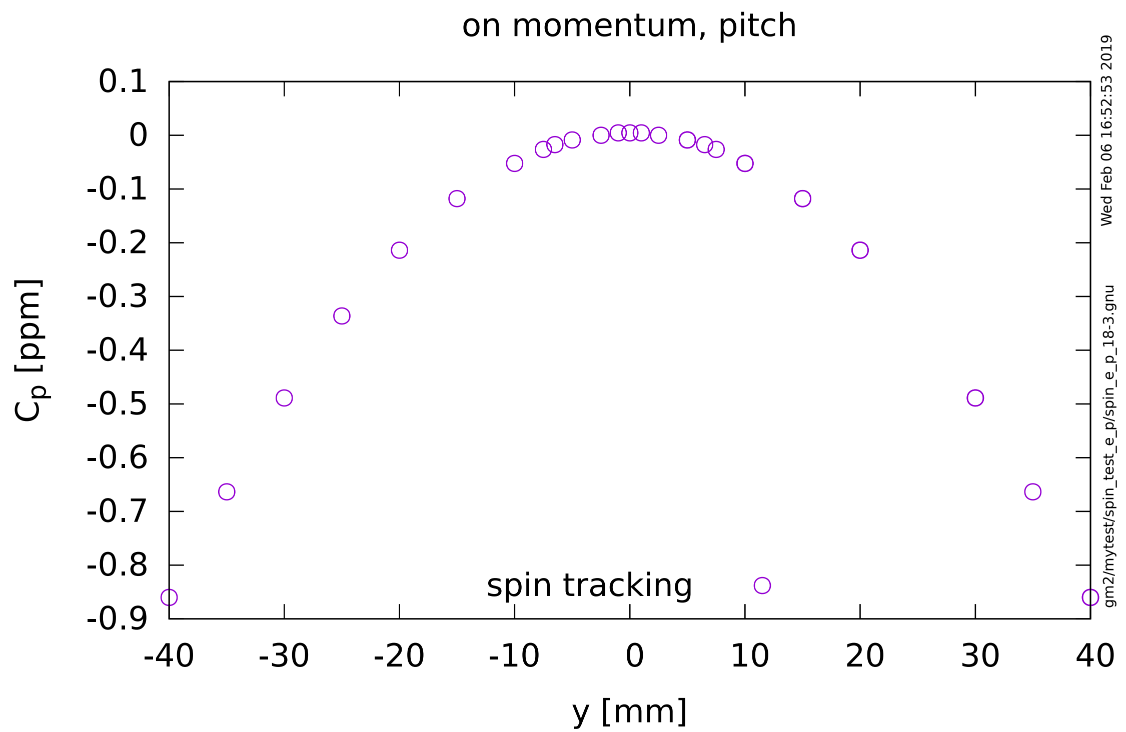
- Spin tracking − *Includes everything, but ppb precision requires many turns*
- Integration along trajectory − *very good approximation far from resonances*

$$C_p = \frac{1}{T}\int_0^T (1 - \hat{\beta} \times \hat{\mathbf{B}})dt$$

- Measurement of vertical amplitude − *assumes quad linearity*

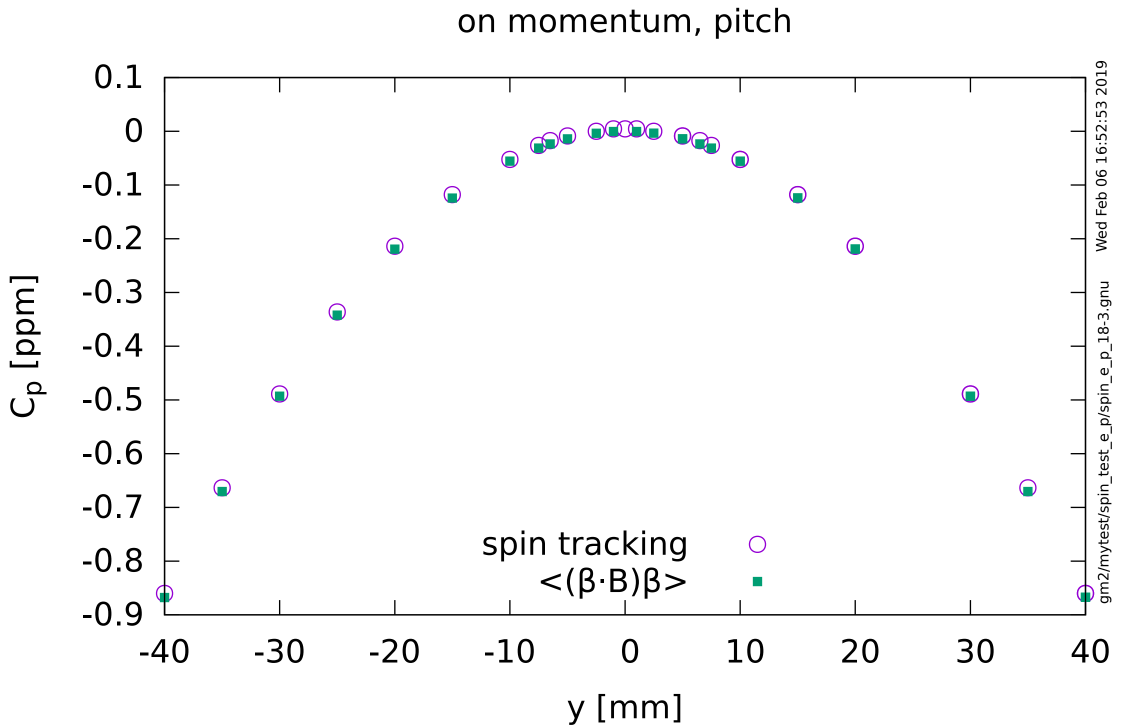
$$C_p = -\frac{n\langle y^2 \rangle}{2R_0^2} = -\frac{\langle y^2 \rangle}{2\beta_y^2} = -\frac{\langle \psi^2 \rangle}{2}$$

