Fast rotation status

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on behalf of the fast rotation group

Elba collaboration meeting
May 30, 2019

Introduction

Spin precession equation and ESQs E–field

For perfect ESQs E-field, perfect vertical magnetic field and for anti-muons with no betatron oscillations and no vertical velocity:

$$\vec{\omega}_a = -\frac{Qe}{m}\left[\boxed{a_\mu \vec{B}} - \boxed{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}}\right]$$

intrinsic spin precession

E-field contribution due to ESQs

If momentum is so-called “magic momentum” of 3.09 GeV/c, i.e. γ = 29.3 → E–field contribution vanishes because of $a_\mu = 1/(\gamma^2 - 1)$

Realistically: stored beam has a momentum spread, a non-magic average momentum and a non-zero betatron oscillation → E–field contribution ≠ 0

Therefore: need to measure γ (momentum distribution) and $\vec{\beta} \times \vec{E}$

E–field correction to ω_a

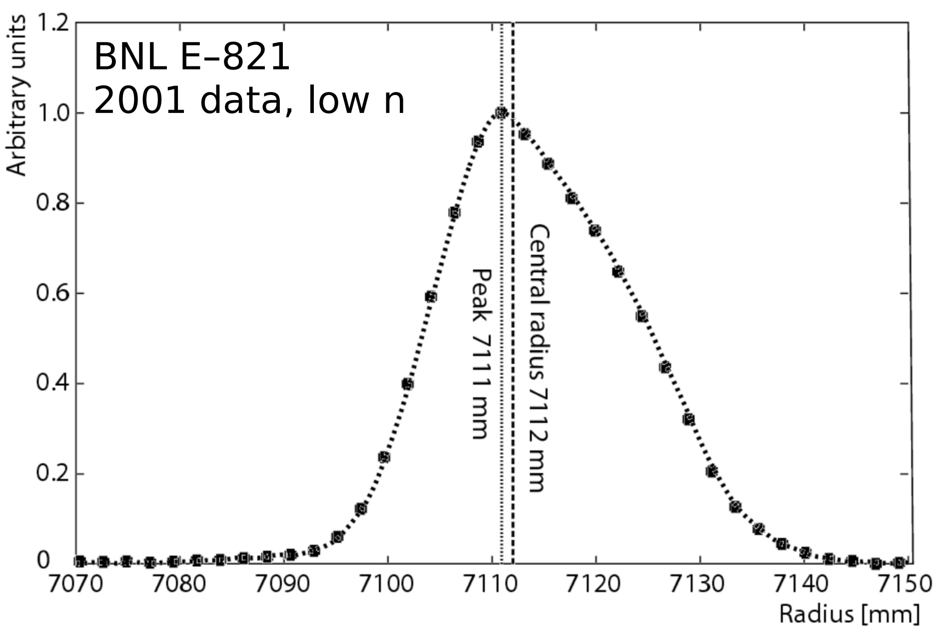
For perfect magnetic and electric fields, no betatron oscillation, correction to ω_a:

$$C_E = \frac{\Delta\omega}{\omega} = -2n(1-n)\beta^2 \frac{\langle x_e^2 \rangle}{R_0^2} \quad \text{where} \quad \langle x_e^2 \rangle = x_e^2 + \sigma^2$$

x_e: equilibrium radius of the beam, i.e. average radial position of the beam

σ : radial width of the beam

→ E-field correction estimation relies on reconstructing the beam radial distribution



E–field correction for BNL E–821 2001 data:

C_E = -470 ± 50 ppb

Compared to BNL E–821 2001:

$\Delta\omega_a$ = 700 ppb

→ need to estimate C_E very accurately and precisely for Fermilab E–989 given 120 ppb precision goal for ω_a

Two key questions

1. How accurately and precisely can we reconstruct the radial distribution by the mean of the Fourier (or FT) and χ^2 (or CERN-III) methods?

Antoine, Alex K., James M., Rob, Kim Siang, Hogan...

2. How accurately and precisely can we estimate the E-field correction given the correct reconstructed radial distribution?

David R., David T., Renee...

Only one for this talk

1. How accurately and precisely can we reconstruct the radial distribution by the mean of the Fourier (or FT) and χ^2 (or CERN III) methods?

THIS TALK

will use:

$$\boxed{C_E = \frac{\Delta\omega}{\omega} = -2n(1-n)\beta^2 \frac{\langle x_e^2 \rangle}{R_0^2}}$$

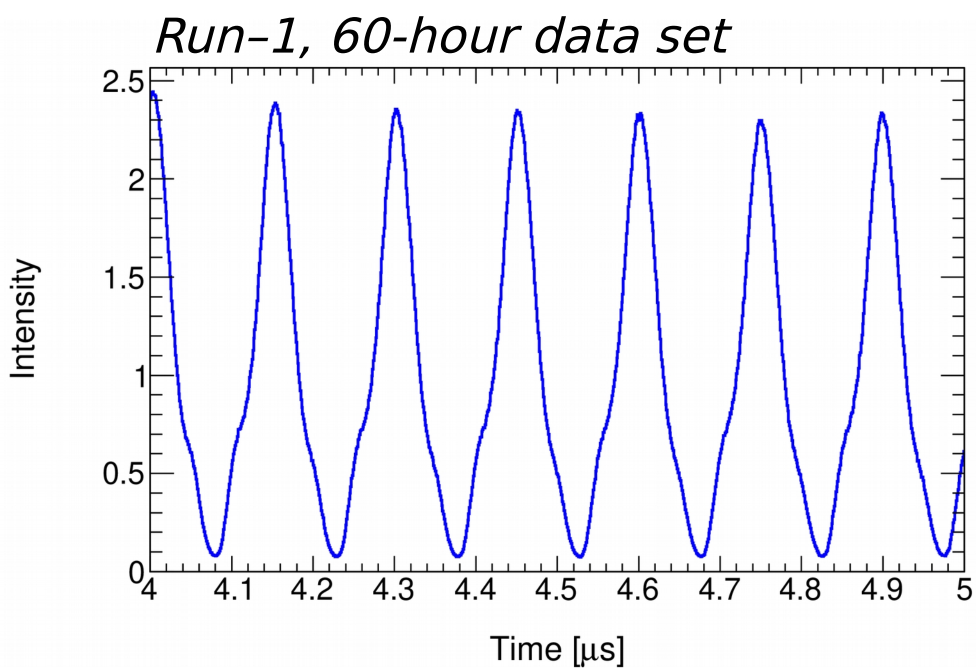
Cornell Fourier method: **Antoine**, David R., Josh F., Daniel S.

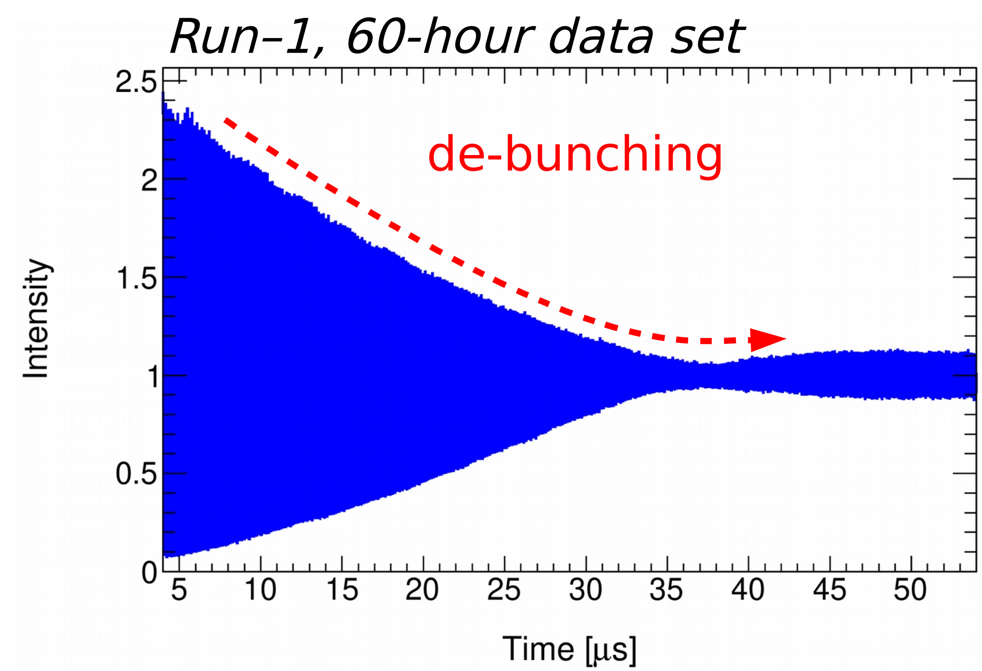
χ^2 method: **Alex K.**, James M., Rob

Fast rotation methods

Beam de-bunching

Beam injected bunched (~200 ns wide) → allow to reconstruct periodic signal from calorimeters data: so-called “fast rotation” signal





Beam de-bunches due to momentum spread:

different momenta → different radii → different cyclotron frequencies

Fourier method

Weak focusing ring: **frequency ↔ radius**

end time of available signal

For fast rotation S(t), frequency distribution is:

$$\Phi(\omega, t_s = t_0, t_m) = \int_{t_0}^{t_m} S(t) cos\left[\omega(t - t_0)\right] dt$$

start time of available signal

center of mass of the beam passing the detector the first time

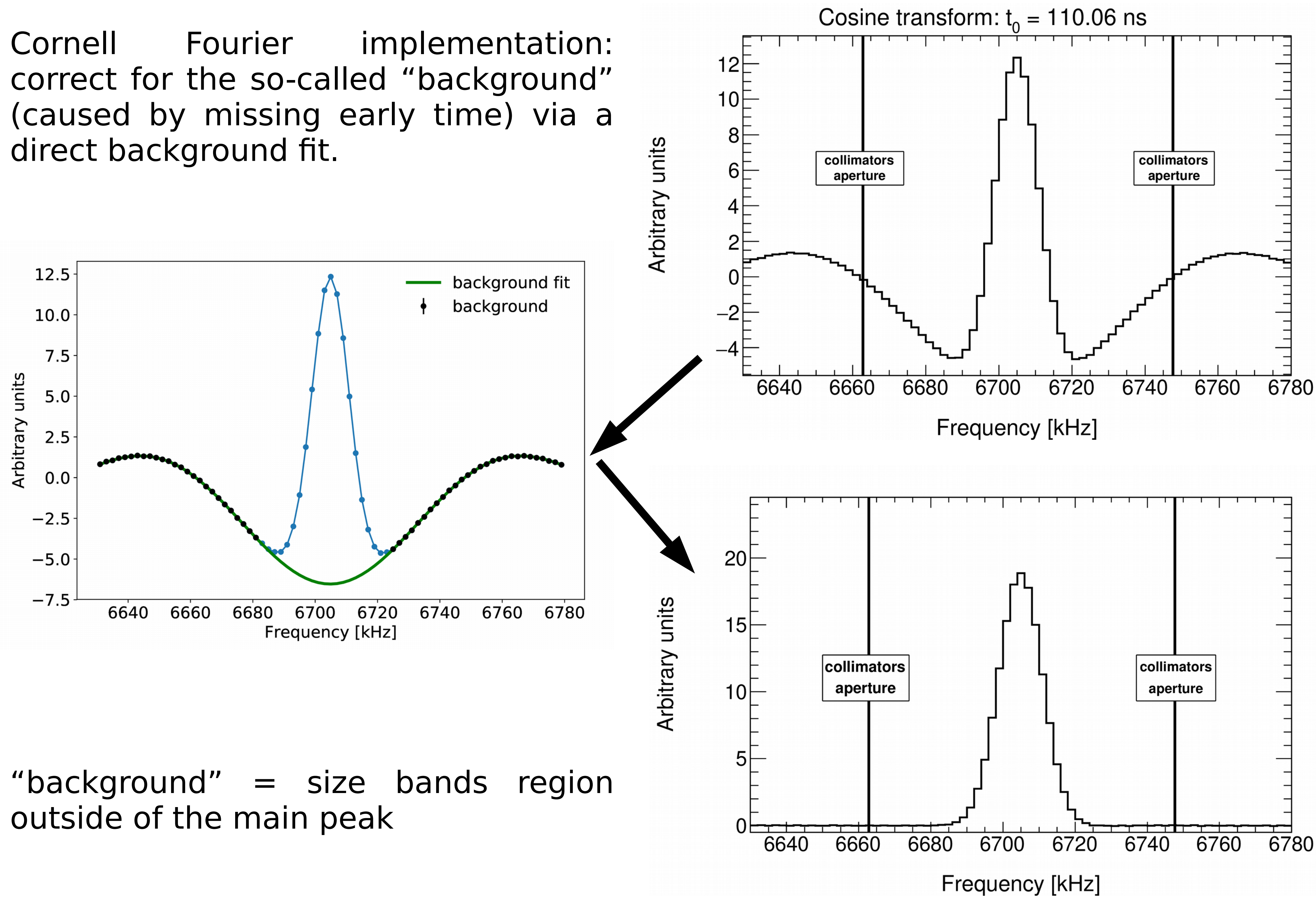
For ***t_s > t_0*** (first turn data not available):

$$\Phi(\omega) = \int_{t_s}^{t_m} S(t) cos\left[\omega(t - t_0)\right] dt + \boxed{A \cdot \int_{\omega^-}^{\omega^+} S_{app}(\omega') \frac{\sin\left[(\omega-\omega')(t_s-t_0)\right]}{\omega-\omega'} d\omega' + B}$$

cardinal sine correction term to account for missing time between $t_0 \rightarrow t_s$

Cornell Fourier: example with toy MC

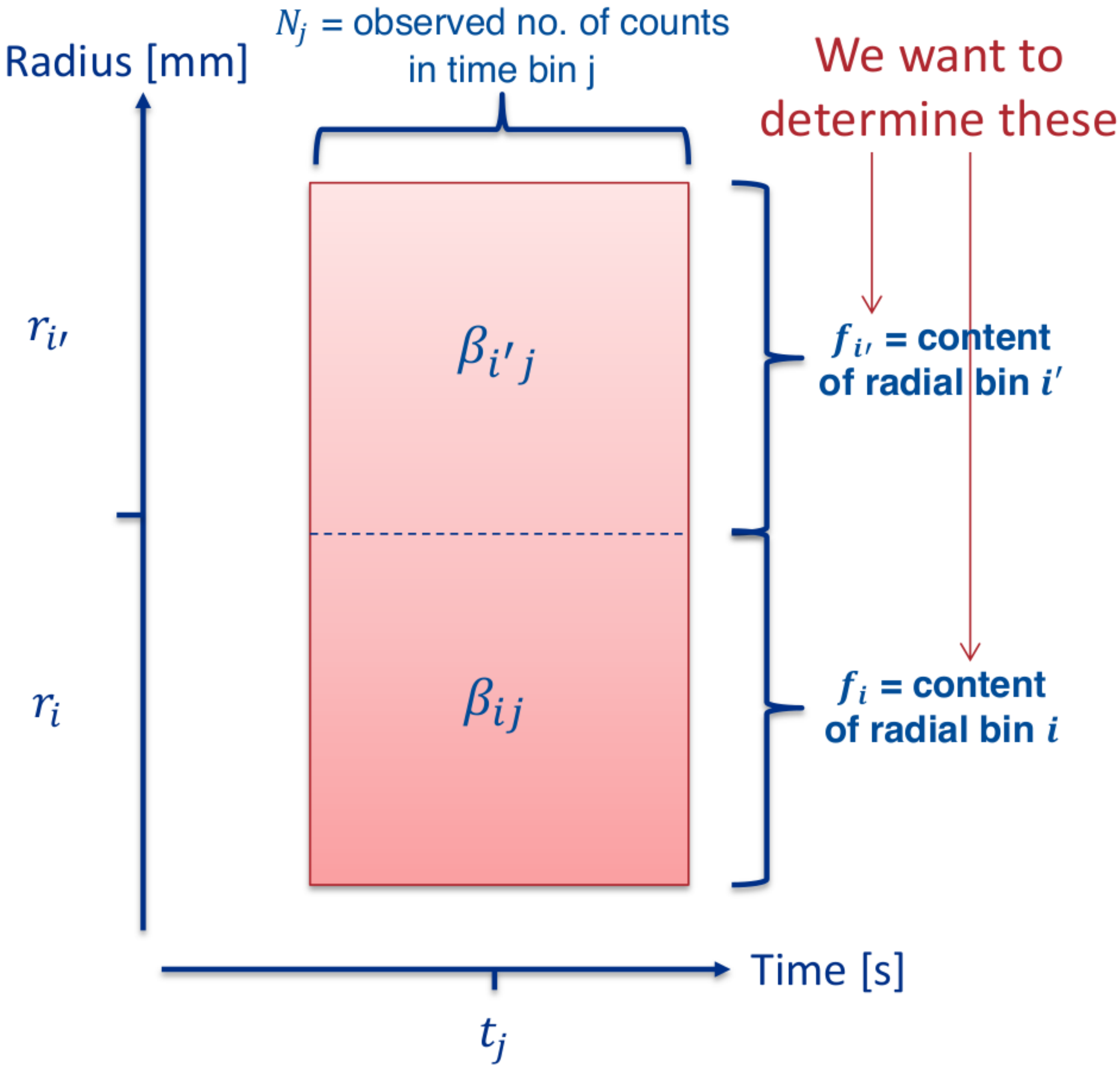
Cornell Fourier implementation: correct for the so-called “background” (caused by missing early time) via a direct background fit.



“background” = size bands region outside of the main peak

χ^2 method

Let's assume only two radial bins:



- β_{ij} are geometry factors which encode the contribution from the radial bin i to the time bin j
- The expected number of counts in the time bin j is given by:

$$C_j = f_i \beta_{ij}$$

- Then, to solve for f_i, we minimize the χ^2 function:

$$\chi^2 = \frac{\sum_j \left(\sum_i f_i \beta_{ij} - N_j\right)^2}{C_j}$$

(Note: this is the original CERN III function… to be updated in a few slides)

χ^2 method

→ Propagate the "injection pulse" bin content, I_k, which is chosen pulse at ~ 4 μs

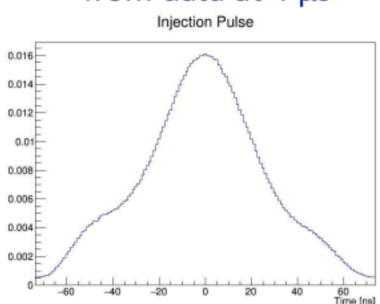
$$\chi^2 = \frac{\sum_j \left(\sum_i f_i \beta_{ij} - N_j\right)^2}{C_j} \longrightarrow \chi^2 = \frac{\sum_j \left(\sum_{i,k} f_i \beta_{ijk} I_k - N_j\right)^2}{C_j}$$

1) The χ^2 function is weighting by the bin errors, σ_j

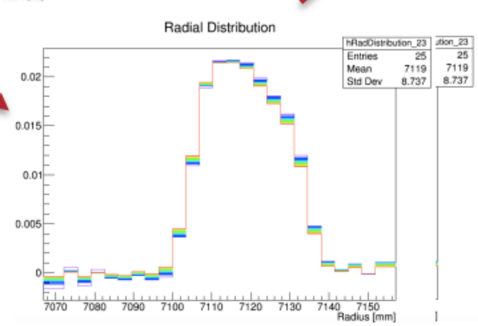
$$\chi^2 = \frac{\sum_j \left(\sum_{i,k} f_i \beta_{ijk} I_k - N_j\right)^2}{C_j} \longrightarrow \chi^2 = \frac{\sum_j \left(\sum_{i,k} f_i \beta_{ijk} I_k - N_j\right)^2}{\sigma_j^2}$$

2) And, once we've solved for f_i, we then solve for I_k and iterate until convergence…

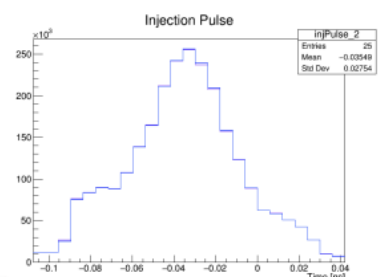
Take injection pulse from data at 4 μs



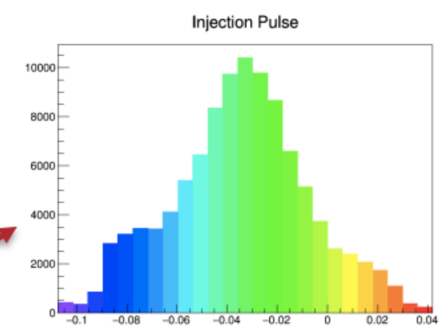
Solve for radial distribution

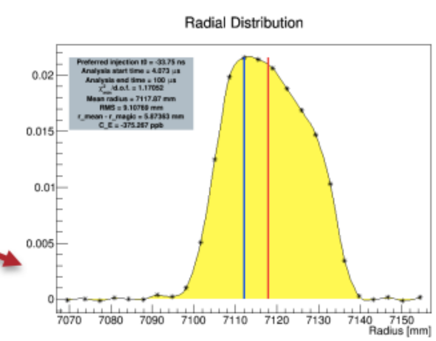


Solve for injection pulse at t_0



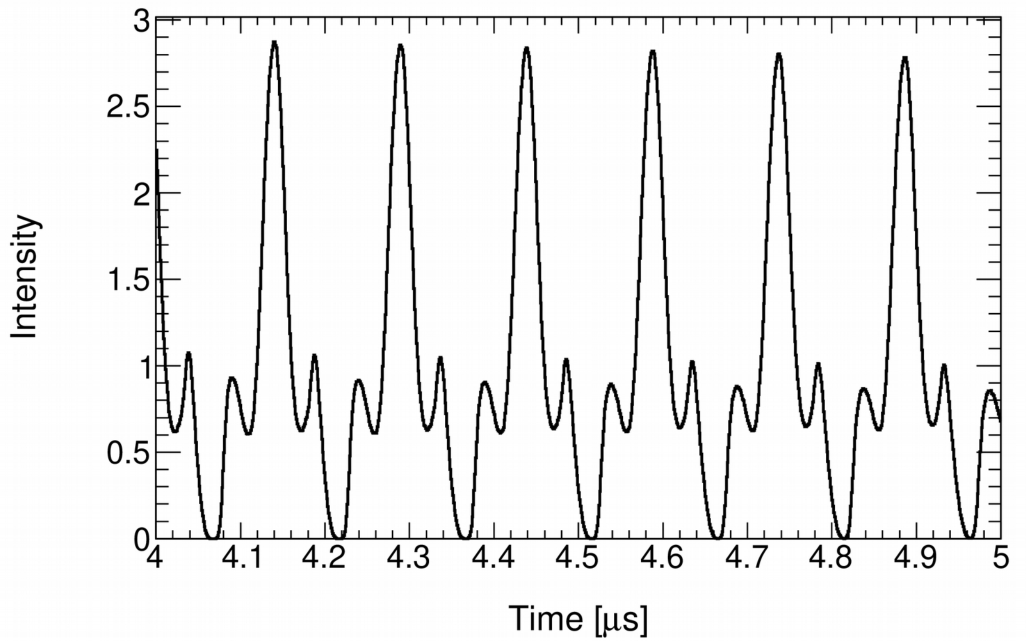
Reach convergence between χ^2_{min}/d.o.f. of radial distribution and injection pulse