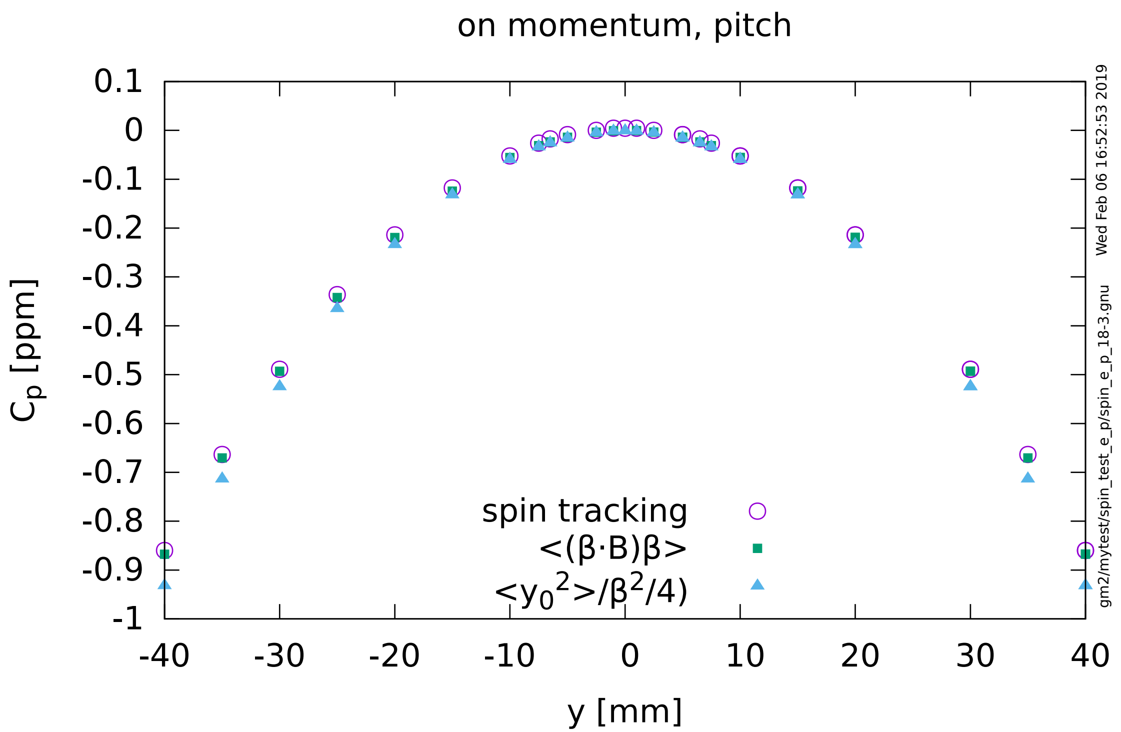
Spin tracking using BMT



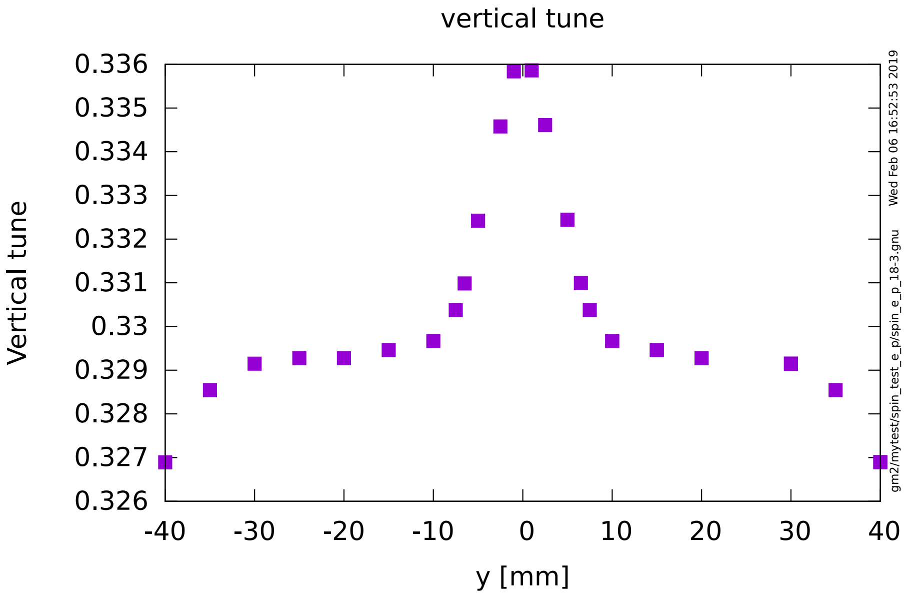
Pitch correction vs vertical amplitude

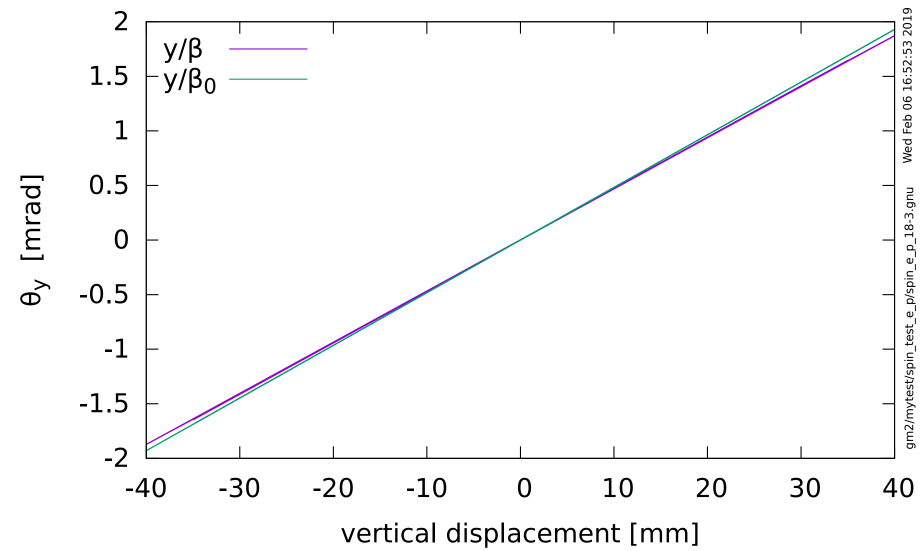
Spin tracking and 'integration' are in good agreement