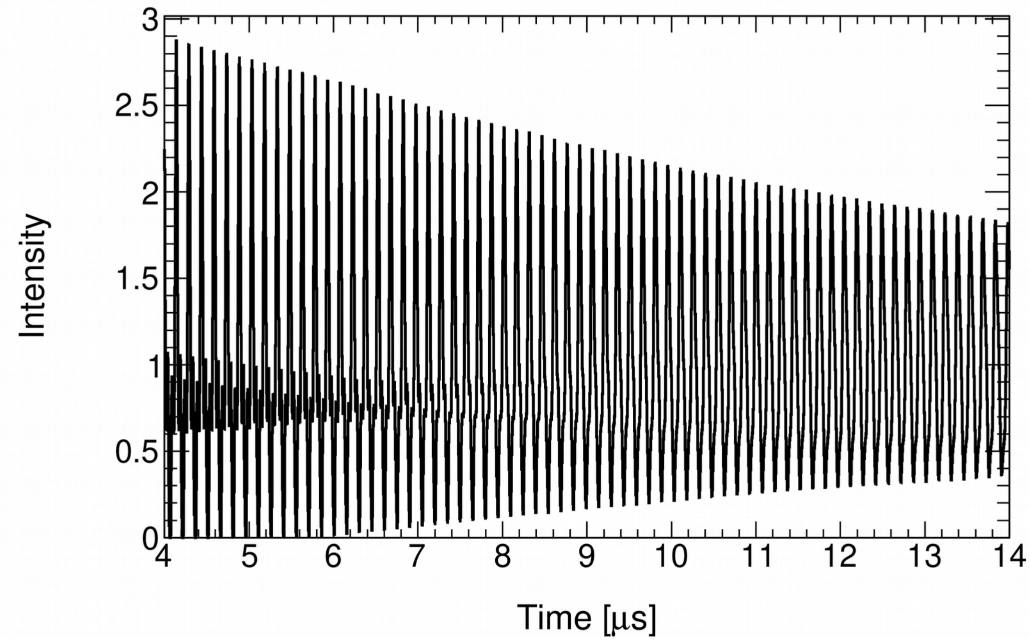


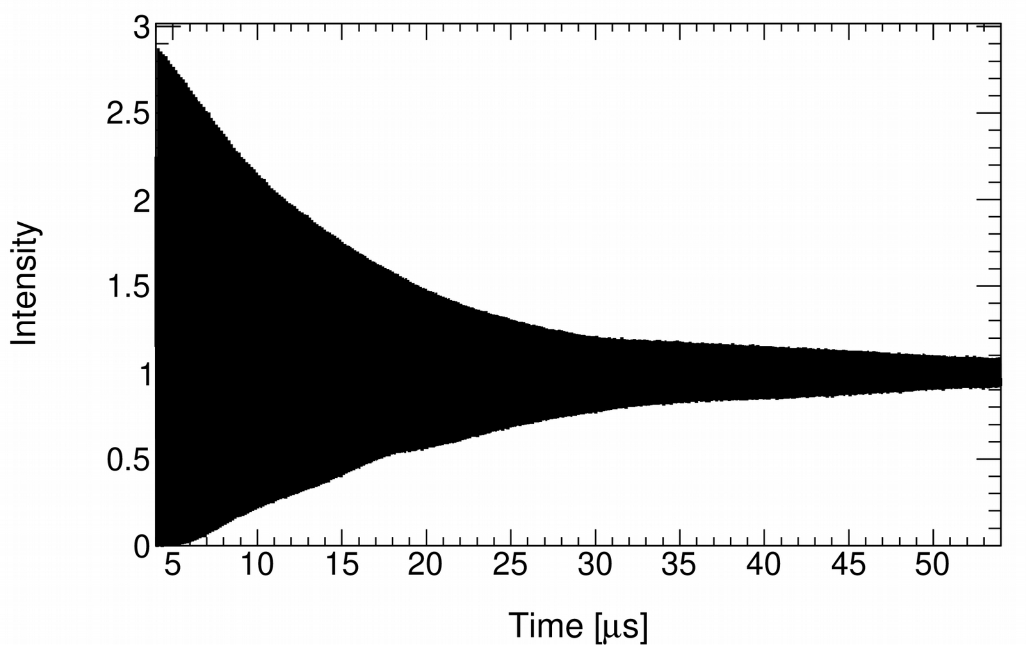


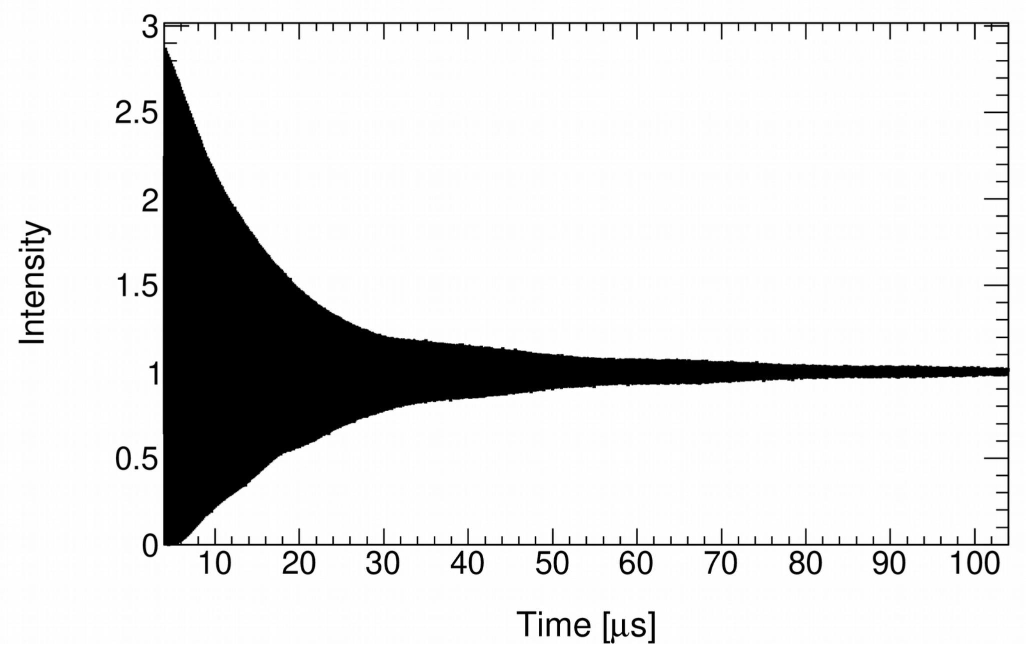
Performance study with toy Monte Carlo simulation

60–hour like simulation: pure fast rotation signal without any beam/detector effects



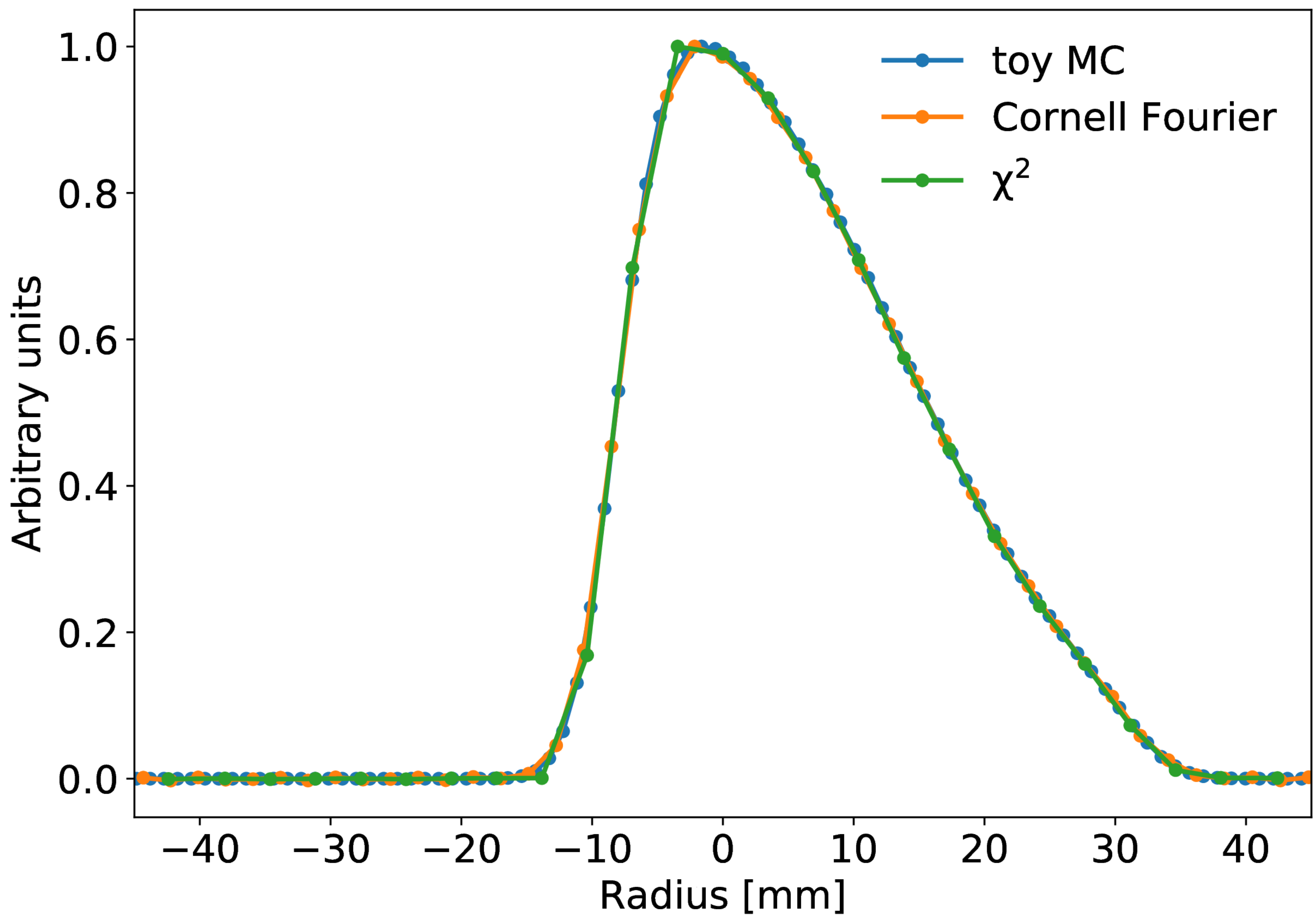




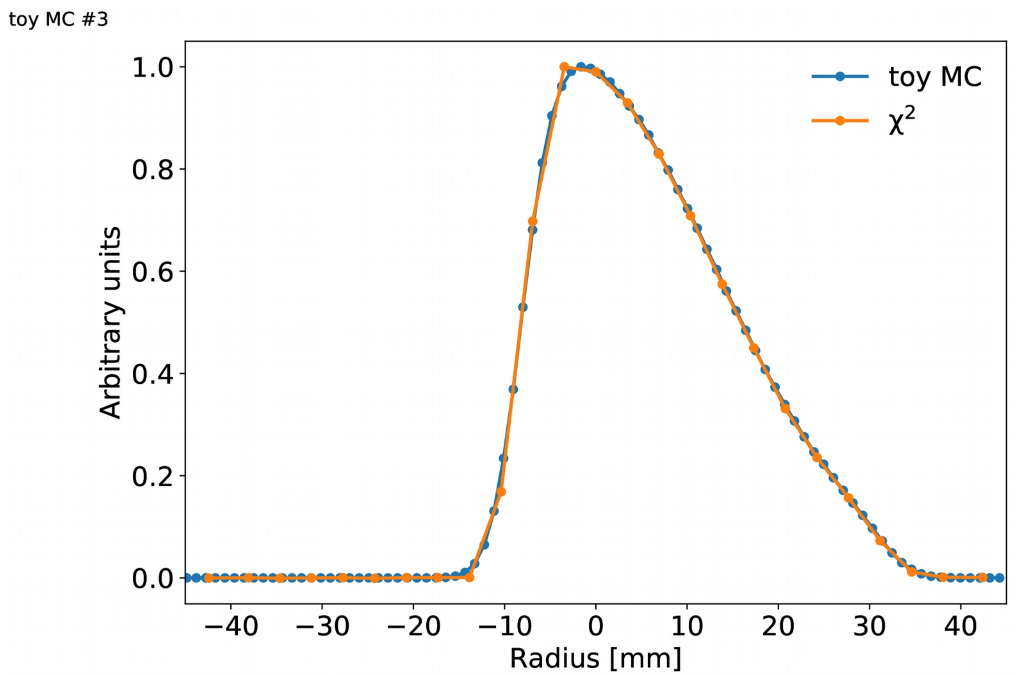


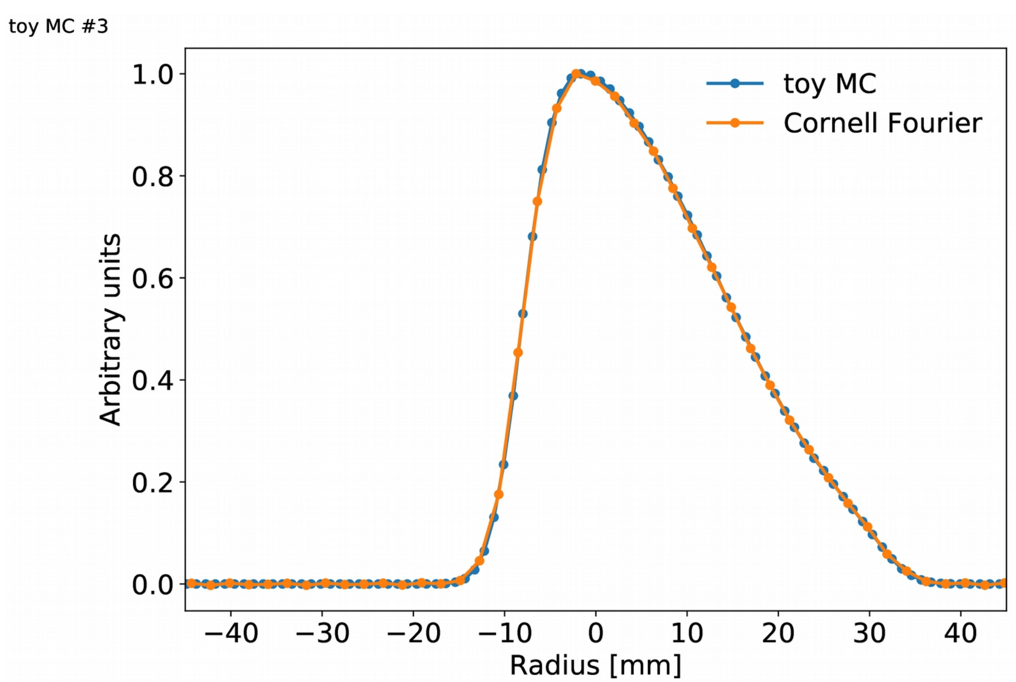
Performance study with toy Monte Carlo simulation

toy MC #3



Performance study with toy Monte Carlo simulation

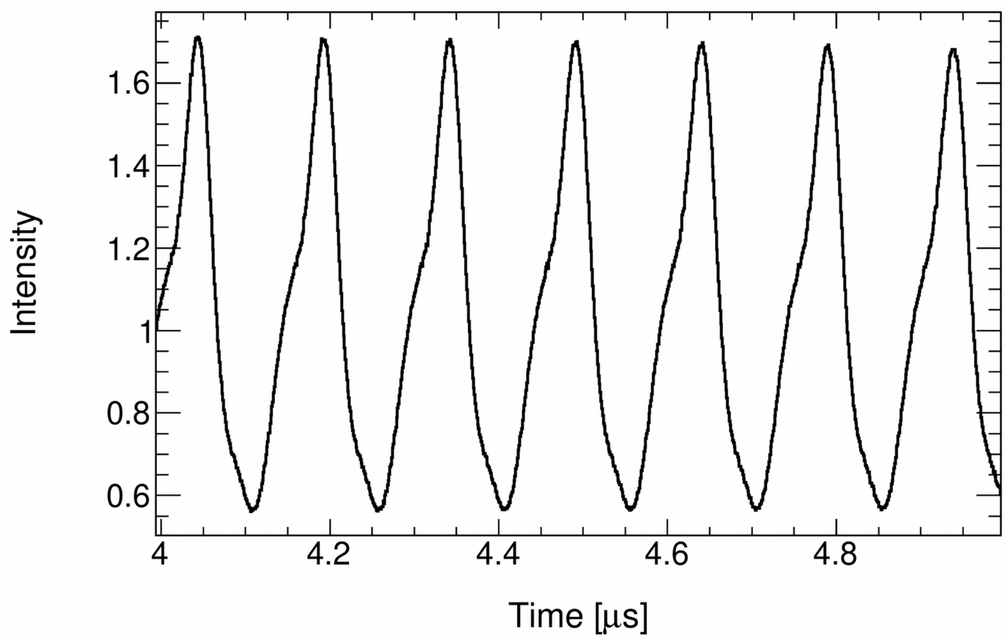


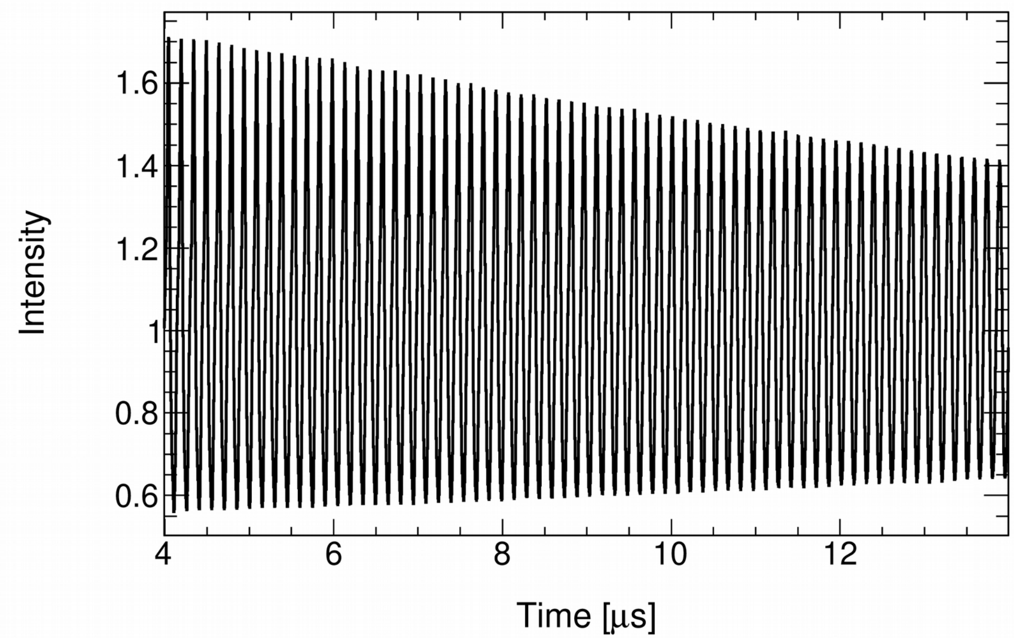


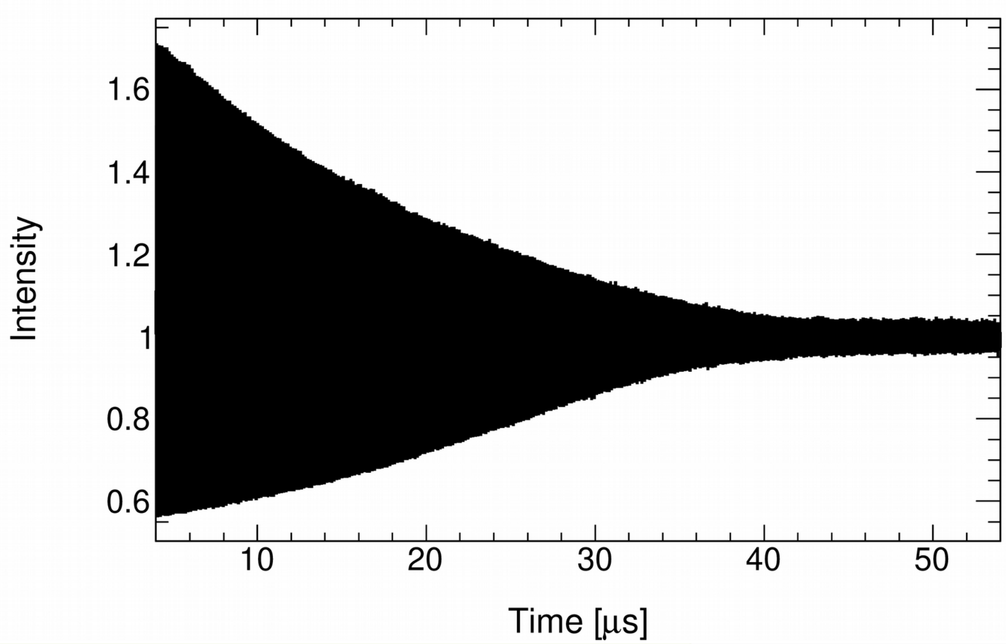
	truth	Fourier	χ^2
x_e [mm]	5.80	5.80	5.83
σ [mm]	9.87	9.89	9.76

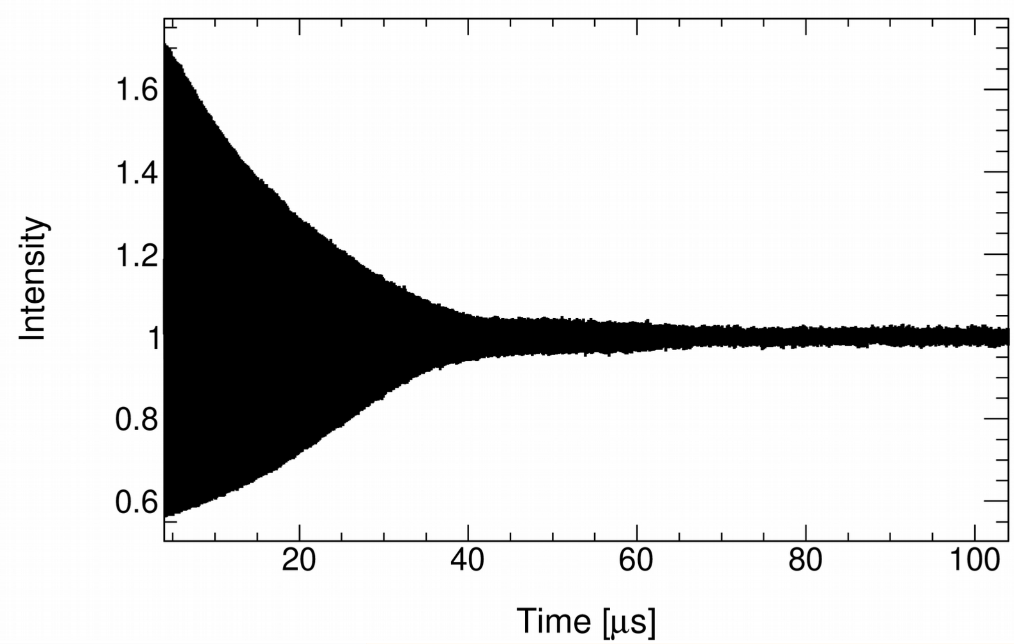
Performance study with BMAD simulation

60–hour like simulation: beam bells/whistles, no inflector injection/detector effects