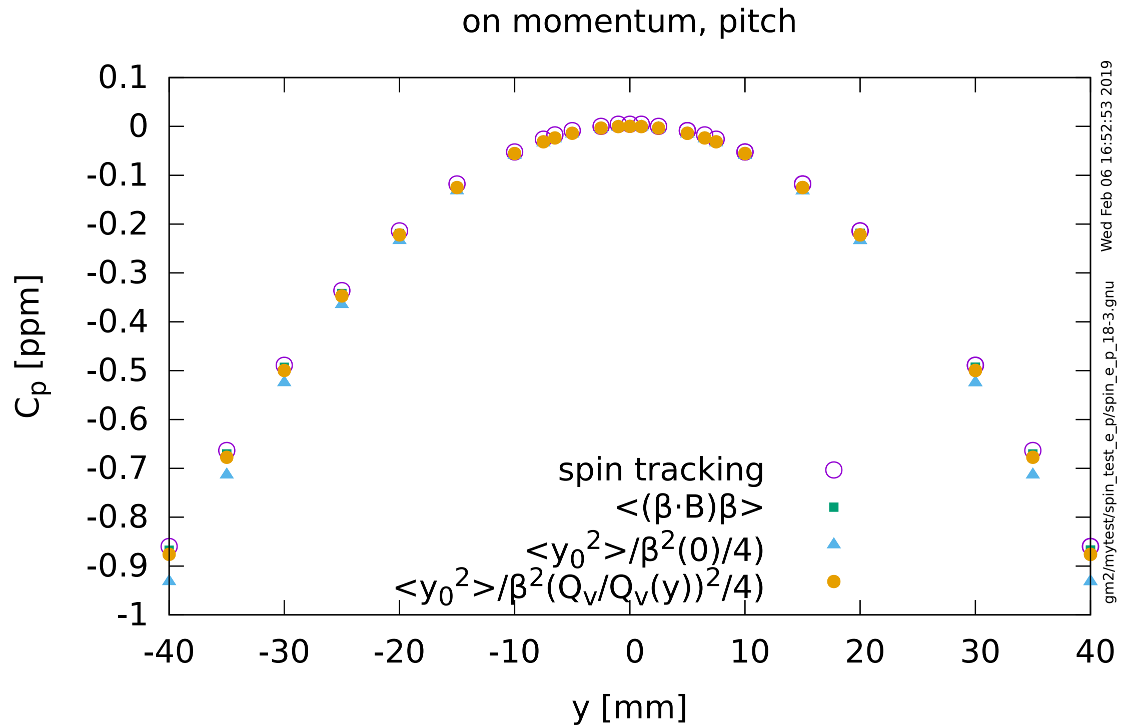
Vertical amplitude $(y_0)/\beta_v$ is not a good measure of angle ψ at large amplitude



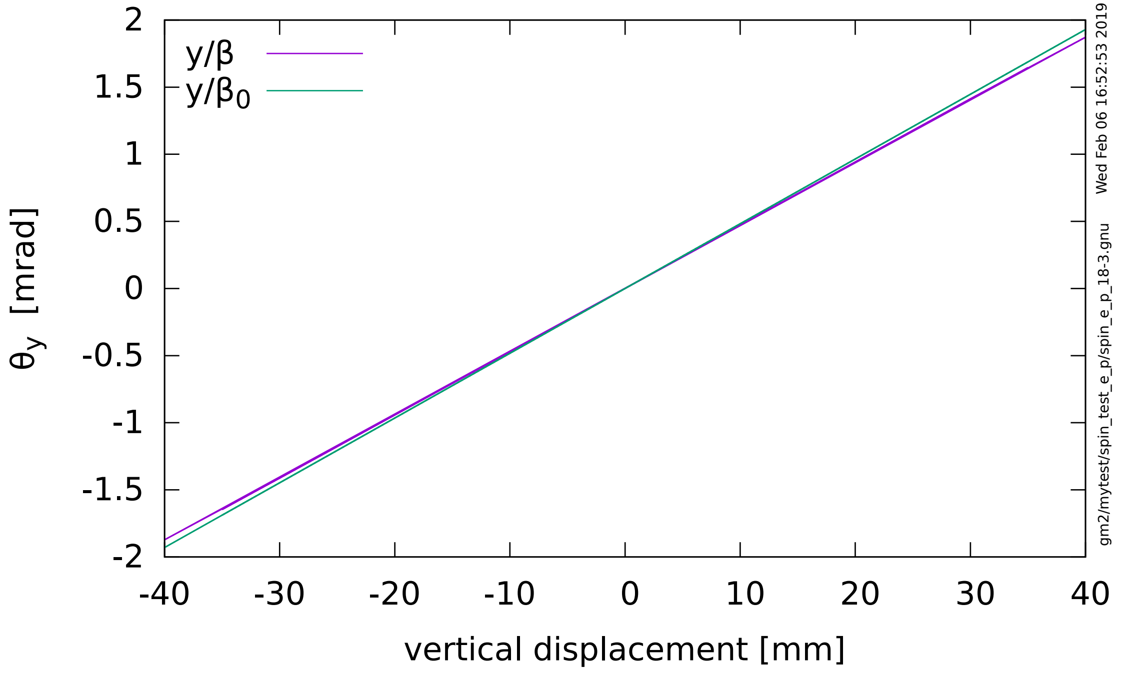


Quad nonlinearity => amplitude dependence of tune and β
And nonlinear dependence of ψ on y_0

$$\beta(y) = \left(\frac{Q_v(0)}{Q_v(y)^2}\right)^2 \beta_0$$



- We can correct for the amplitude dependence by measuring the vertical tune
- Alternatively, measure the angular $\langle\psi^2\rangle$ distribution directly


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
θy [mrad]
y/β
y/β0
-40
-30
-20
-10
0
10
20
30
40
vertical displacement [mm]
Wed Feb 06 16:52:53 2019
gm2/mytest/spin_test_e_p/spin_e_p_18-3.gnu

E field correction

3 ways to compute E-field contribution to ω_a

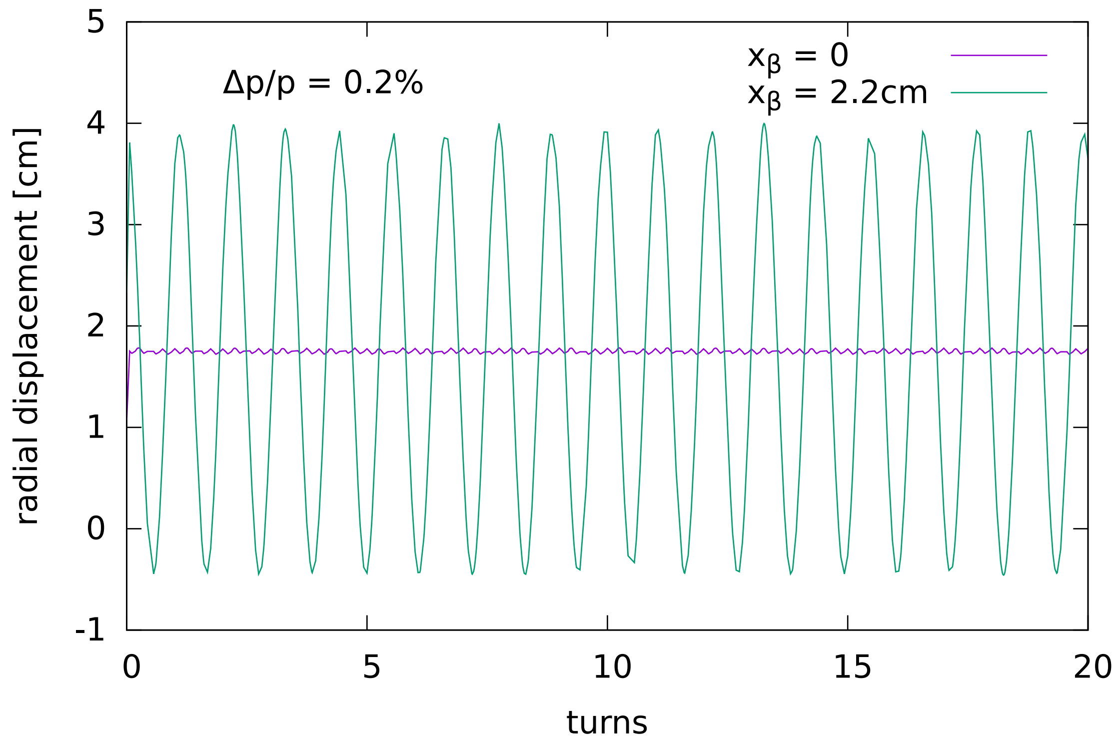
1. Spin tracking (BMT equation)
2. Integration $$\vec{C}_e(T) \sim 2\frac{\Delta p}{p}\frac{1}{T}\int_0^T \frac{\vec{\beta}\times\vec{E}}{Bc}dt$$
   a) Integration along trajectory (includes betatron oscillations)
   b) Integration along closed orbit ($x = \eta\delta$)