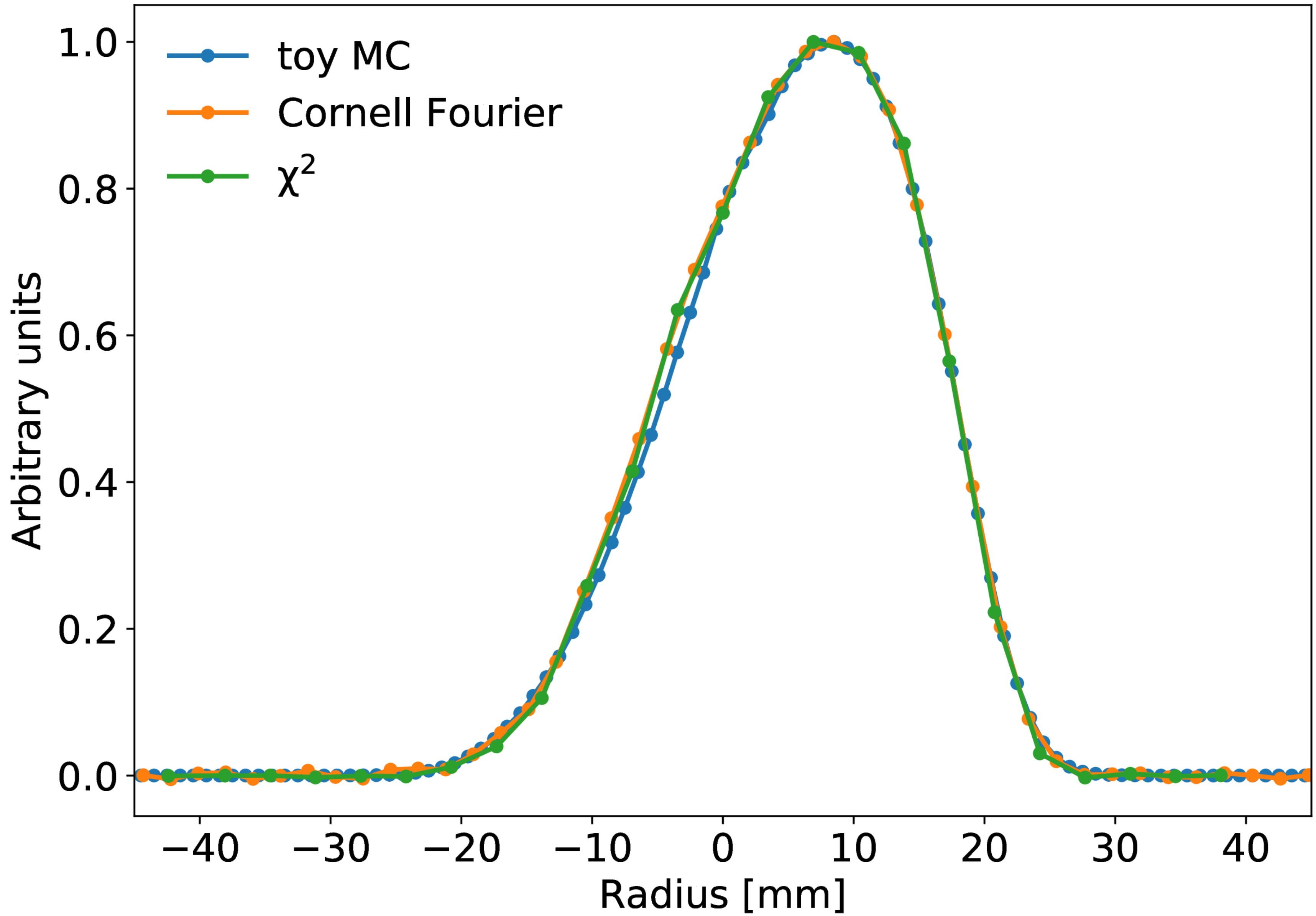




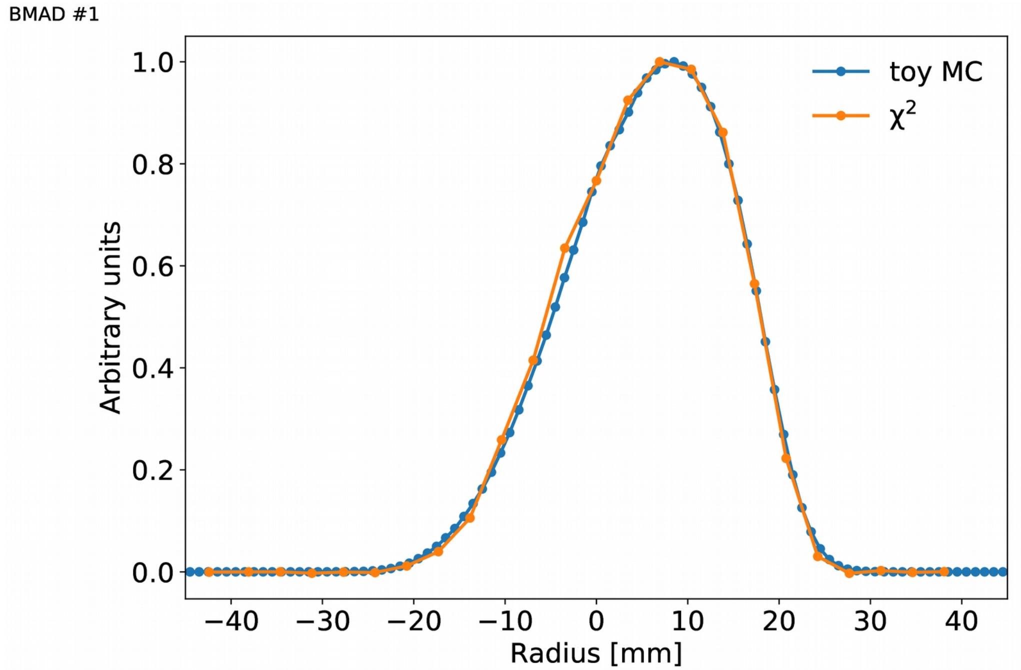


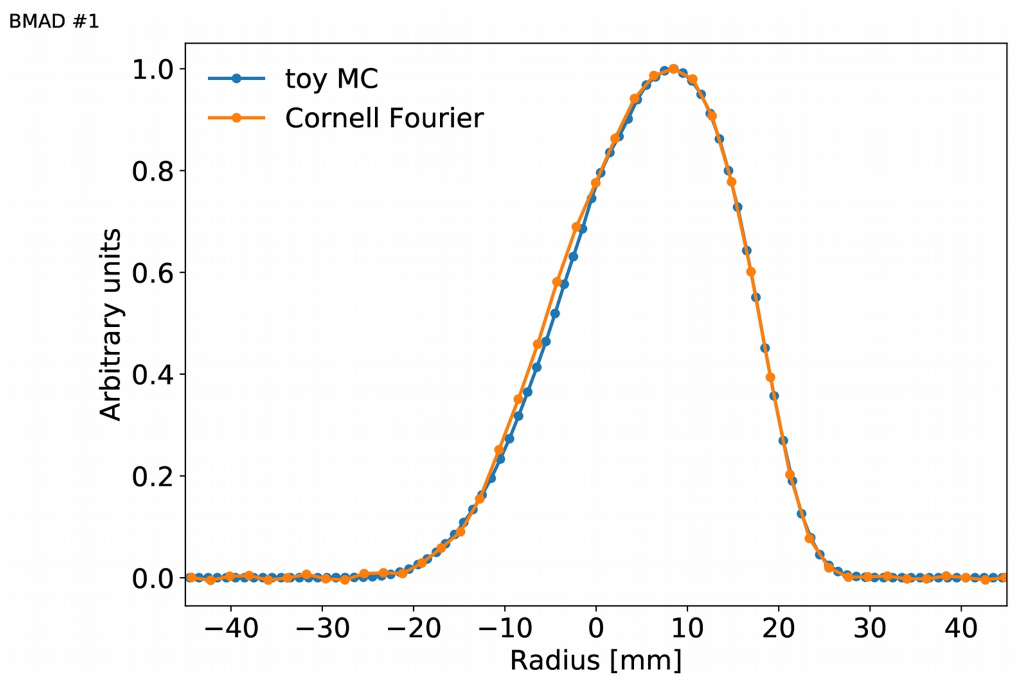
Performance study with BMAD simulation

BMAD #1



Performance study with BMAD simulation





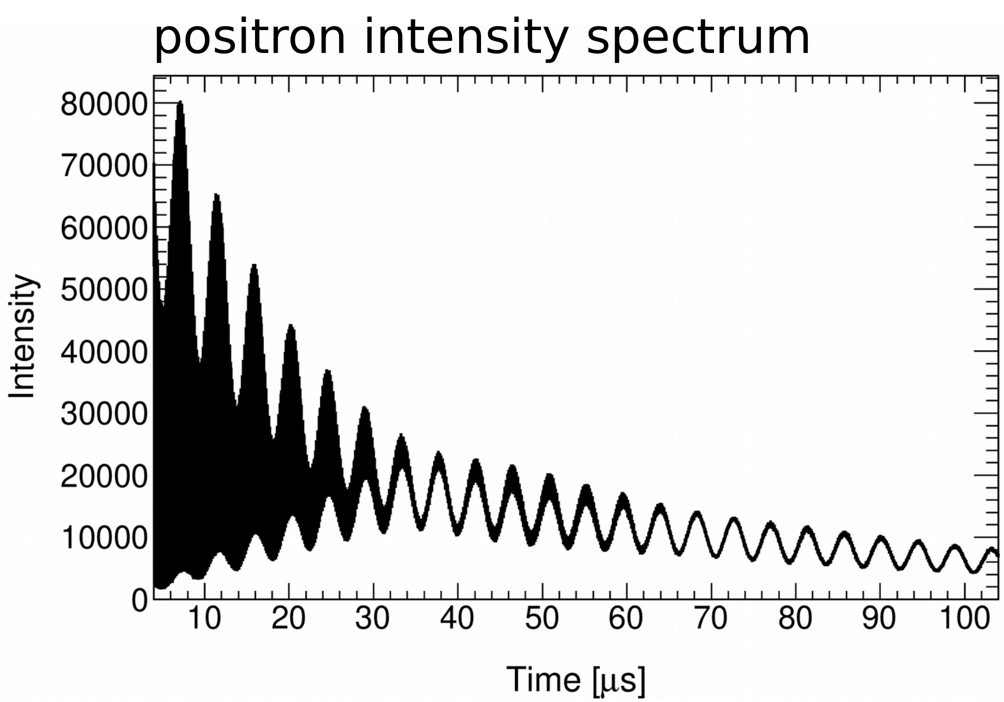
	truth	Fourier	χ^2
x_e [mm]	5.62	5.37	5.49
σ [mm]	8.80	8.80	8.53
C_E [ppb]	-413	-403	-390
δC_E [ppb]	–	10	23

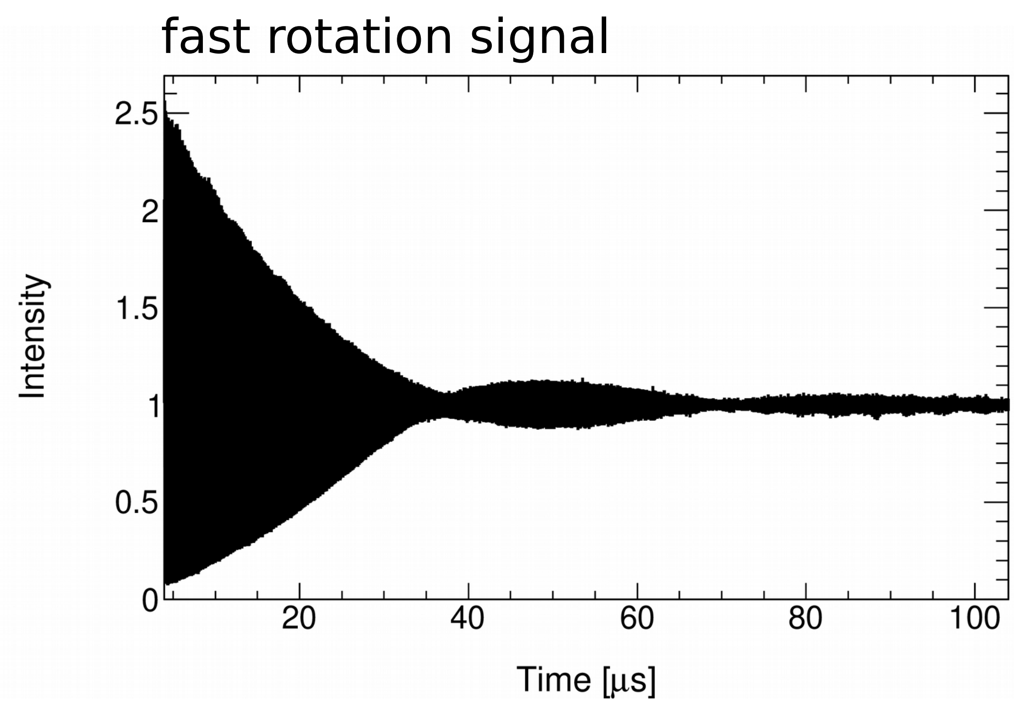
Producing the fast rotation signal: Cornell Fourier method

Apply > 1.5 GeV energy cut and time-align all calorimeters

Perform 9-parameter fit: τ_μ, ω_a, CBO

Correct positron intensity spectrum by 9-parameter fit to factor out τ_μ, ω_a, CBO

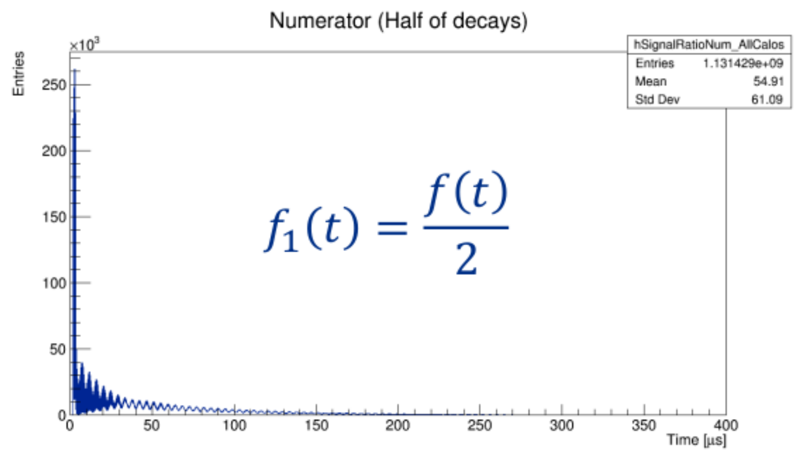




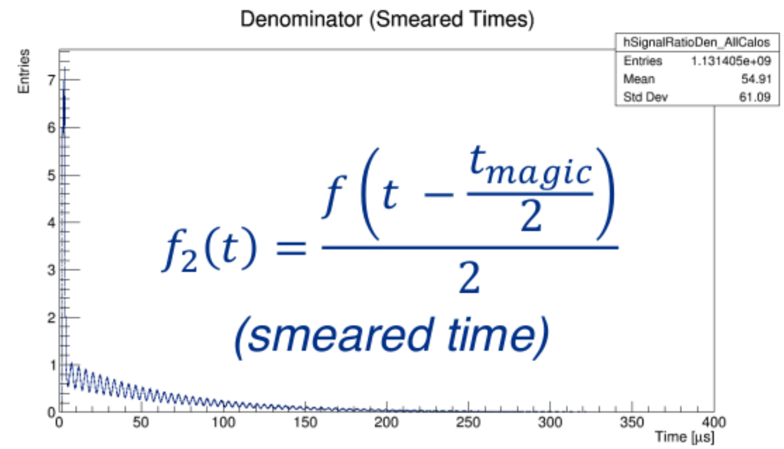
Producing the fast rotation signal: χ^2 method

Apply > 1.5 GeV energy cut and time-align all calorimeters

Split decay positron spectrum into 2:



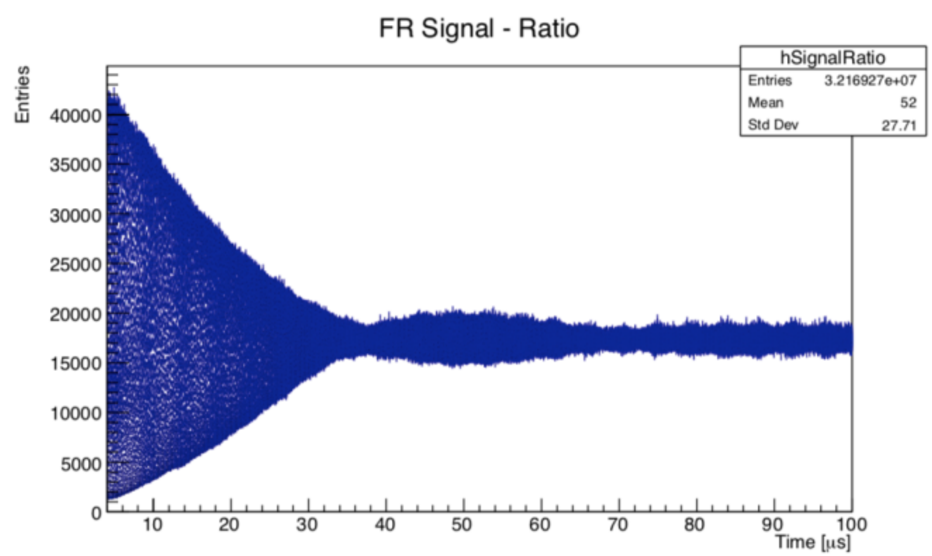
$$f_1(t) = \frac{f(t)}{2}$$



$$f_2(t) = \frac{f\left(t - \frac{t_{magic}}{2}\right)}{2}$$

(smeared time)

And divide to remove “background” signals $\rightarrow \omega_a, \tau_\mu$, CBO... :



Run–1 data set

Run–1 data set

60–hour

- `gm2pro_daq_full_run1_60h_5039A_goldList`
- field index of 0.108

9–day

- `gm2pro_daq_full_run1_9d_5039A_goldList`
- field index of 0.12

End game

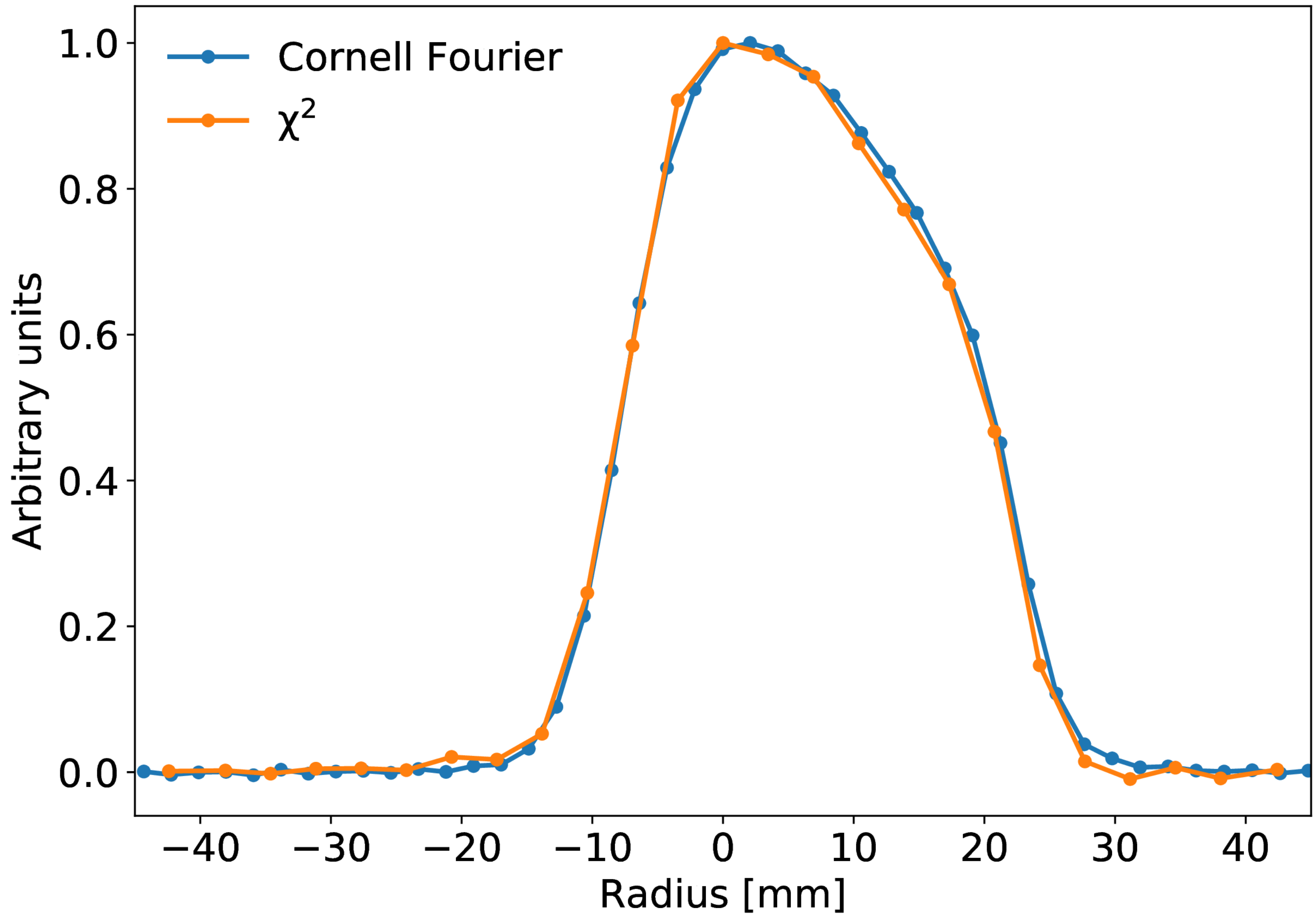
- `gm2pro_daq_full_run1_EndGame_5039A_goldList`
- field index of 0.108

For all the data set:

- nominal positron energy threshold of 1.5 GeV
- use recon west data product
- fast rotation signal time range, Cornell Fourier: 4-300 μs, : χ^2: 4-100 μs