Note that method 2b) is most nearly equivalent to the 'classic' method, namely

$$C_E = -2\beta^2 n(1-n)\frac{\langle x_e^2\rangle}{r_0^2}$$

Compare the 3 methods in simulation to determine

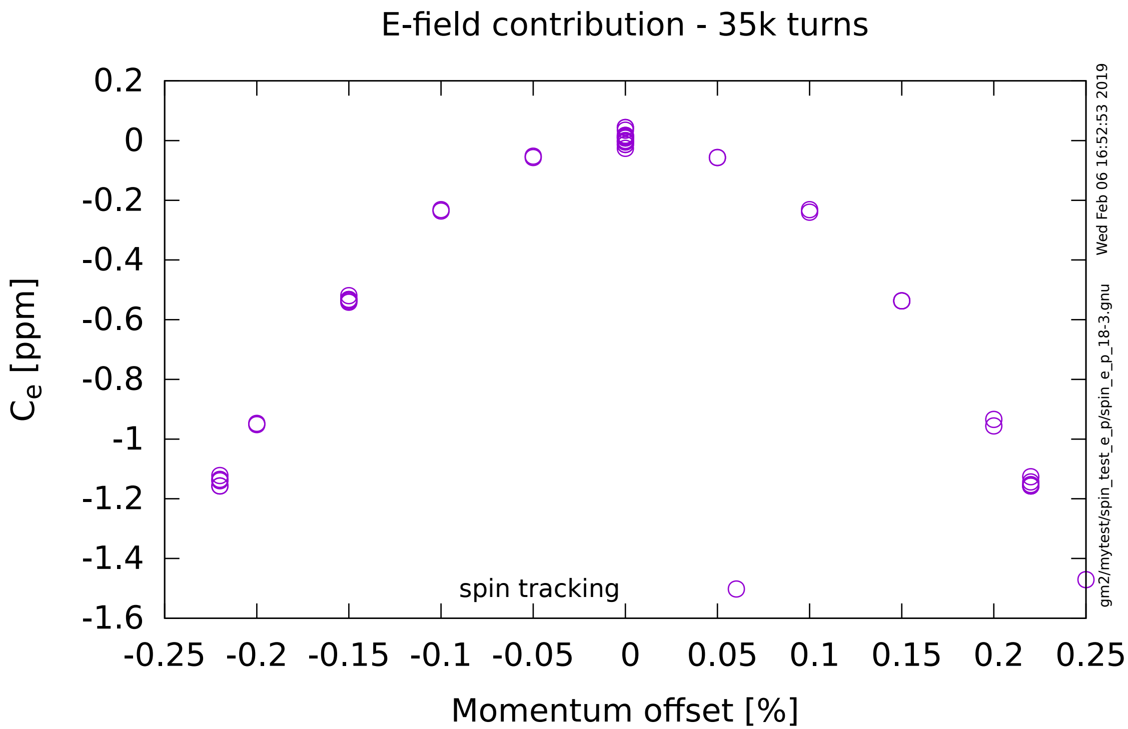
1. If integration is a reliable proxy for spin tracking
2. The size of the contribution from finite betatron oscillation amplitude
3. Effect of quad nonlinearity



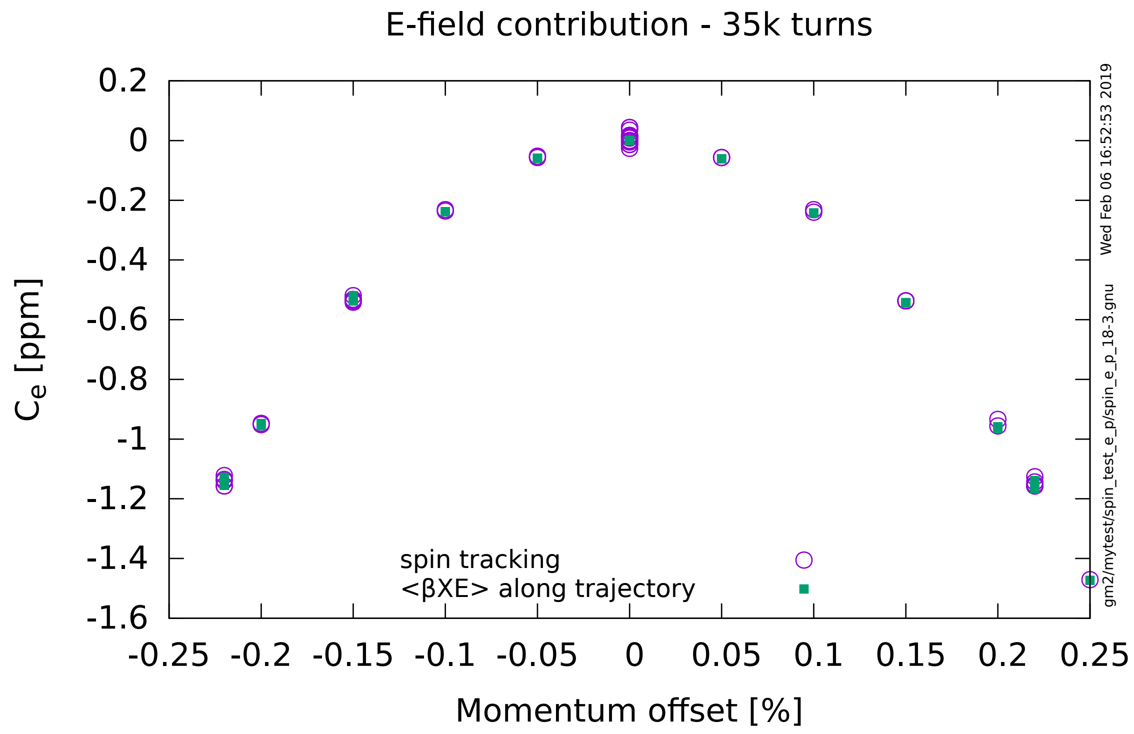
Distinct trajectories with common momentum offset

- For trajectory compute ω_a by spin tracking and by integration

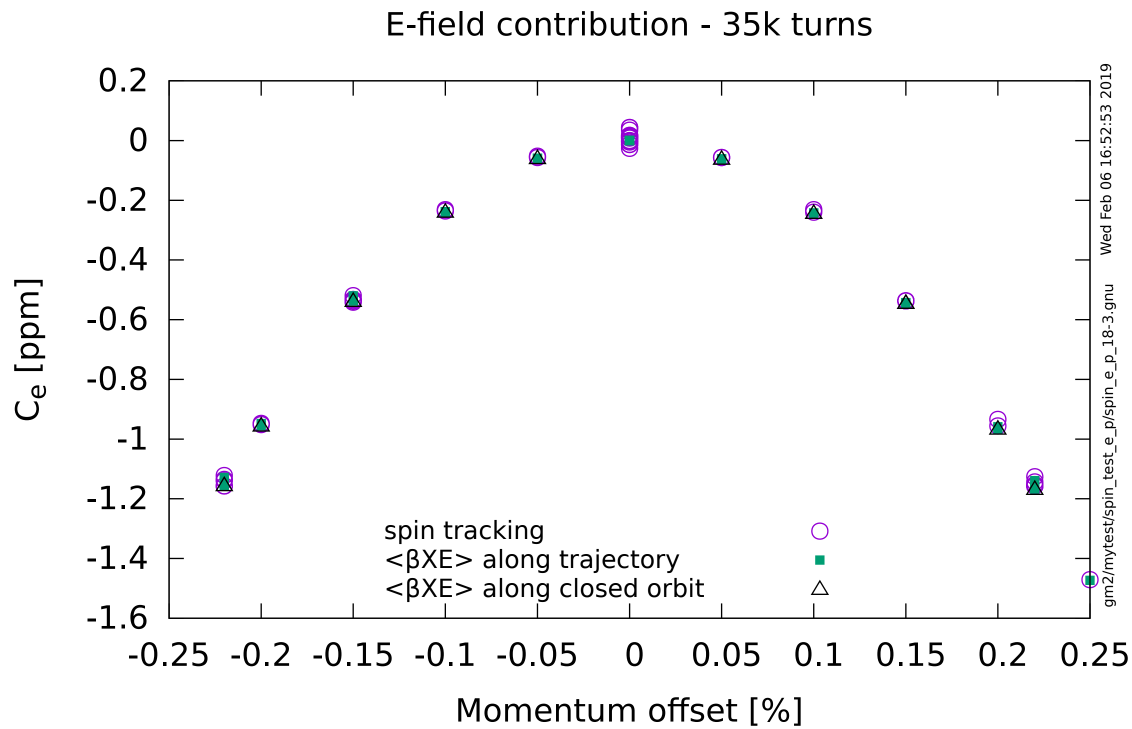
Is the Efield correction independent of the betatron amplitude?



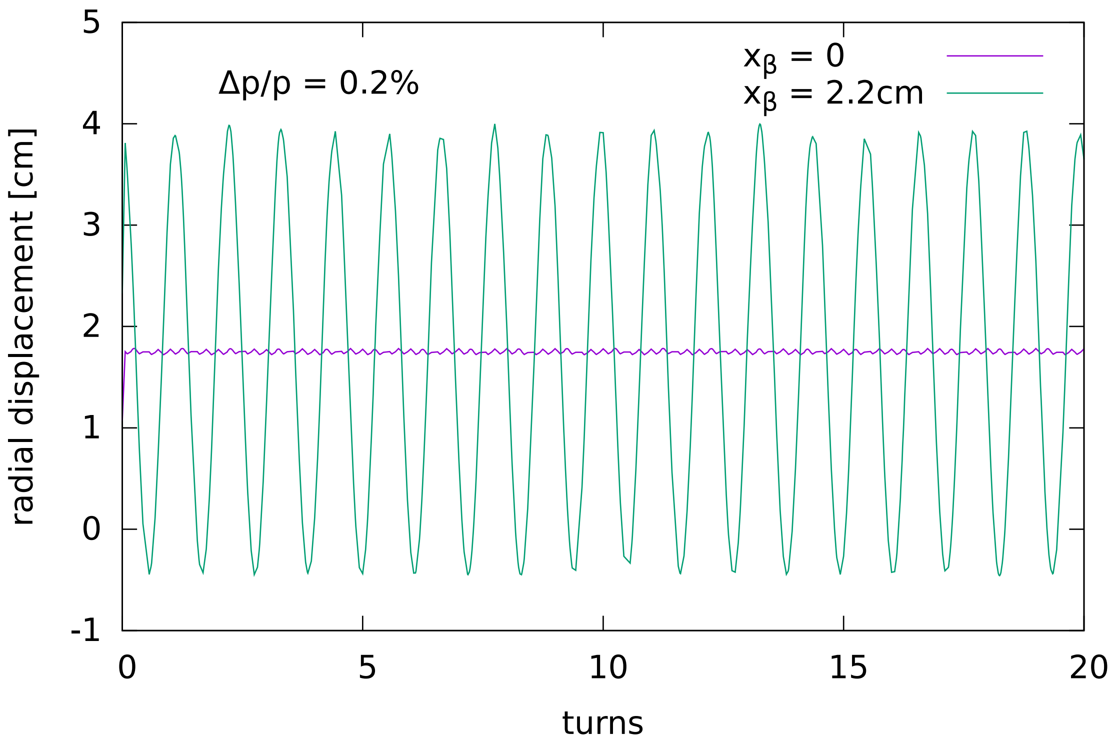
- Multiple points at each momentum correspond to different betatron amplitudes
- The spread at momentum zero is a measure of the accuracy of the simulation (since the E-field correction is nominally zero at the magic momentum)