60–h data set

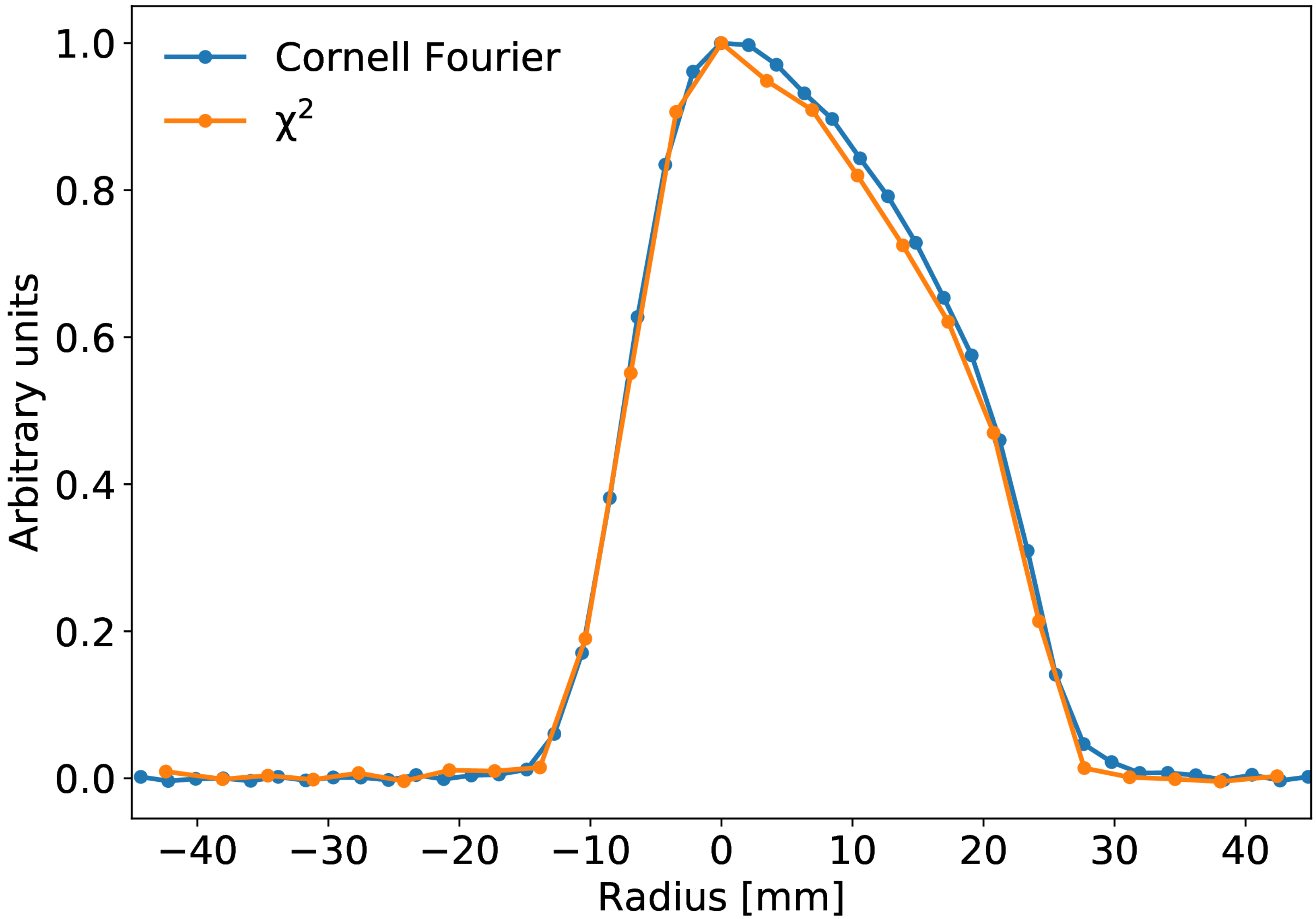
Run-1
60-hour



	x_e [mm]	σ [mm]	C_E [ppb]
Fourier	6.12	9.21	-465
χ^2	5.57	9.20	-440

9–day data set

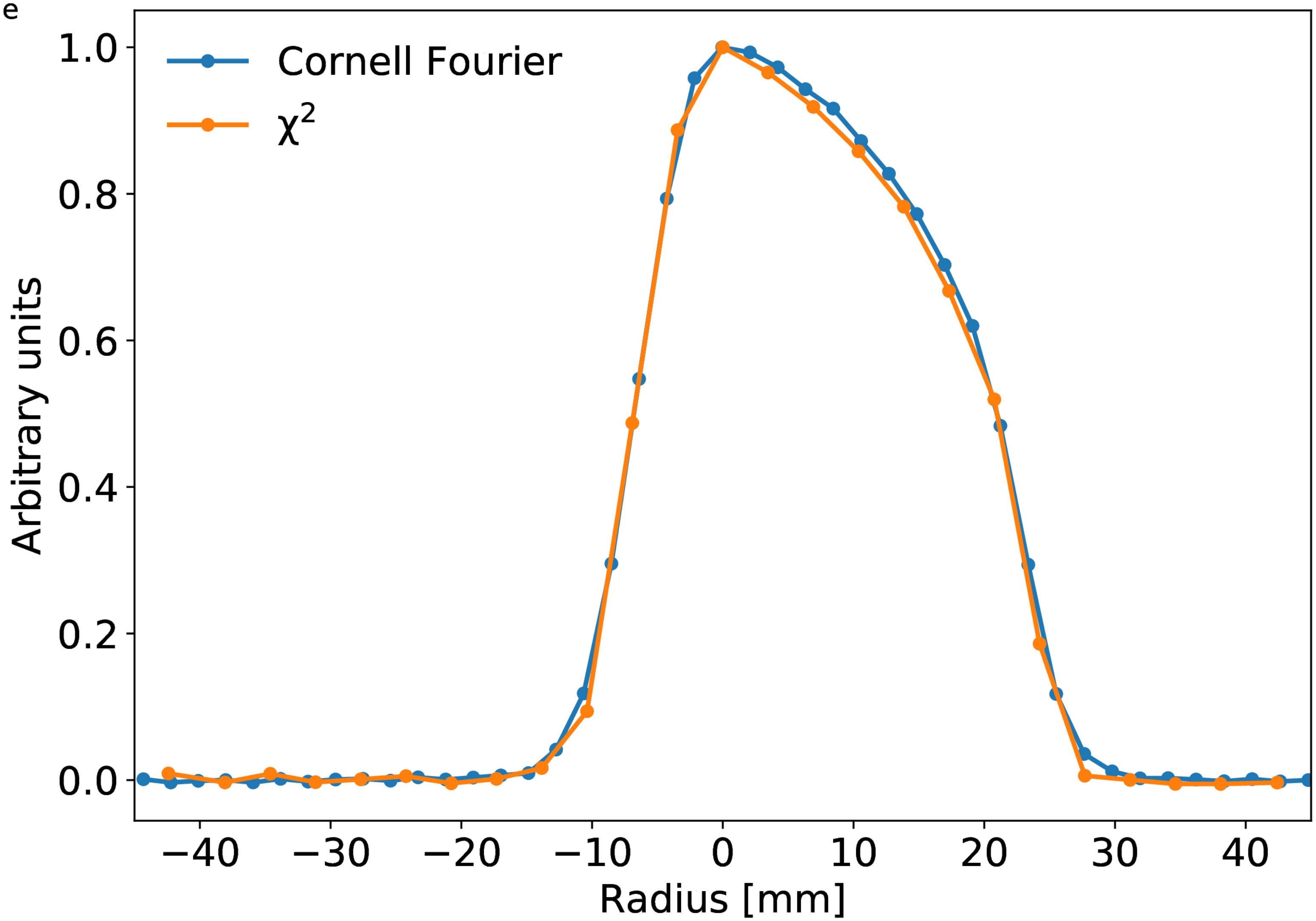
Run-1
9-day



	x_e [mm]	σ [mm]	C_E [ppb]
Fourier	6.35	9.19	-520
χ^2	5.95	9.24	-503

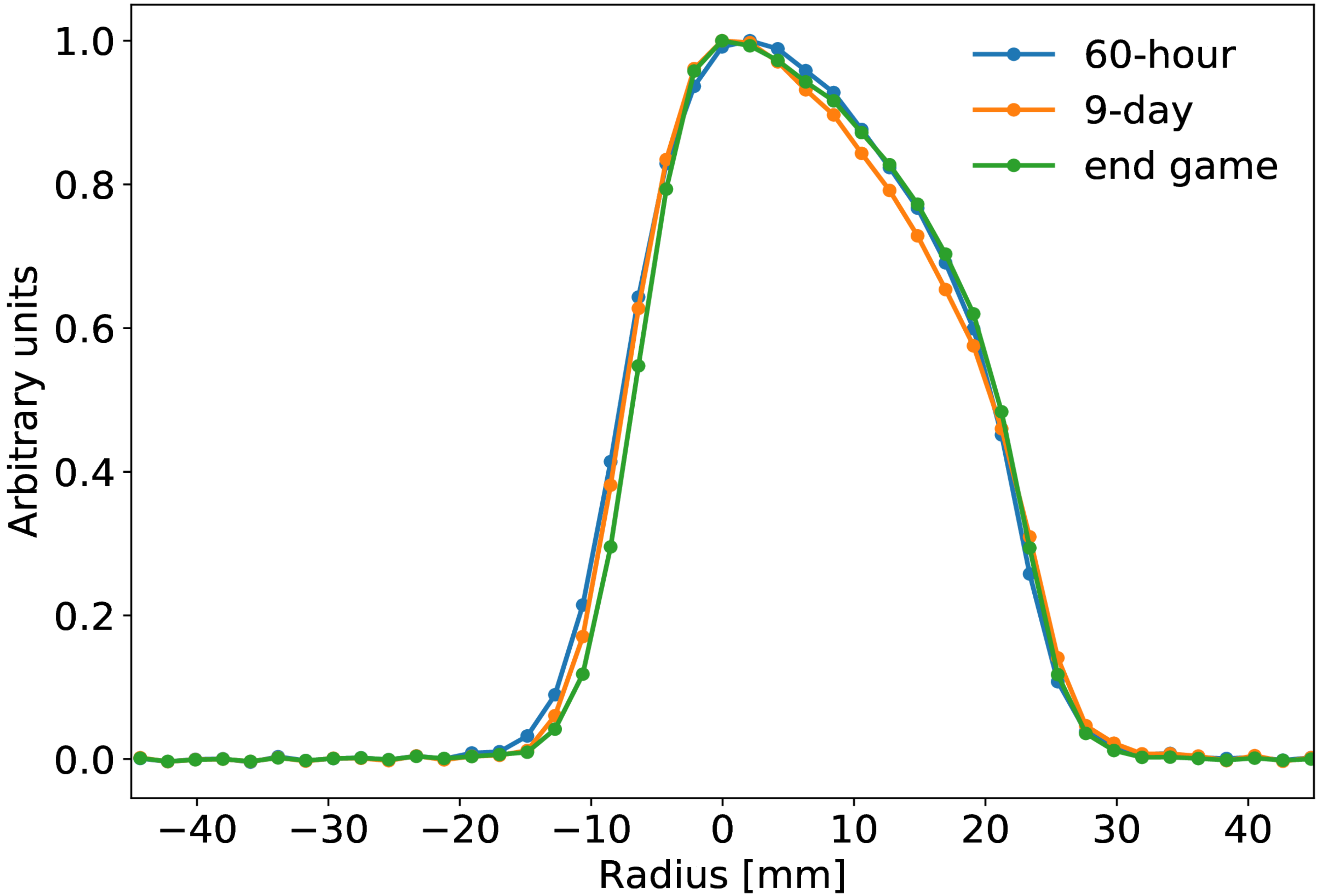
End game data set

Run-1
end game

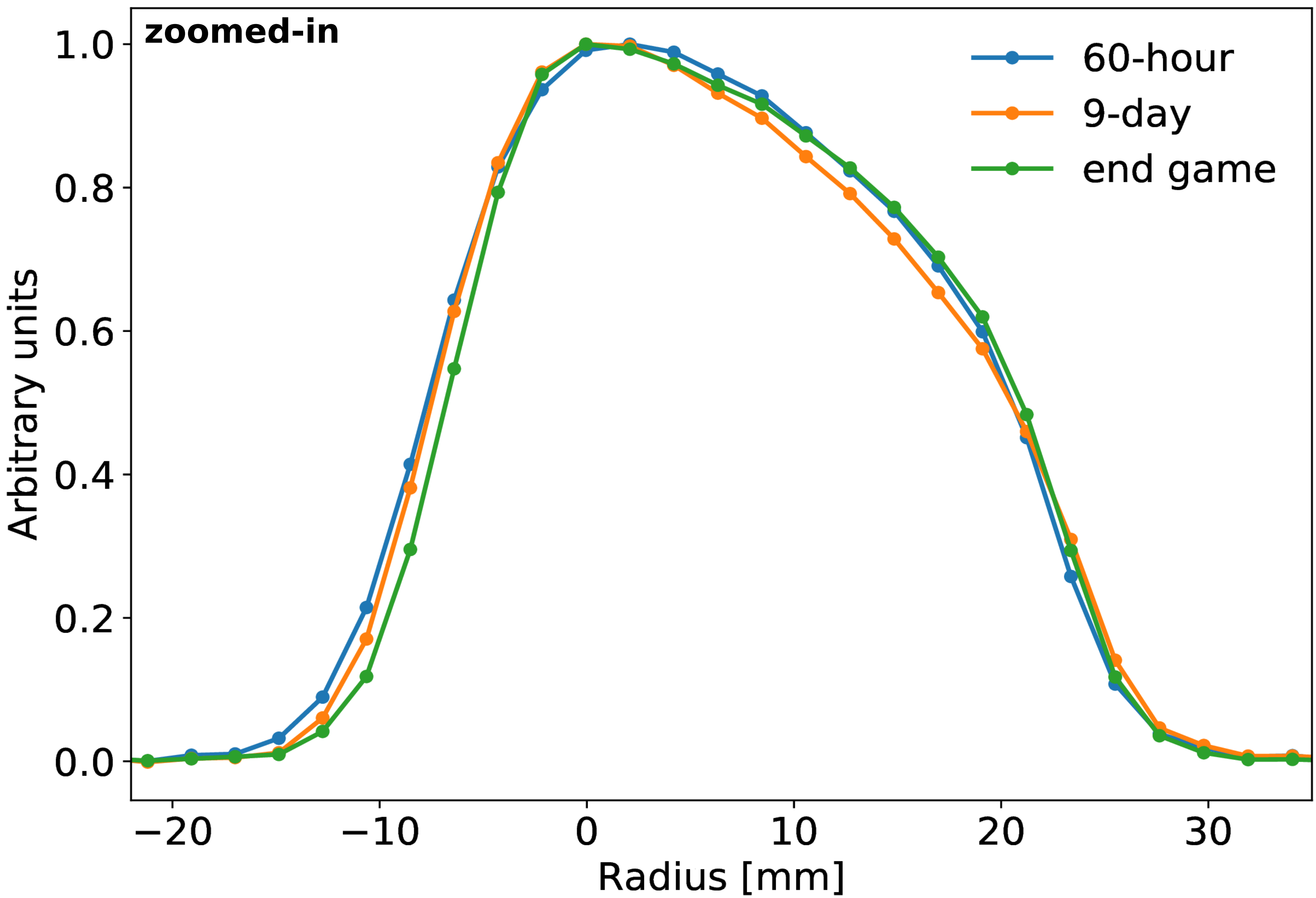


	x_e [mm]	σ [mm]	C_E [ppb]
Fourier	6.71	8.90	-473
χ^2	6.46	8.84	-456

Overlaid data set (Fourier)

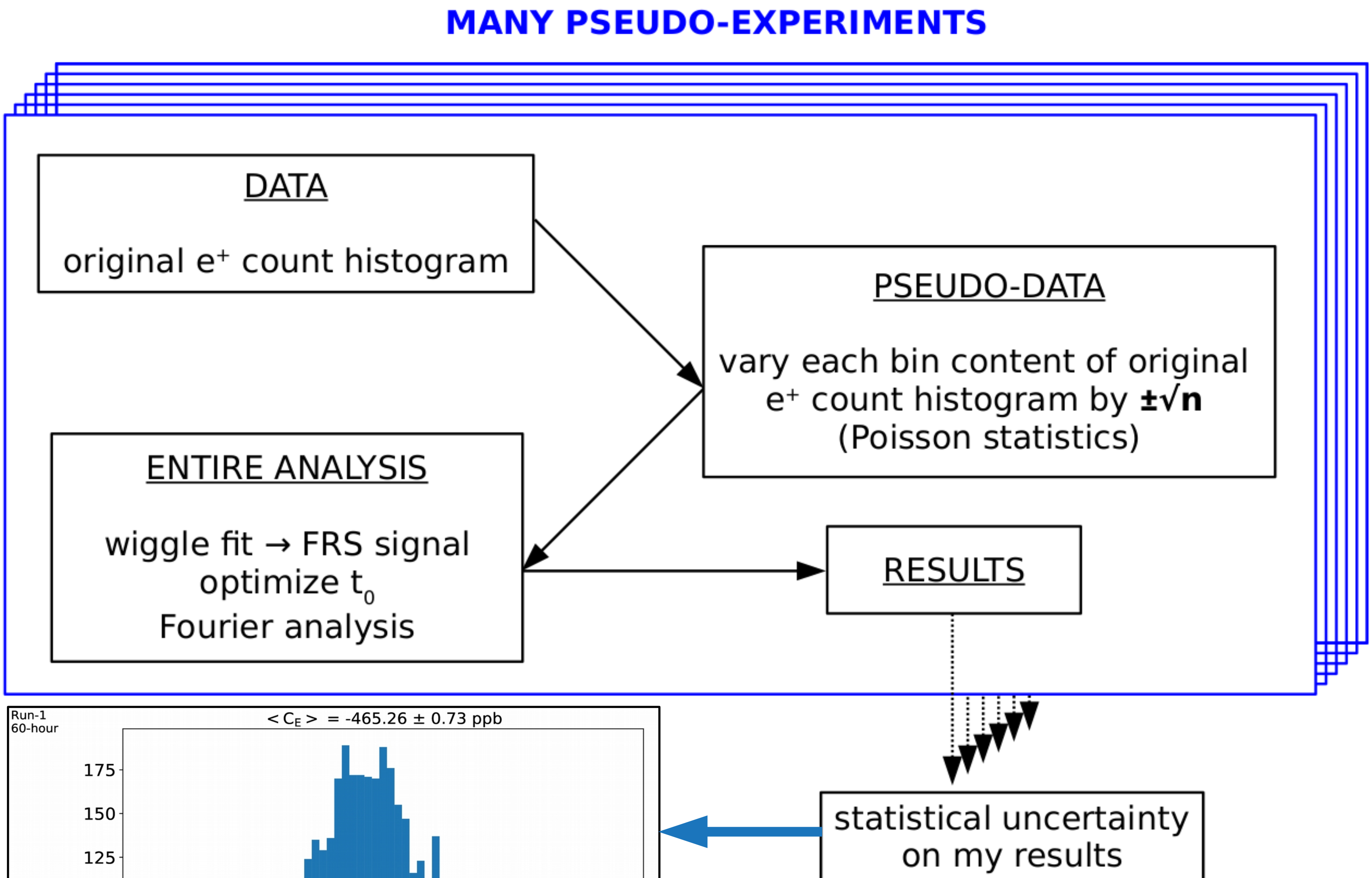


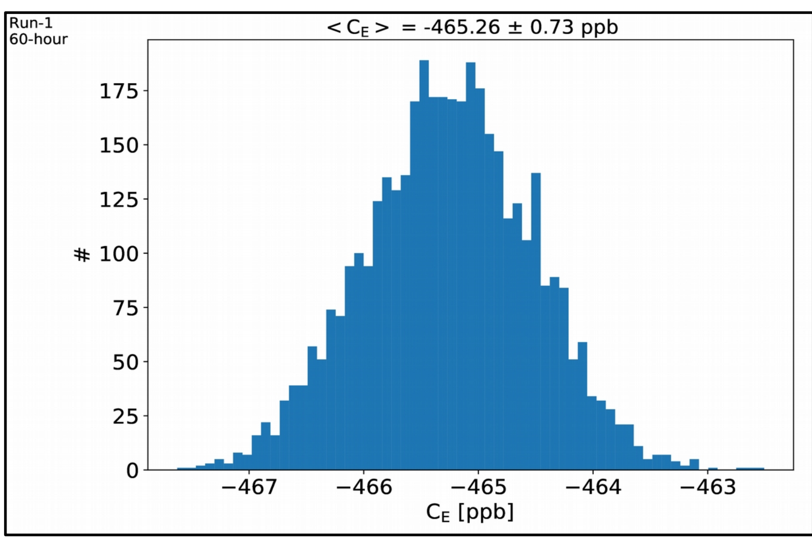
Overlaid data set (Fourier)



Statistical uncertainty

Statistical uncertainty: Cornell Fourier pseudo-experiments





Statistical uncertainty: Fourier error propagation

Kim Siang: docdb 17890, docdb 17773

Estimating statistical uncertainty using time bin uncertainty

First order frequency distribution

For the first order frequency distribution F_i, the variance of a single frequency bin $\sigma_{F_i}^2$ can be determined by considering its dependence on each of the time bins, which have independent statistical fluctuations:

$$\sigma_{F_i}^2 = \sum_j \sigma_{T_j}^2 \left(\frac{\partial F_i}{\partial T_j}\right)^2 . \tag{6}$$

Here, σ_{T_j} can be related to the original positron time histogram P_j and 5-parameter fit function $N(t)$ using

$$\sigma_{T_j} \sim \left(\frac{\sigma_{P_j}}{N(t_j)}\right) , \tag{7}$$

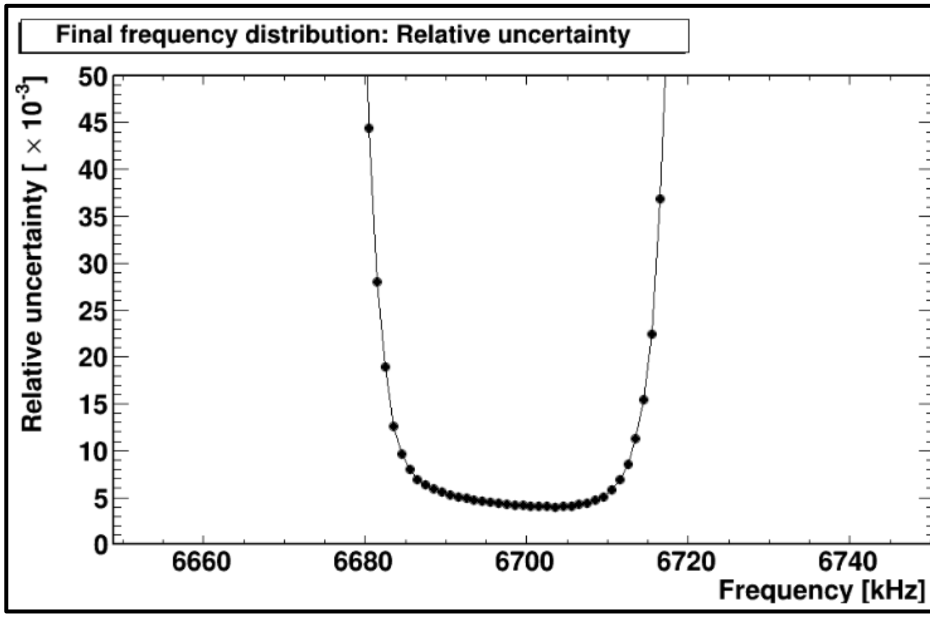
with a sufficiently precise fit of $N(t)$ to P_j. The derivative in Eq. 6 can be calculated as

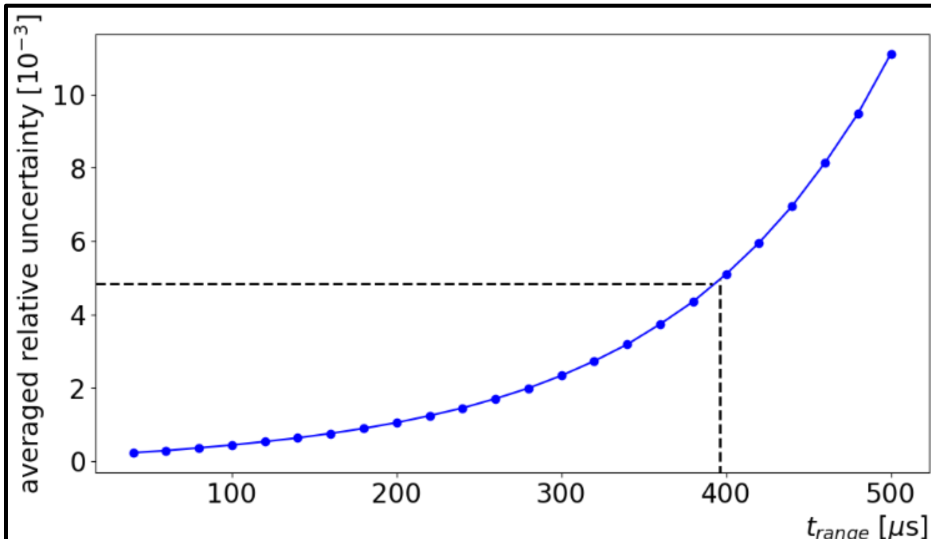
$$\begin{aligned}\frac{\partial F_i}{\partial T_j} &= \sum_k \delta_{kj} \cdot \cos\left[2\pi f_i(t_k - t_0)\right] \cdot \Delta t \\ &= \cos\left[2\pi f_i(t_j - t_0)\right] \cdot \Delta t .\end{aligned} \tag{8}$$

Combining Eq. 6 and Eq. 8, we get

$$\sigma_{F_i}^2 = \sum_j \sigma_{T_j}^2 \cdot \Delta t^2 \cdot \left(\cos\left[2\pi f_i(t_j - t_0)\right]\right)^2 . \tag{9}$$

So each time bin T_j contributes to the uncertainty of each frequency bin σ_{F_i} with a weight given by $\cos\left[2\pi f_i(t_j - t_0)\right] \cdot \Delta t$. The uncertainty of each frequency bin calculated using method above for the 60h dataset is shown in Fig. 3.