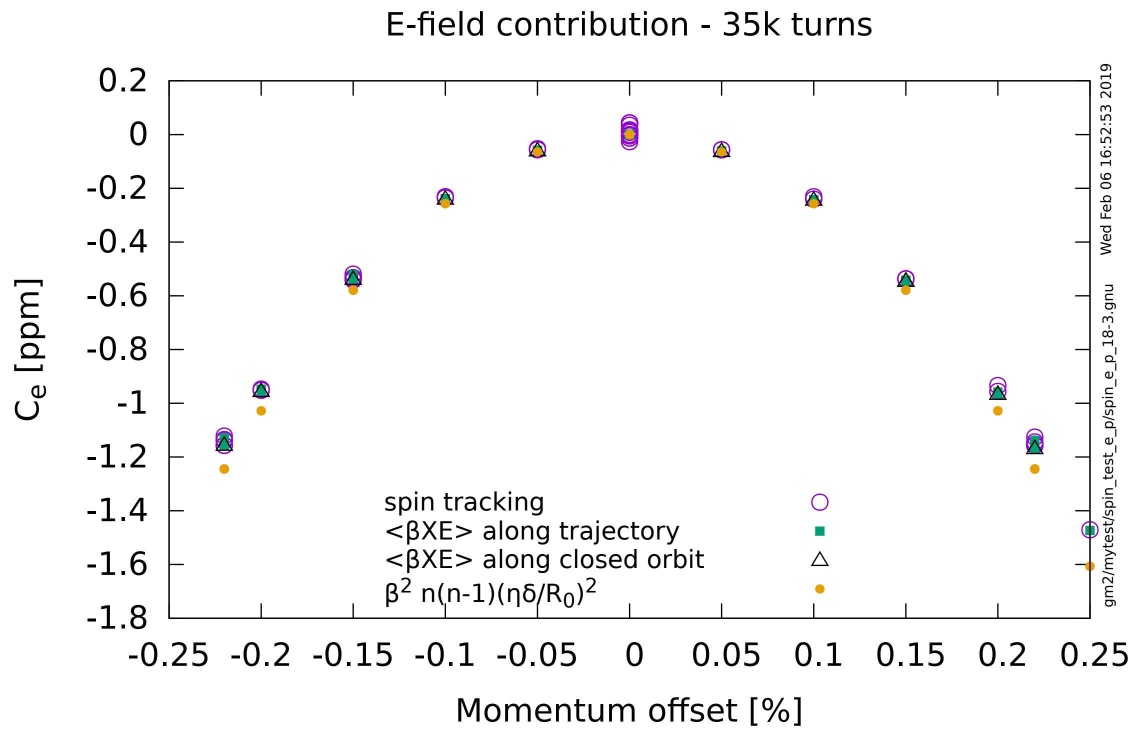
Efield correction computed with spin tracking in good agreement with correction based on $\langle \beta \times \mathbf{E} \rangle$



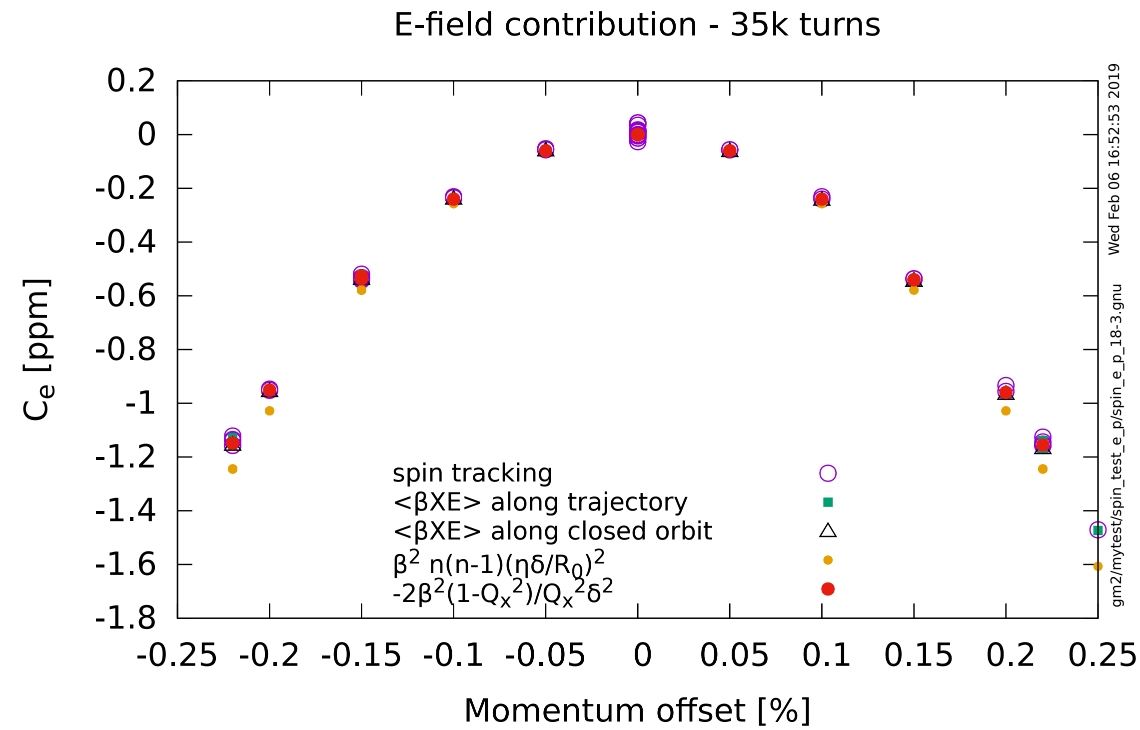
$\langle \beta \times \mathbf{E} \rangle$ along the trajectory is very nearly the same as $\langle \beta \times \mathbf{E} \rangle$ along the closed orbit. (There is little dependence on betatron amplitude)



$\Delta p/p$ = 0.2%
x_β = 0
x_β = 2.2cm
radial displacement [cm]
turns
5
4
3
2
1
0
-1
0
5
10
15
20