Hence for a $C_E \sim 450\,\text{ppb}$ (60h), $\sigma_{C_E}(stat) \sim 4.4\,\text{ppb}$. A proper evaluation of Eq. 17 gives

$$\frac{\sigma_{C_E}}{C_E} \sim 10^{-3} \text{ or } \sigma_{C_E} \sim 0.45\,\text{ppb} .$$

60–hour statistical uncertainty on C_E

Let's compare the statistical uncertainties keeping in mind that the data used do not overlap 100%, i.e., different time ranges used

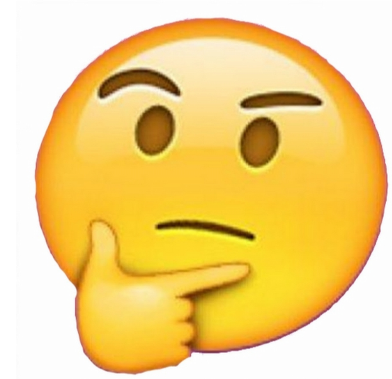
Fourier pseudo-experiments:	0.7 ppb
Kim Siang's error propagation:	0.5 ppb
χ^2 method:	3.2 ppb

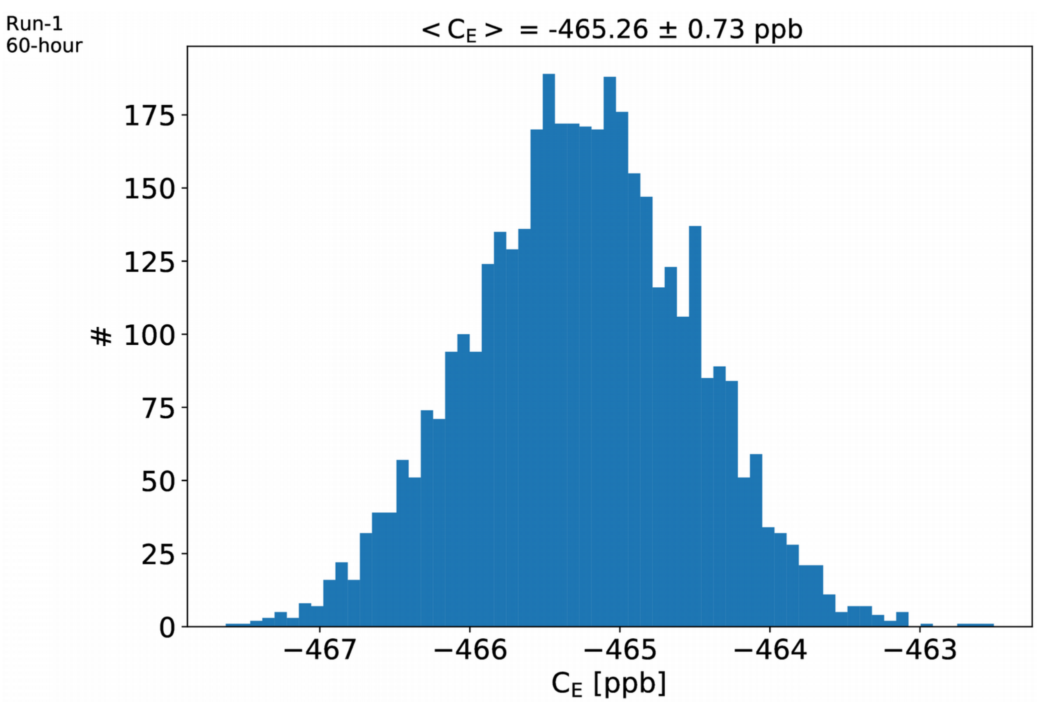
Close to each other and small enough not to care about the difference, but work on-going to improve estimations

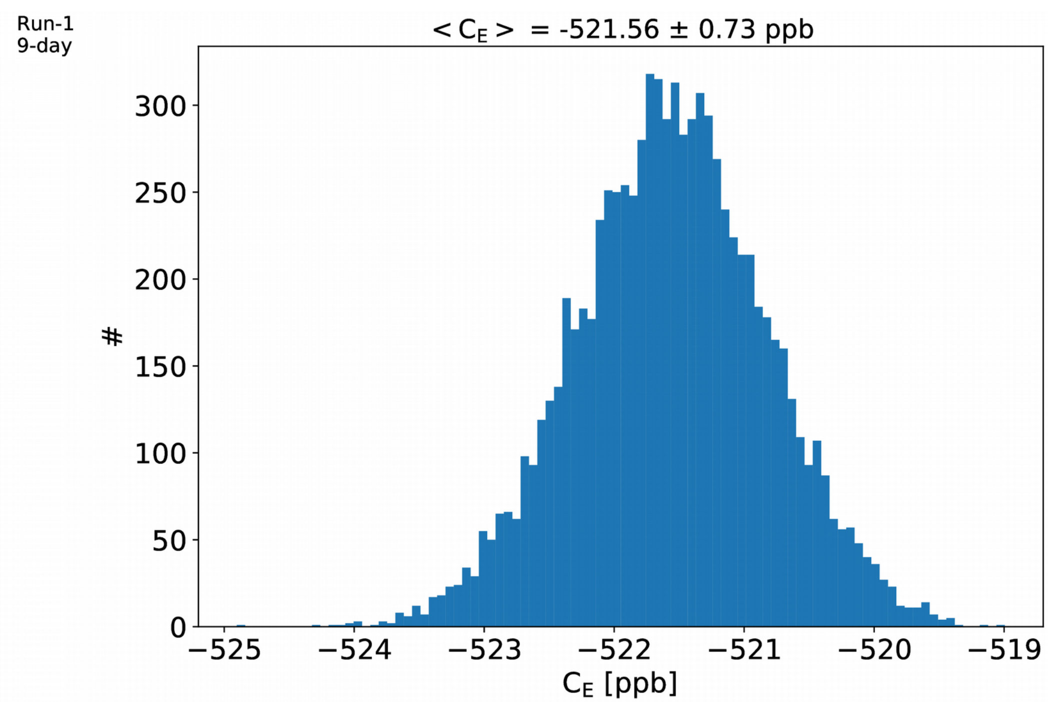
Statistical uncertainty: 60–hour vs 9–day

9–day data set has about twice the 60–hour data set statistics:

- expect a reduction in the statistical uncertainty by √2
- not what we observe... why?

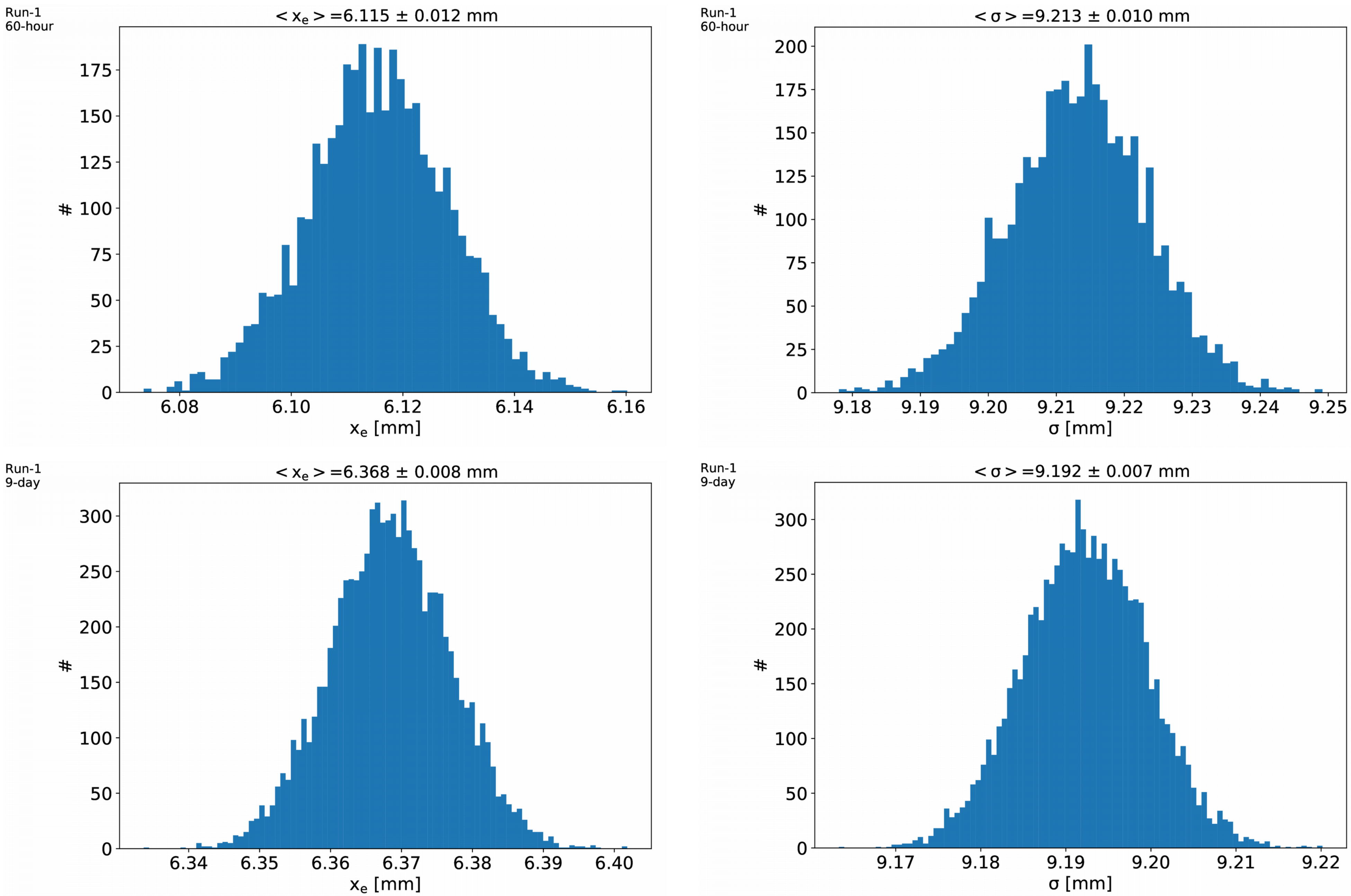






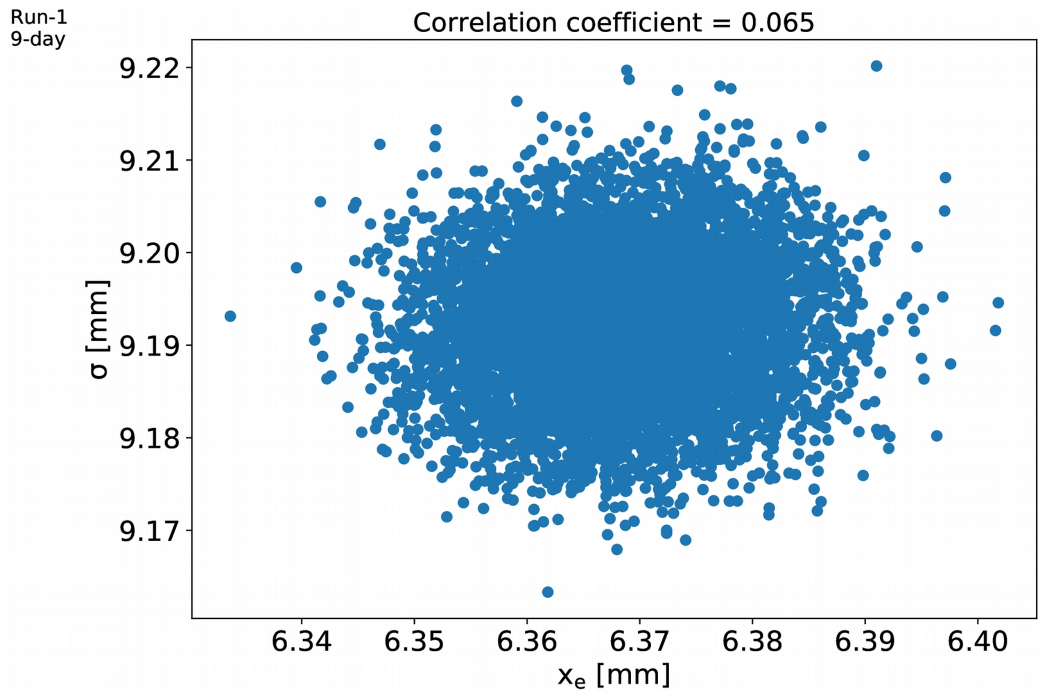
x_e and σ: 60–hour vs 9–day

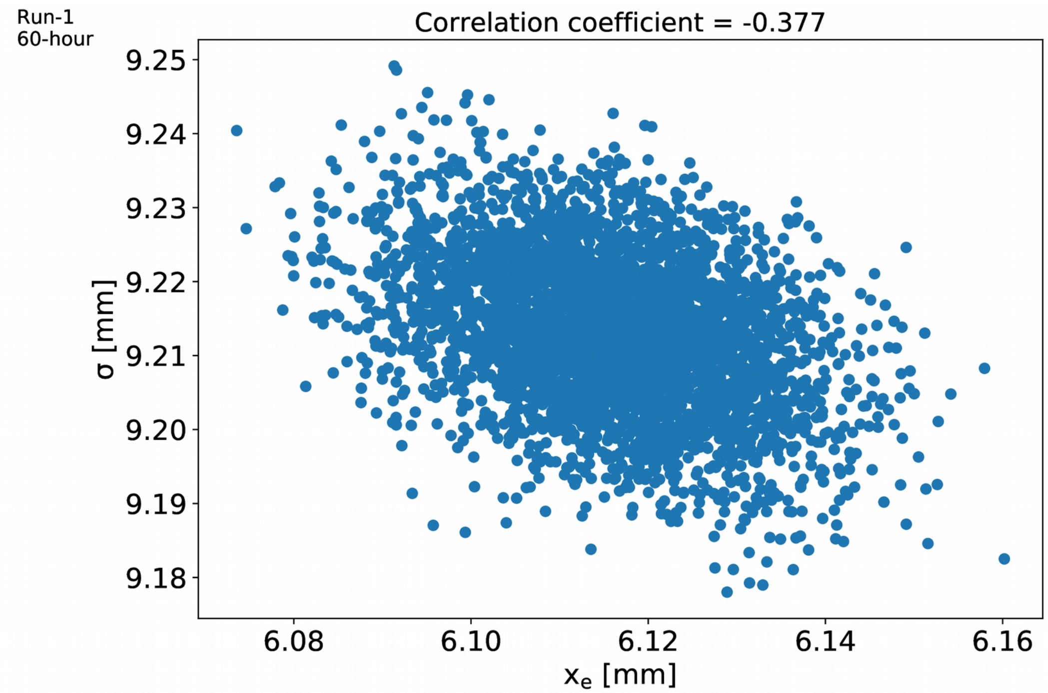
Proper scaling of the statistical uncertainty between the two data set



Correlation between x_e and σ

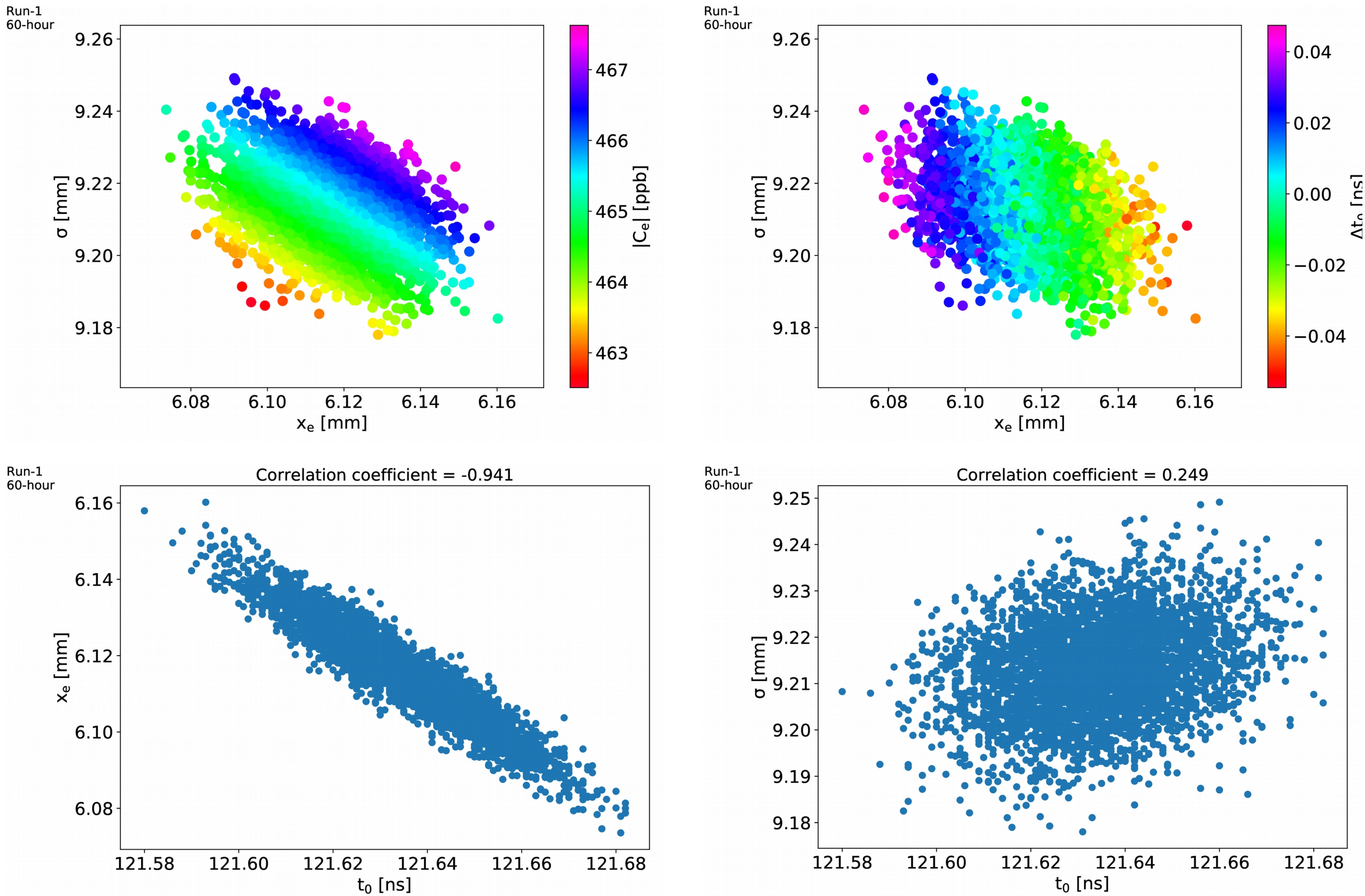
Blame their correlations!





The negative (positive) correlation in the 60–hour (9–day) data reduces (increases) the uncertainty on C_E. The amounts of correlation miraculously yield the same uncertainty on C_E for both data set...