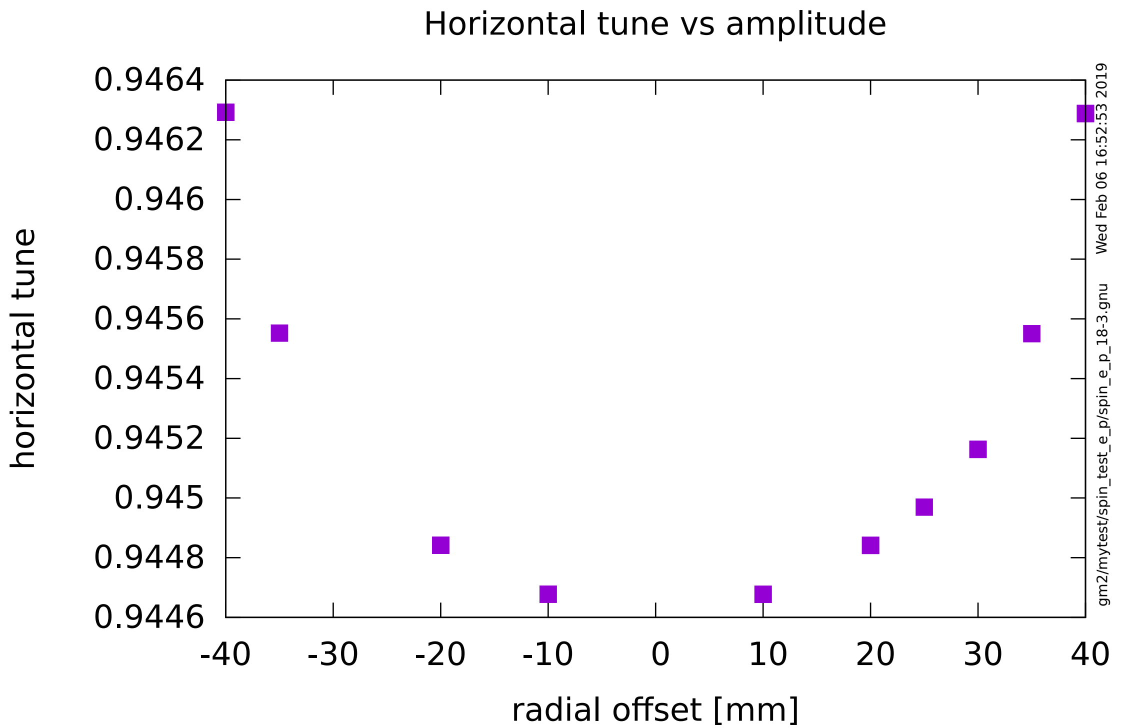


The calculation of the E-field correction that assumes quad linearity, *overestimates* the effect at large momentum offset (where E-field does not increase linearly with displacement)



Replace n, R_0 and η with Q_x and measure Q_x for each momentum => $C_e = -2\beta^2 \frac{(1-Q_x)^2}{Q_x^2}\delta^2$

The effect of quad nonlinearity can be corrected by measuring momentum dependence of horizontal tune



Horizontal tune vs amplitude
horizontal tune
0.9464
0.9462
0.946
0.9458
0.9456
0.9454
0.9452
0.945
0.9448
0.9446
-40
-30
-20
-10
0
10
20
30
40
radial offset [mm]
gm2/mytest/spin_test_e_p/spin_e_p_18-3.gnu
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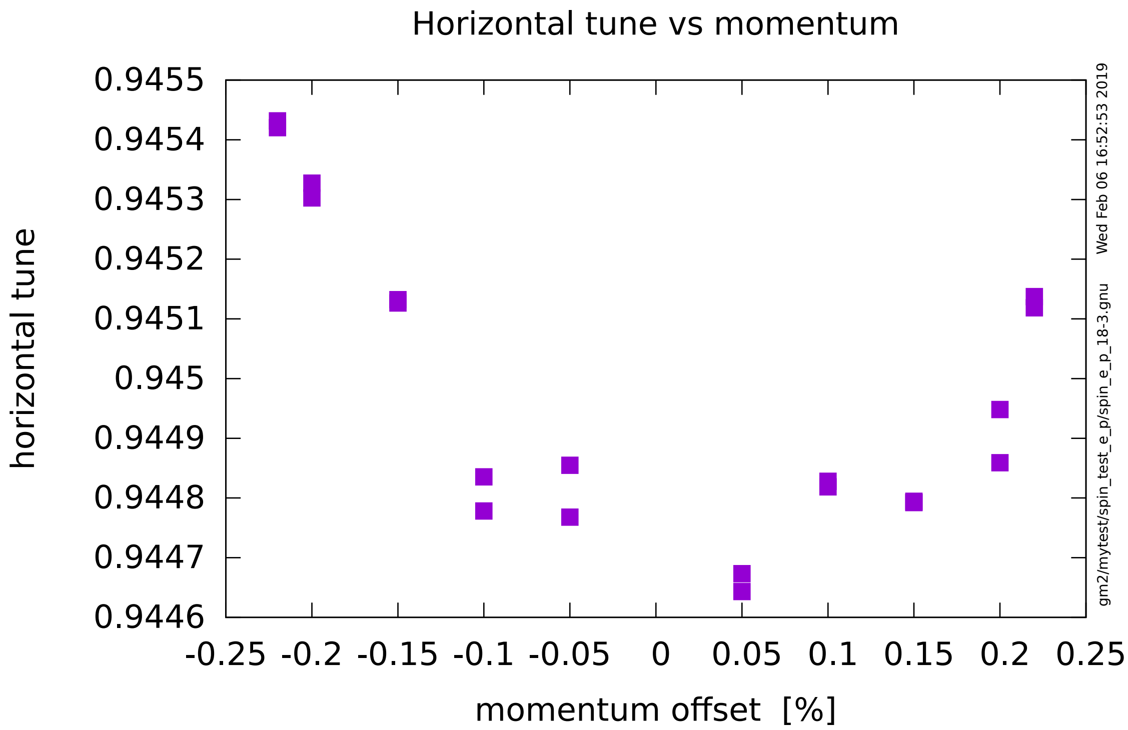
Horizontal tune vs momentum



Comments

- Quad fields are based on an azimuthal slice of 3-D field map with no end effect details
- And perfect relative alignment of plates and absolute alignment about magic radius
- Perfect B-field

Measurement of amplitude/momentum dependence of tunes would be very useful to diagnose quad fields and compensate nonlinearities.