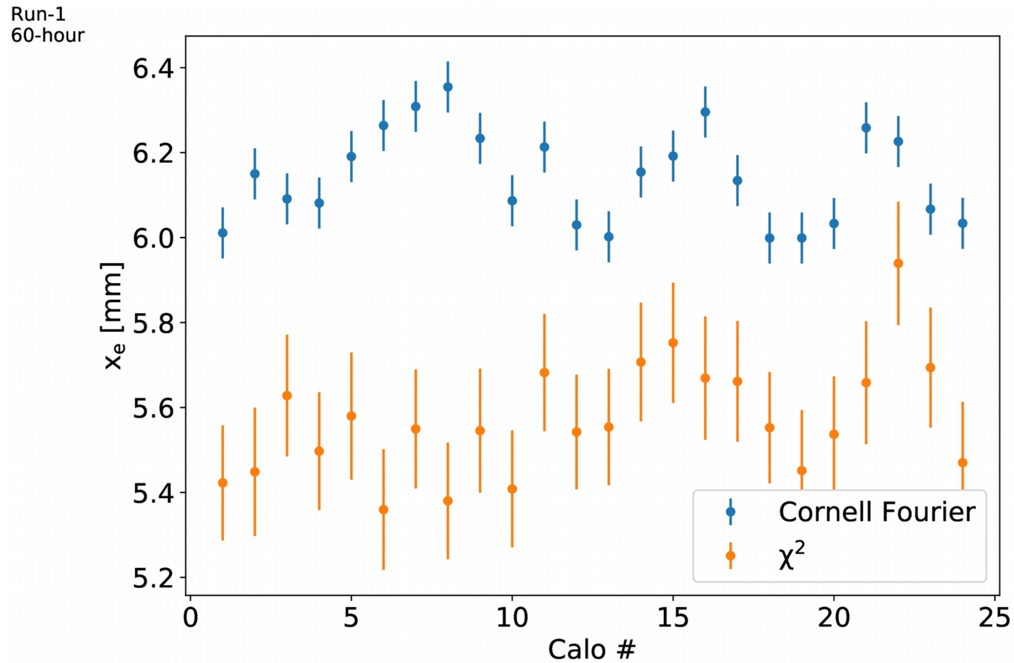
C_E, x_e, σ, and t_0

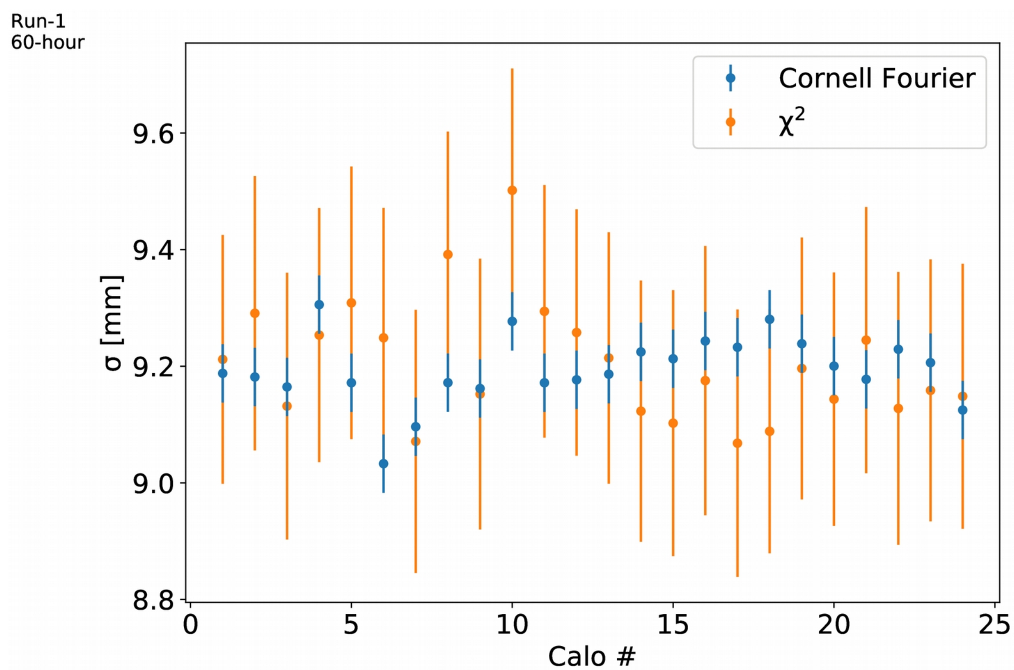


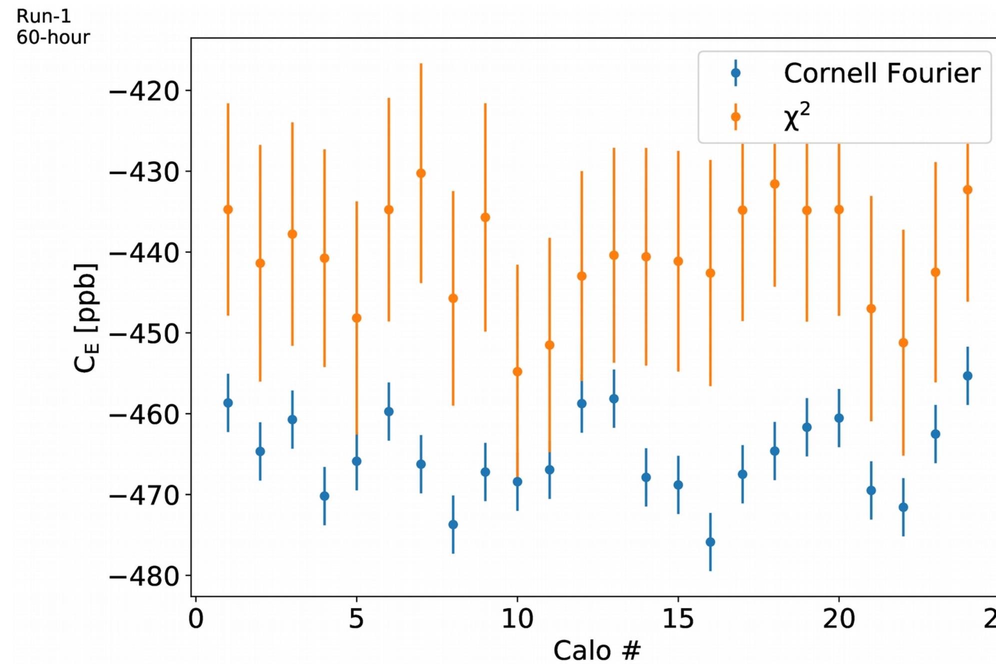
Run-1
60-hour
σ [mm]
9.26
9.24
9.22
9.20
9.18
6.08
6.10
6.12
6.14
6.16
x_e [mm]
$|C_e|$ [ppb]
467
466
465
464
463
Run-1
60-hour
σ [mm]
x_e [mm]
0.04
0.02
0.00
−0.02
−0.04
Δt_0 [ns]
Run-1
60-hour
Correlation coefficient = -0.941
x_e [mm]
t_0 [ns]
121.58
121.60
121.62
121.64
121.66
121.68
Run-1
60-hour
Correlation coefficient = 0.249
σ [mm]
9.25
9.24
9.23
9.22
9.21
9.20
9.19
9.18
t_0 [ns]

60–hour data set: calo-by-calo results

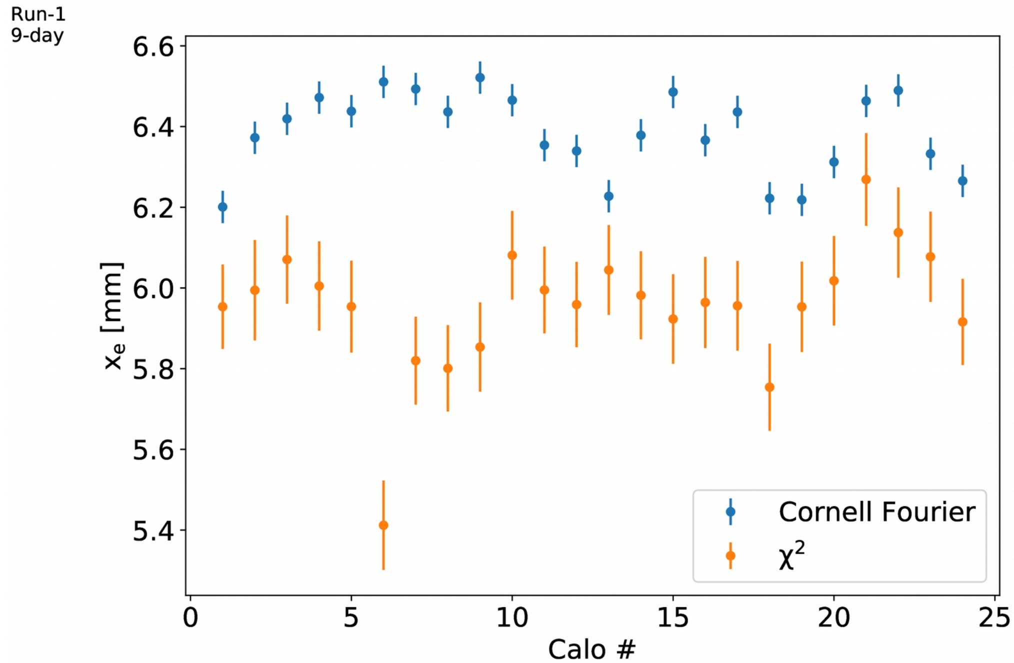
error bar = statistical uncertainty



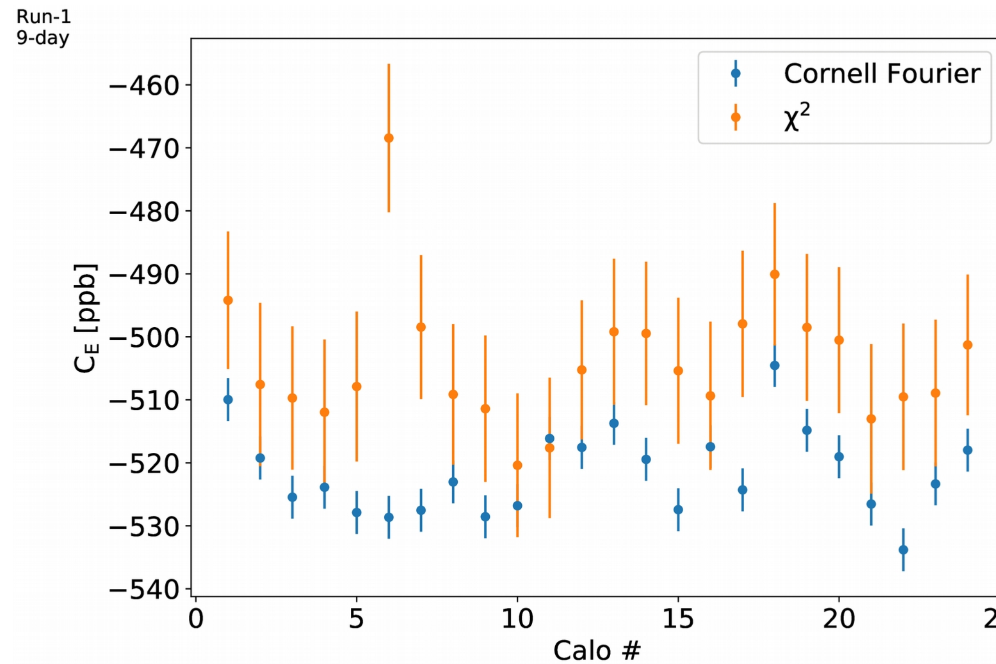


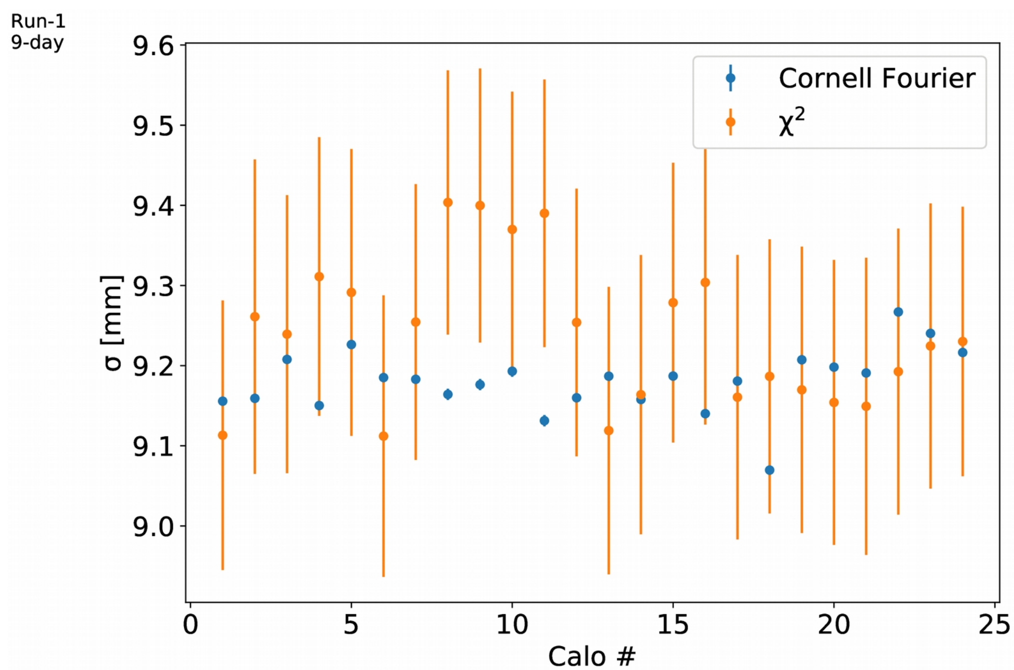


9–day data set: calo-by-calo results



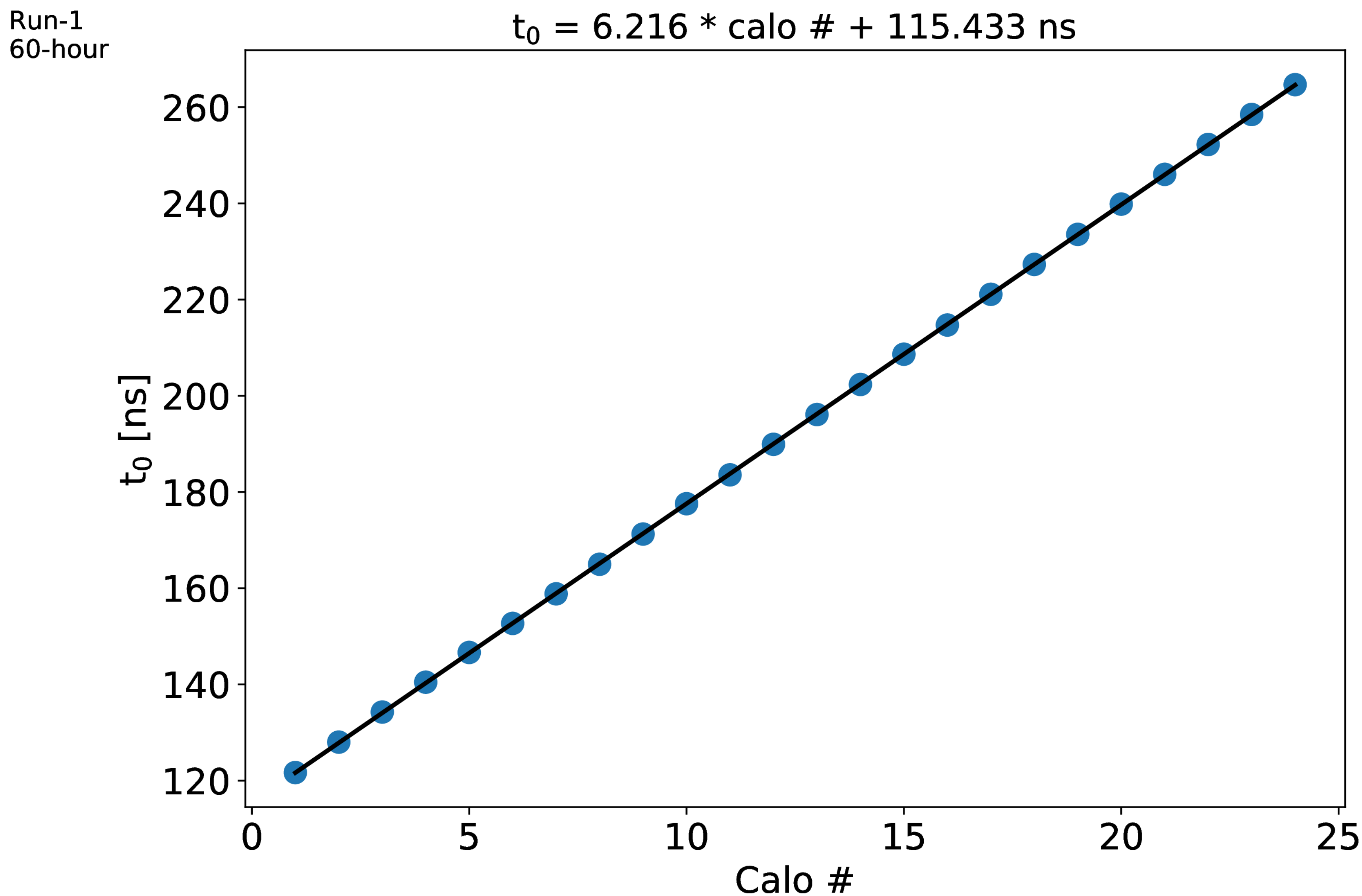
error bar = statistical uncertainty





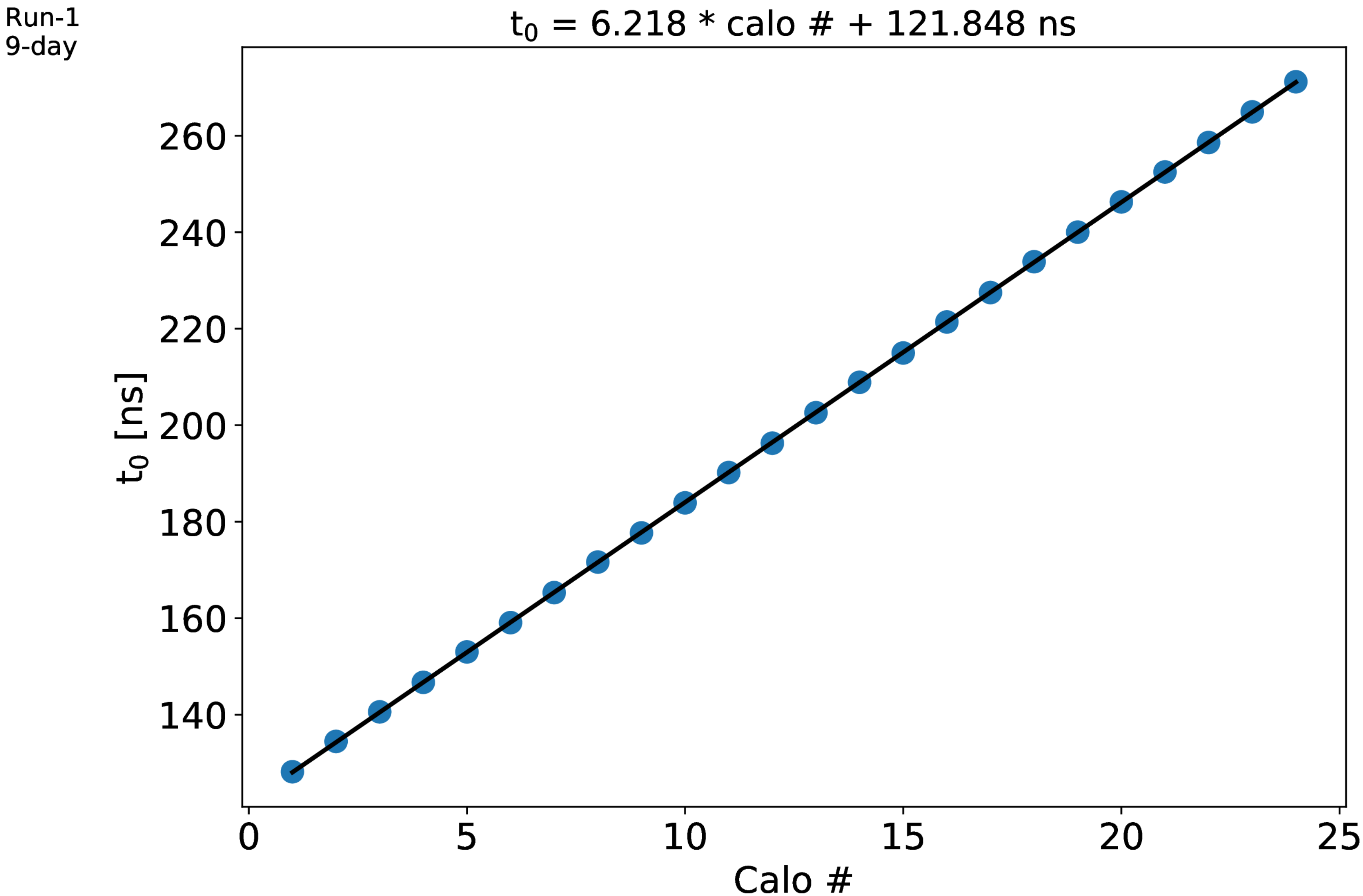
60–hour data set: calo-by-calo results

Time at which each calorimeter sees the beam for the first time (Cornell Fourier)

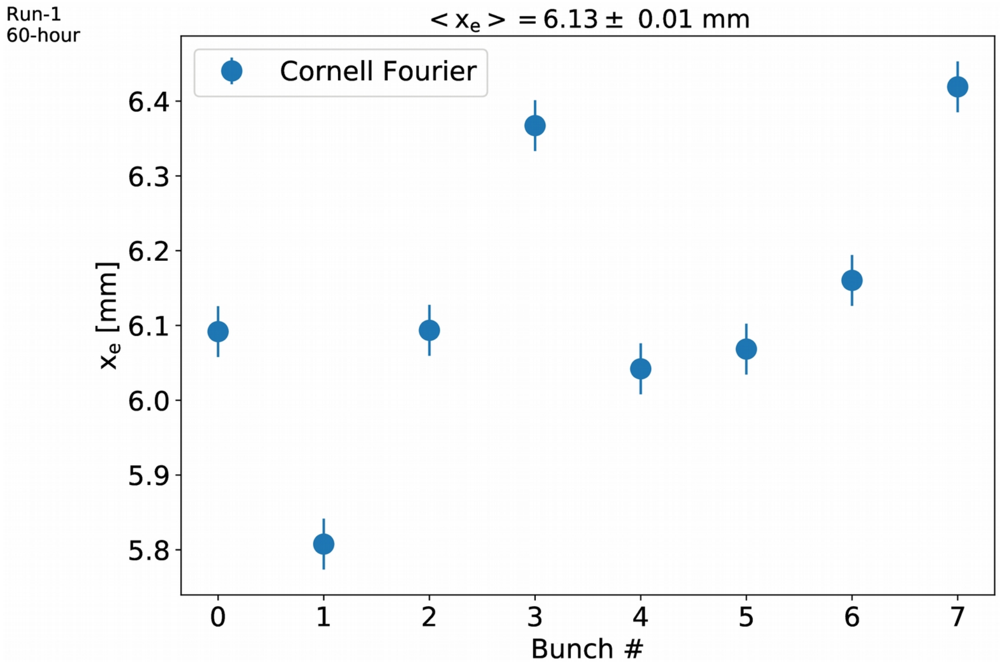


9–day data set: calo-by-calo results

Time at which each calorimeter sees the beam for the first time (Cornell Fourier)

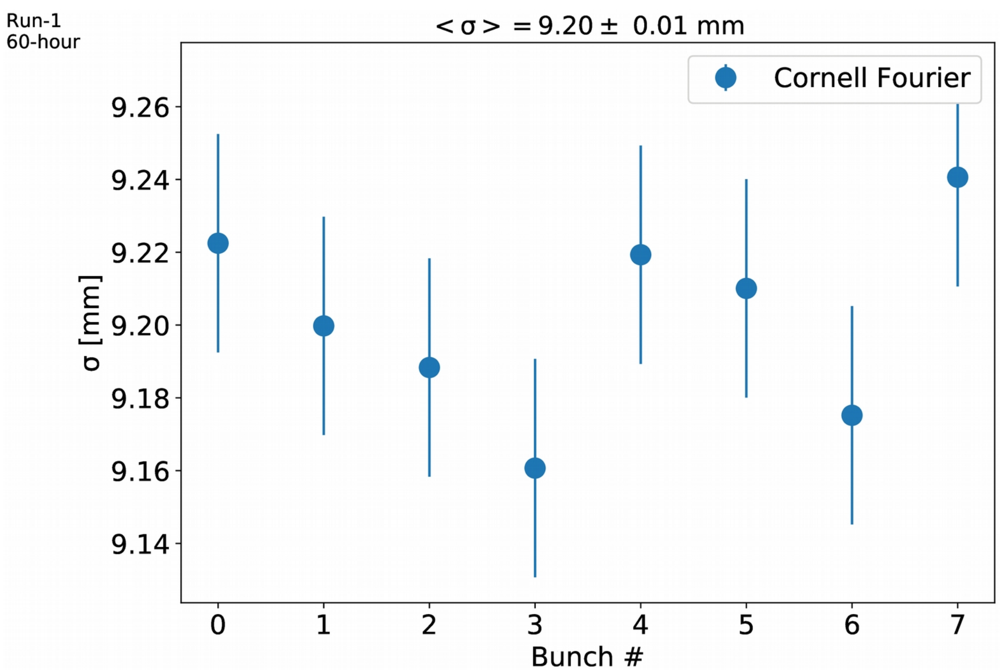


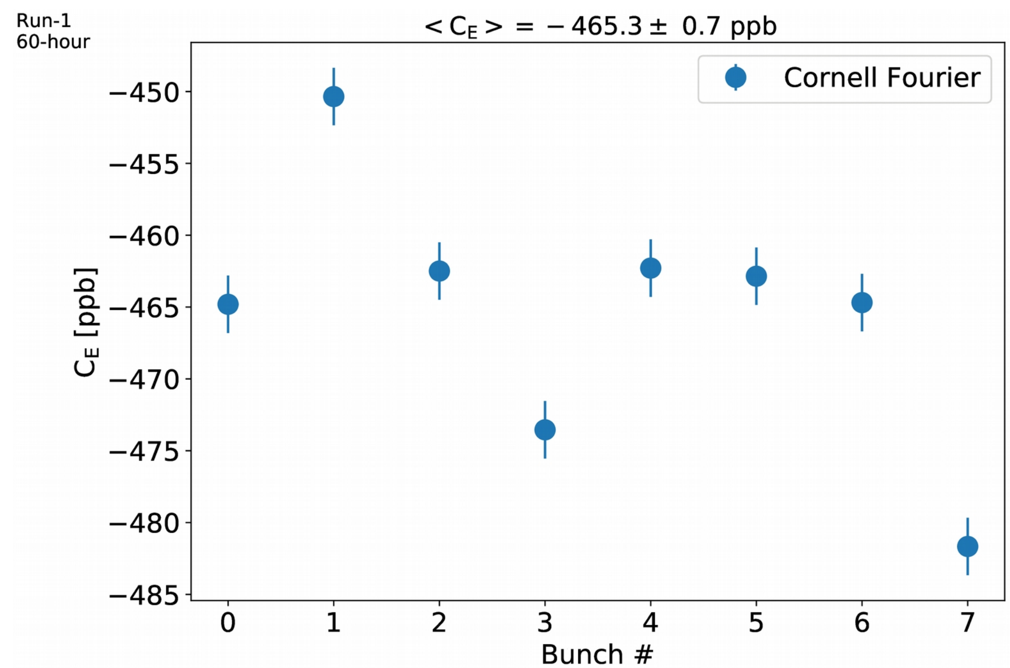
60–hour data set: bunch-by-bunch results



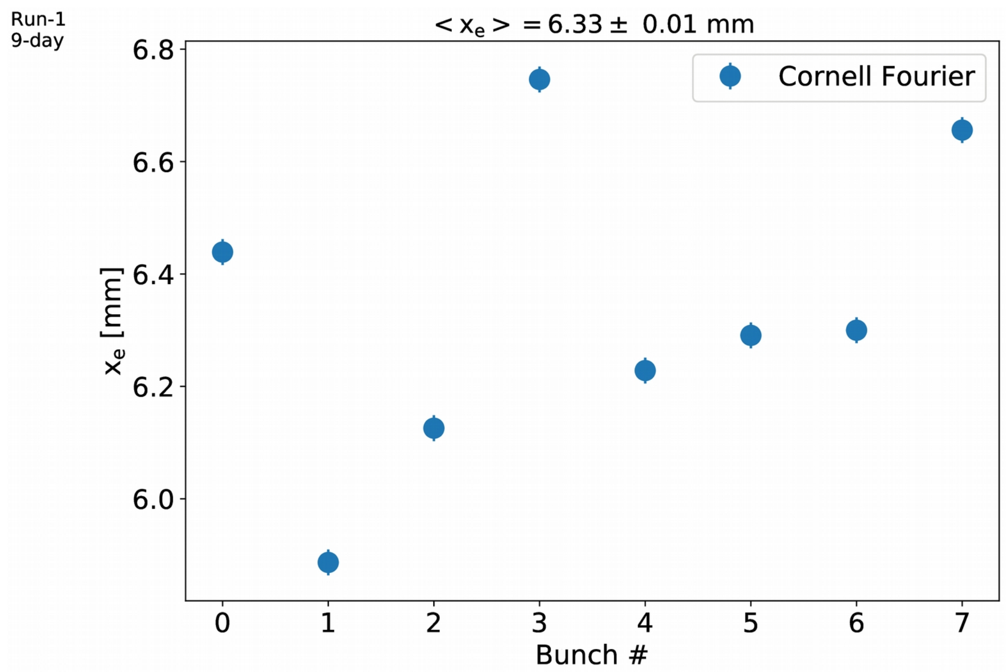
error bar = statistical uncertainty

Averaging results from the 8 bunches yield the same answer as merging the bunches before analysis: it all is linear



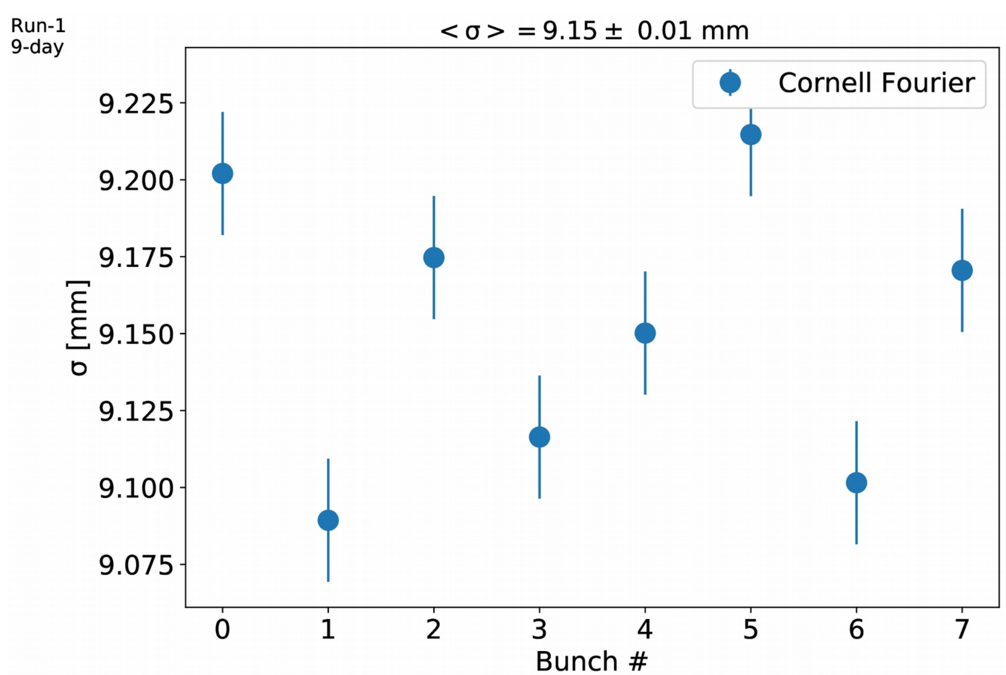


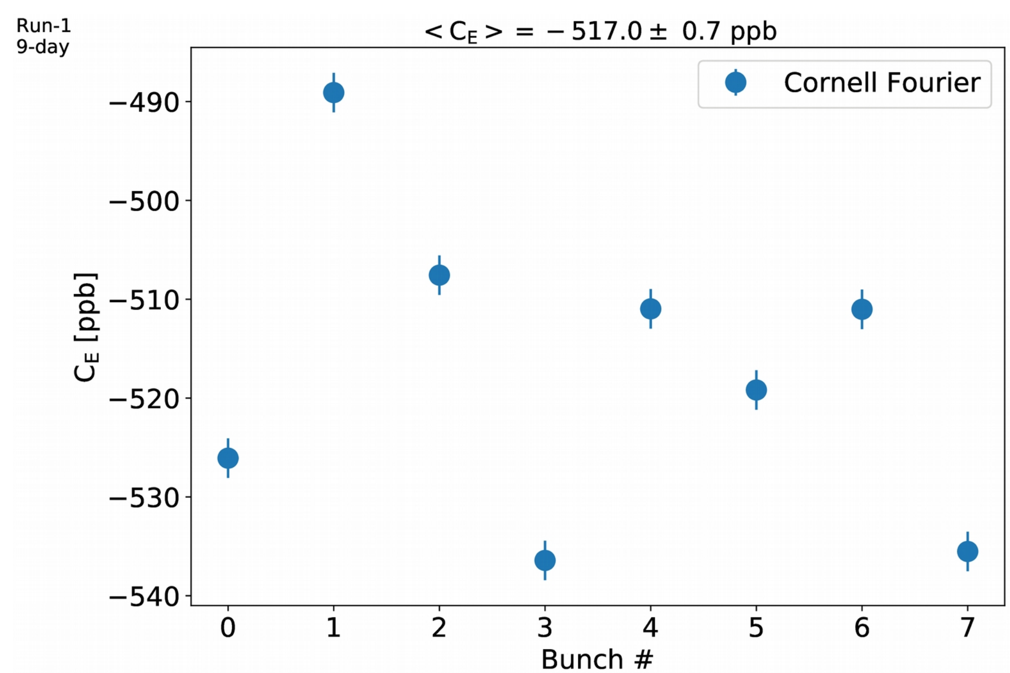
9–day data set: bunch-by-bunch results



error bar = statistical uncertainty

Averaging results from the 8 bunches yield the same answer as merging the bunches before analysis: it all is linear





Results comparison with statistical uncertainty

		x_e [mm]	σ [mm]	C_E [ppb]
60–hour	Fourier	6.12 ± 0.01	9.21 ± 0.01	-465 ± 1
	χ^2	5.57 ± 0.03	9.20 ± 0.05	-440 ± 3
9–day	Fourier	6.35 ± 0.01	9.19 ± 0.01	-520 ± 1
	χ^2	5.95 ± 0.03	9.24 ± 0.05	-503 ± 3
End game	Fourier	6.73 ± 0.01	8.90 ± 0.01	-473 ± 1
	χ^2	6.46 ± 0.02	8.84 ± 0.04	-456 ± 2

Systematic uncertainty

Systematic uncertainty

Statistical uncertainty appears to be small and under control → systematic uncertainty will be the name of the game

Cornell Fourier method:

- intrinsic systematic effects studied with simulated data (toy MC, BMAD)
- systematic uncertainties estimated for Run–1 data

χ^2 method:

- systematic uncertainties not estimated yet (people power issue) but scans available for Run–1 data

Cornell Fourier systematic uncertainties

The fun is in how well the parameters are optimized and how the results hold when varying them:

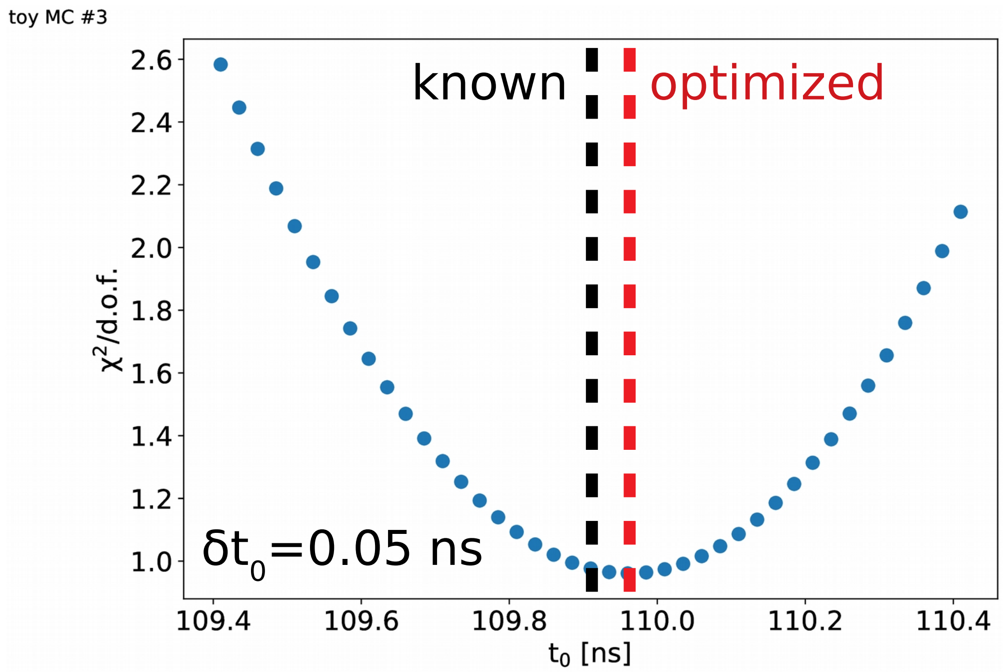
- t_0: beam passing the detector for the first time (first turn in the ring)
- t_s: start time of the fast rotation signal with respect to injection → effect from early in-fill time effect (scraping, gain...)
- t_m: end time of the fast rotation signal with respect to injection → trade off between adding noisier data at late time/improving the frequency resolution

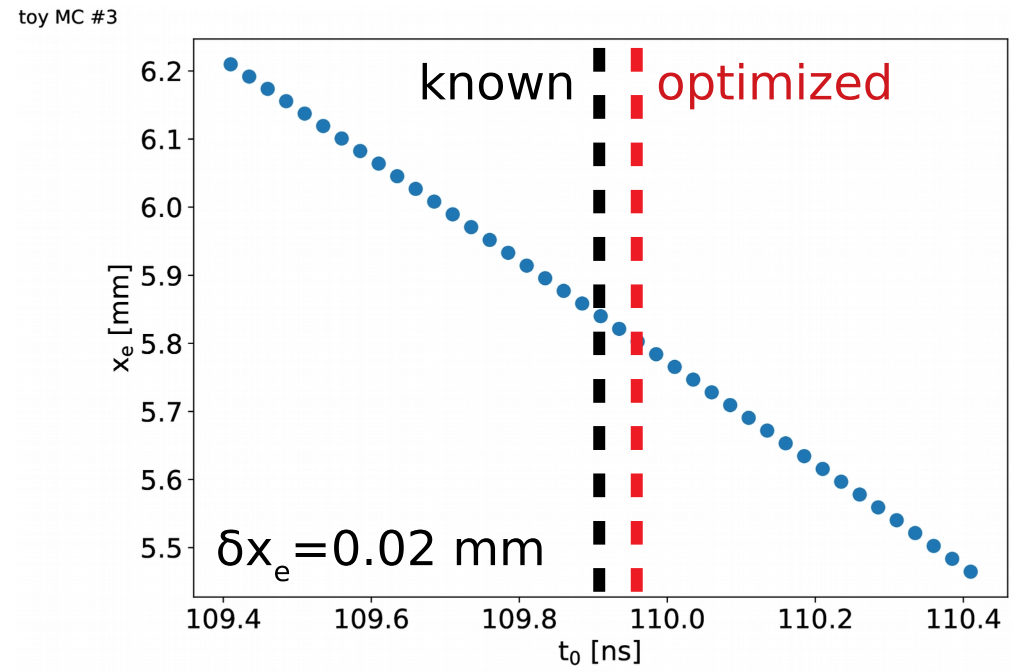
And how is the Fourier method implementation behaving:

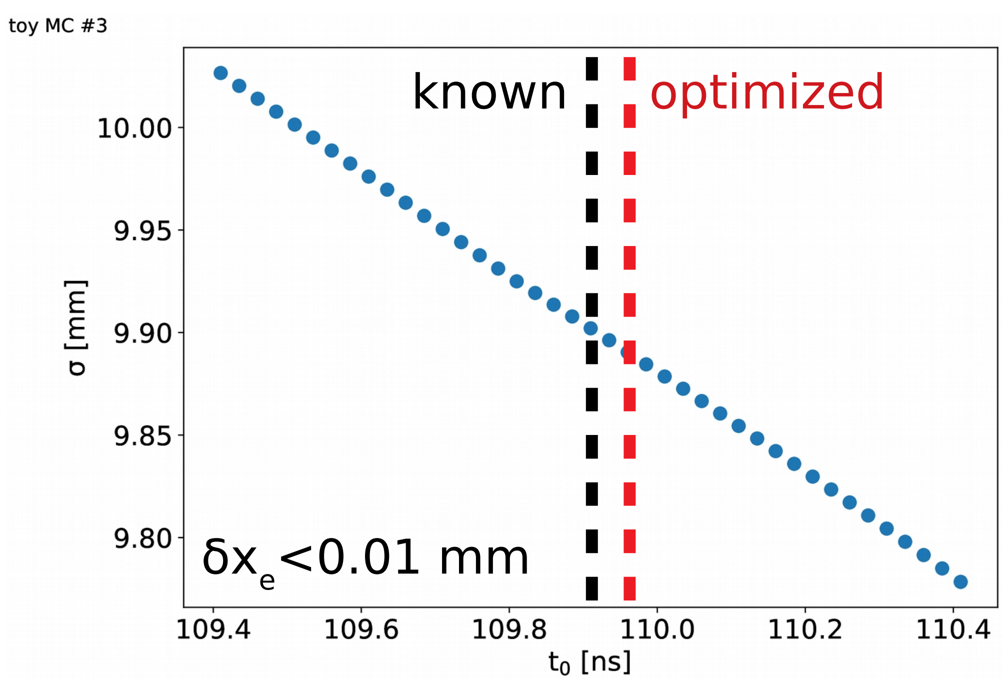
- choice of frequency resolution
- background correction
- background removal
- wiggle fit

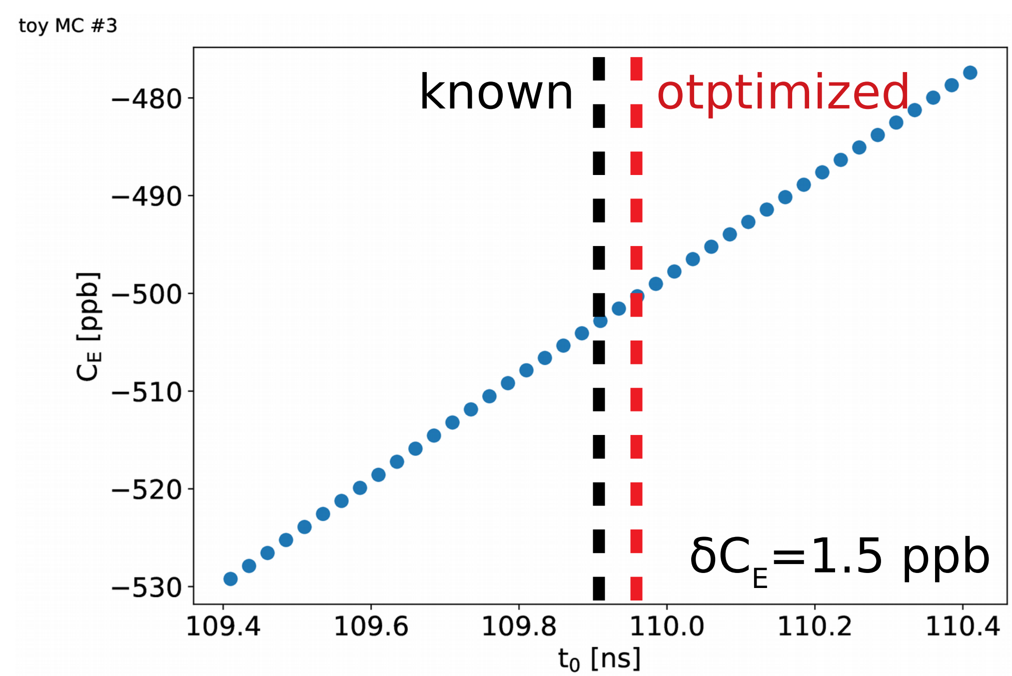
And what the data/beam have in stock

Fourier t_0 scan with simulation



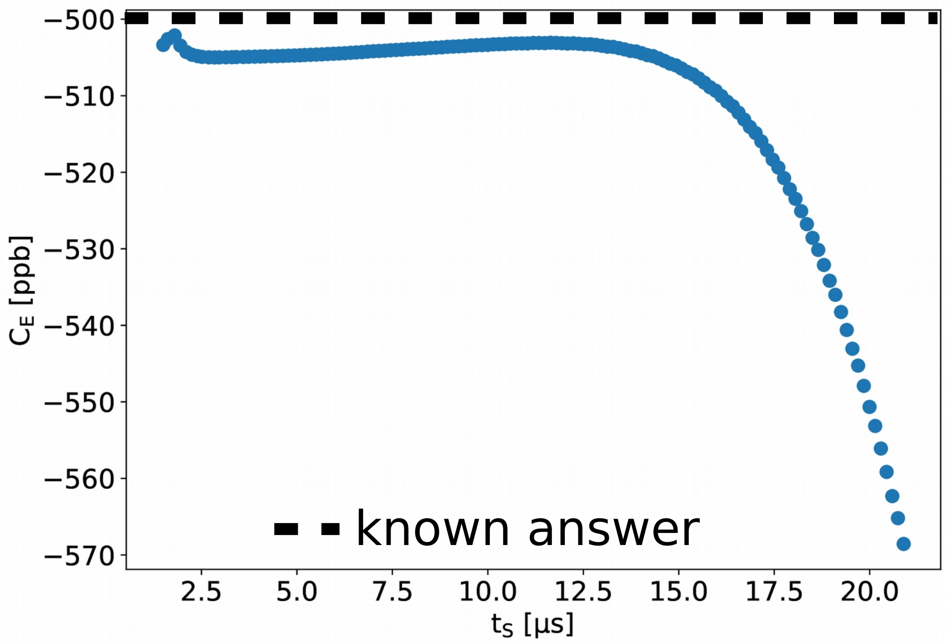




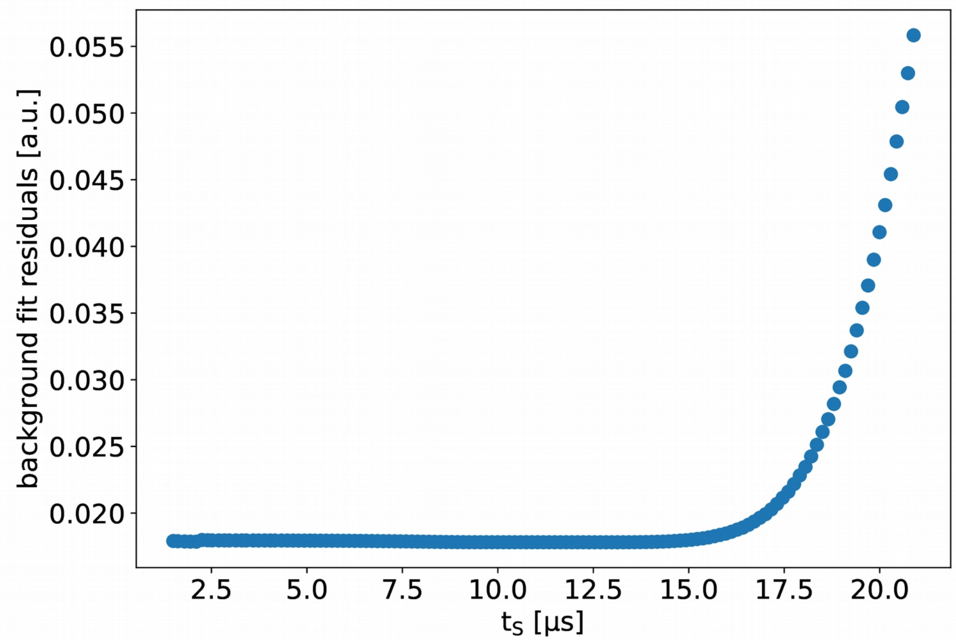


Fourier start time scan with simulation

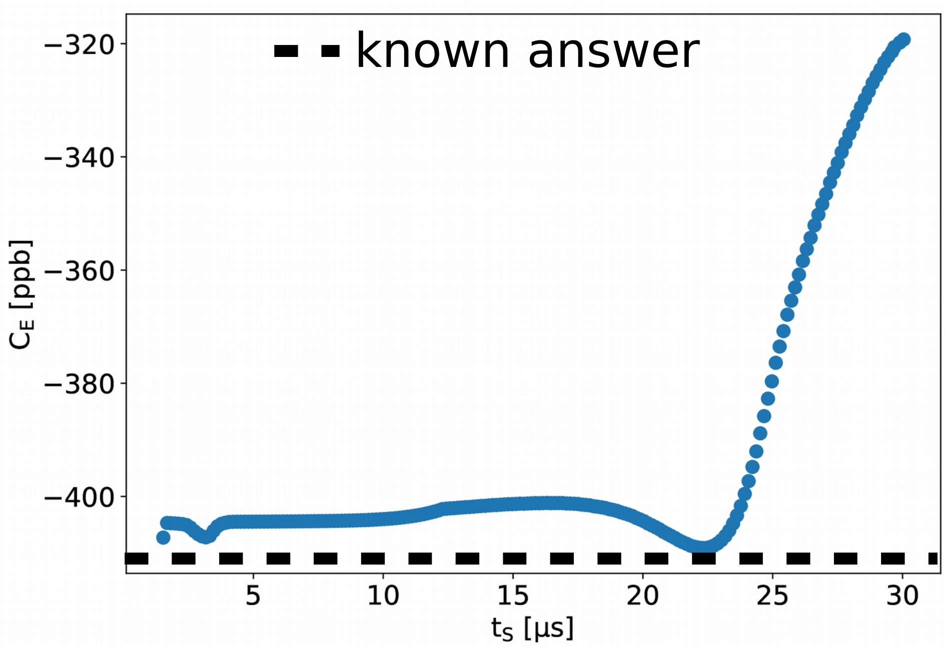
toy MC #3



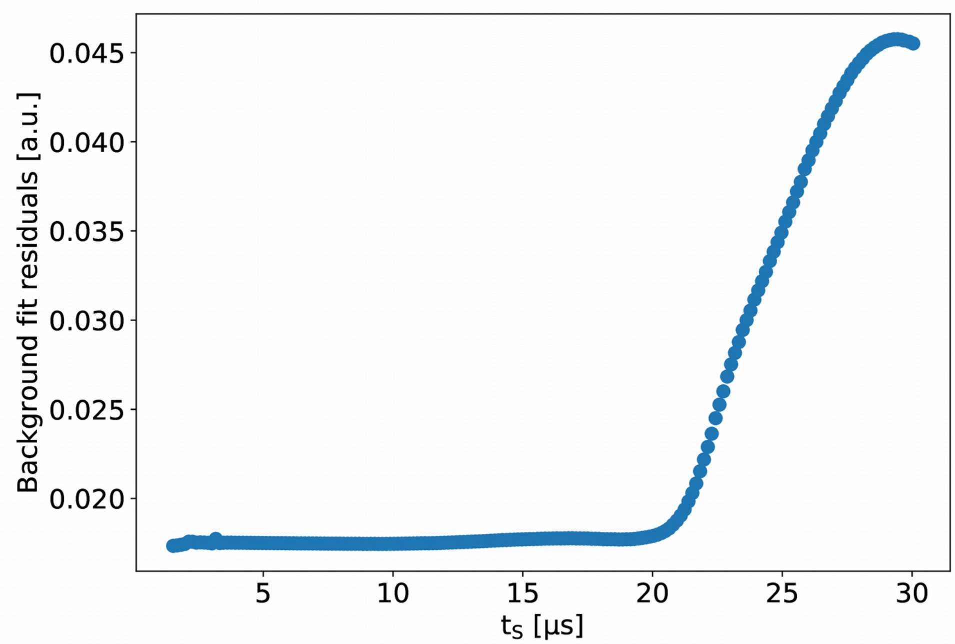
toy MC #3



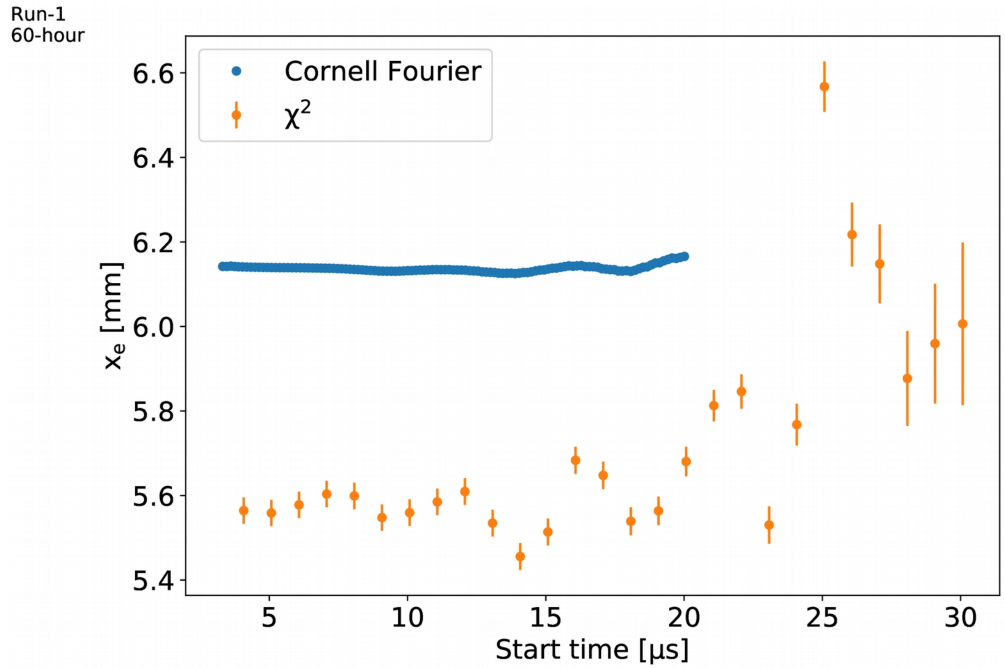
BMAD #1



BMAD #1

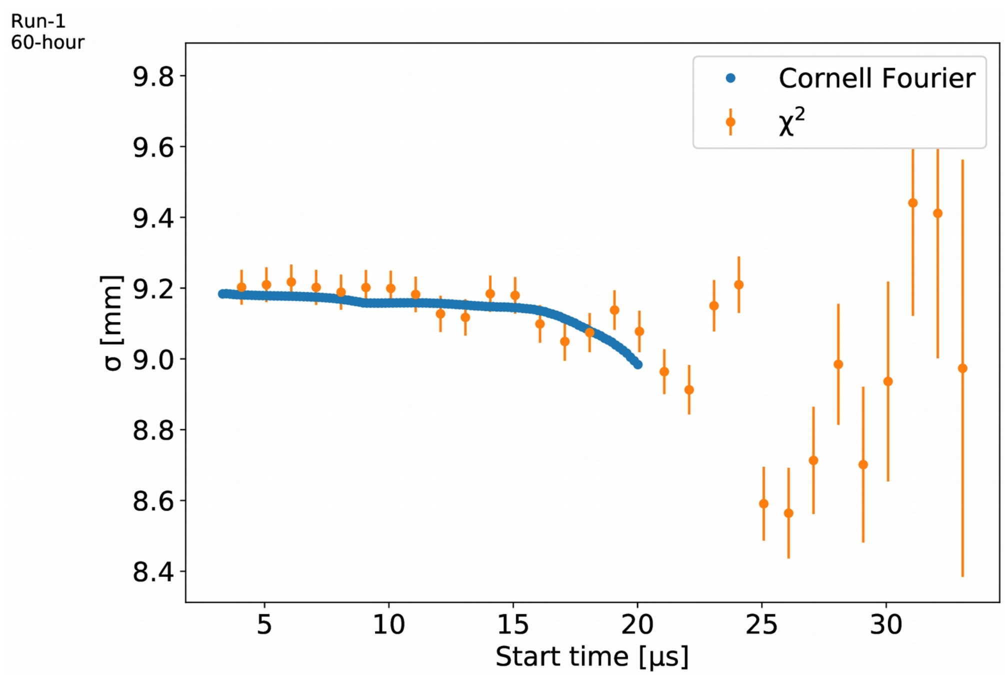


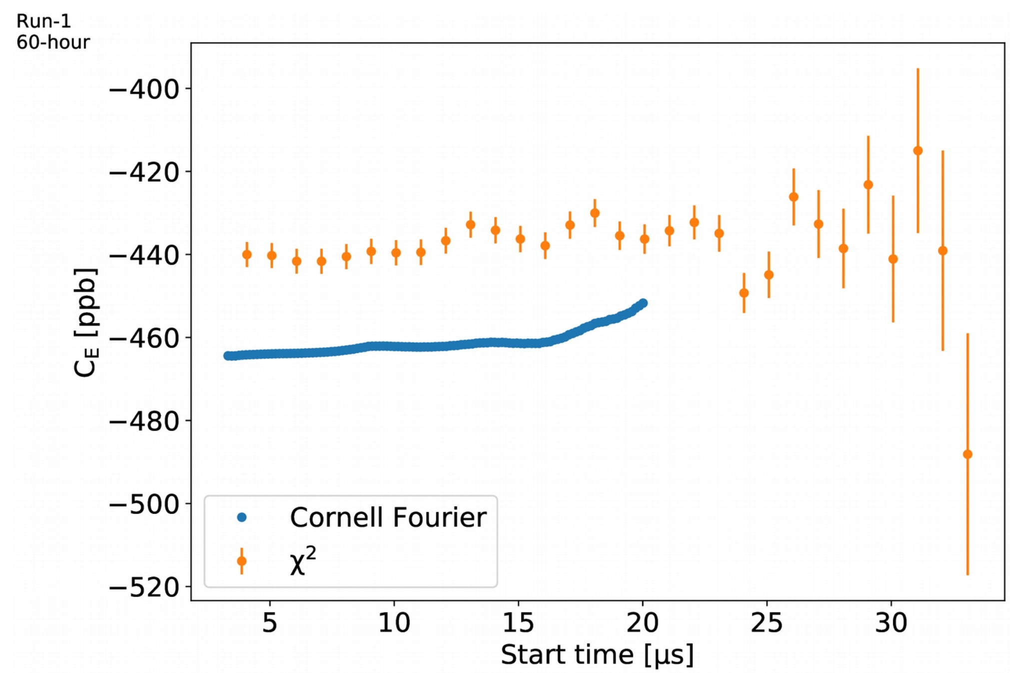
60–hour data set: start time scan



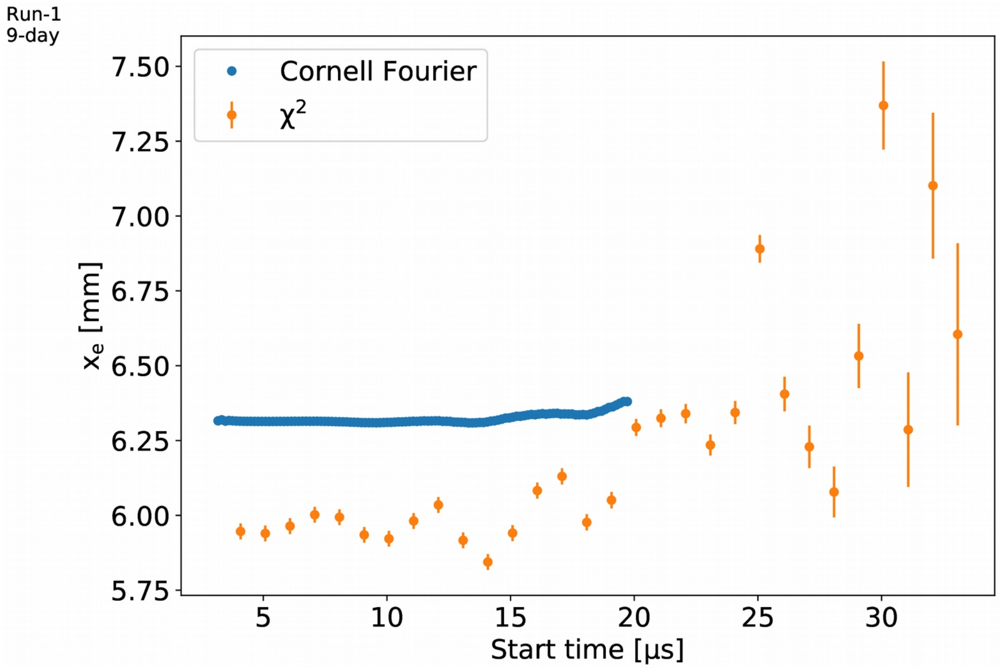
error bar = statistical uncertainty

Similar trends between the two methods



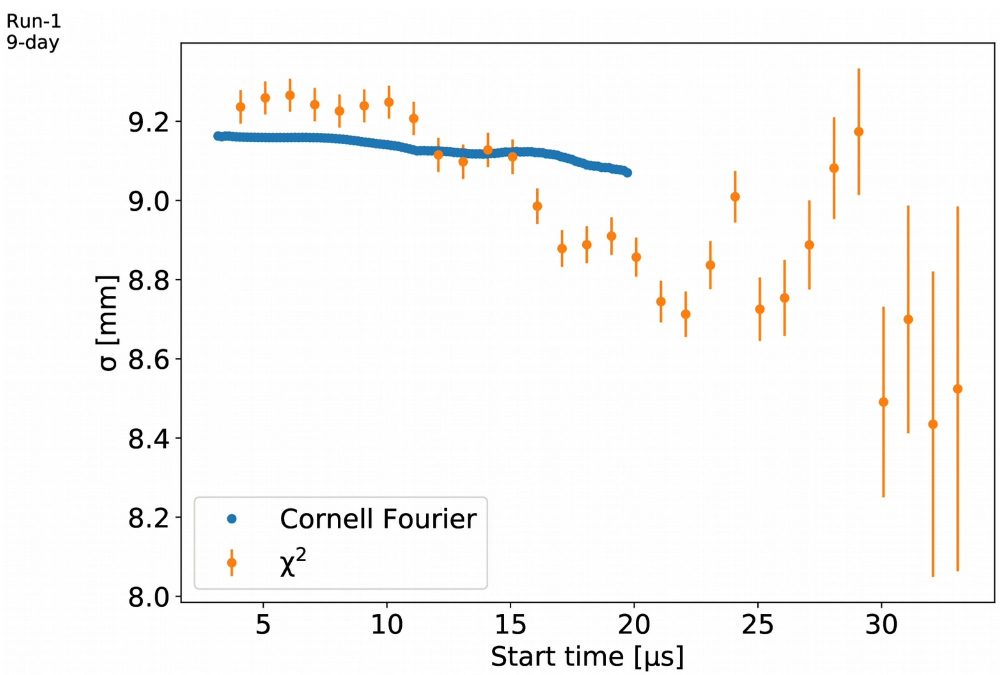


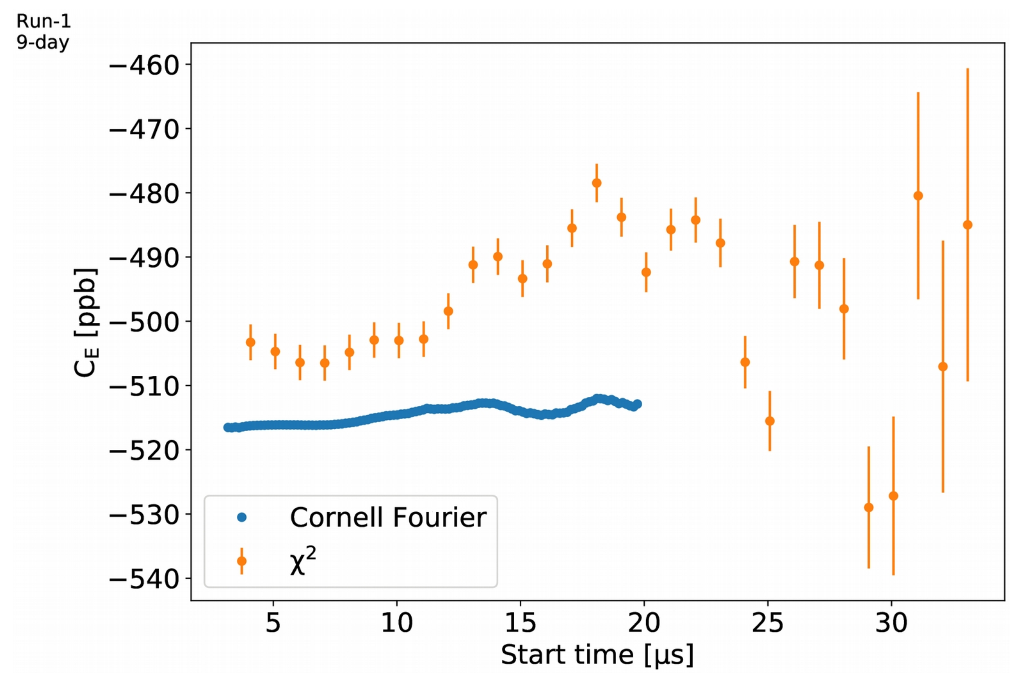
9–day data set: start time scan



error bar = statistical uncertainty

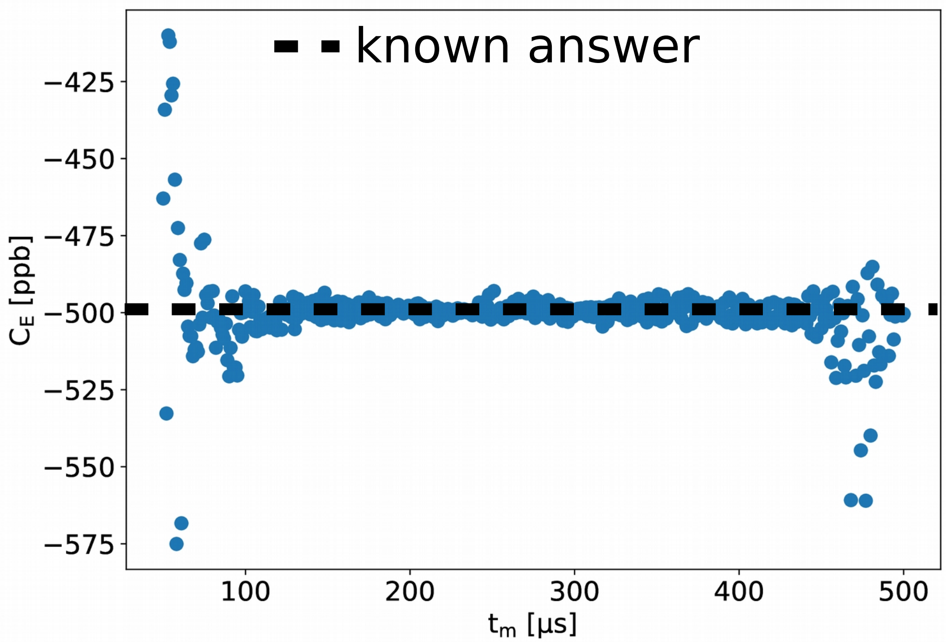
Similar trends between the two methods



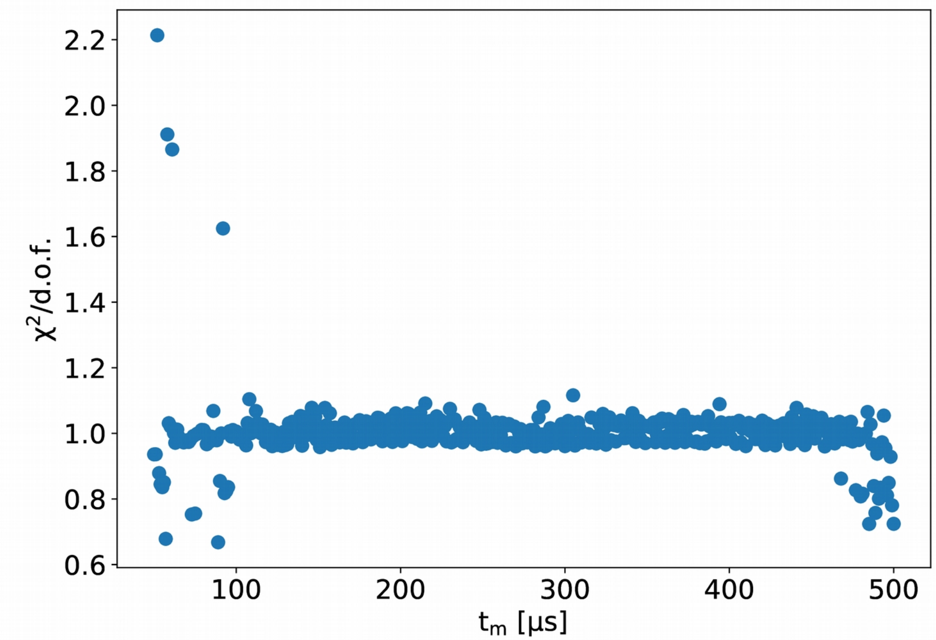


Fourier end time scan with simulation

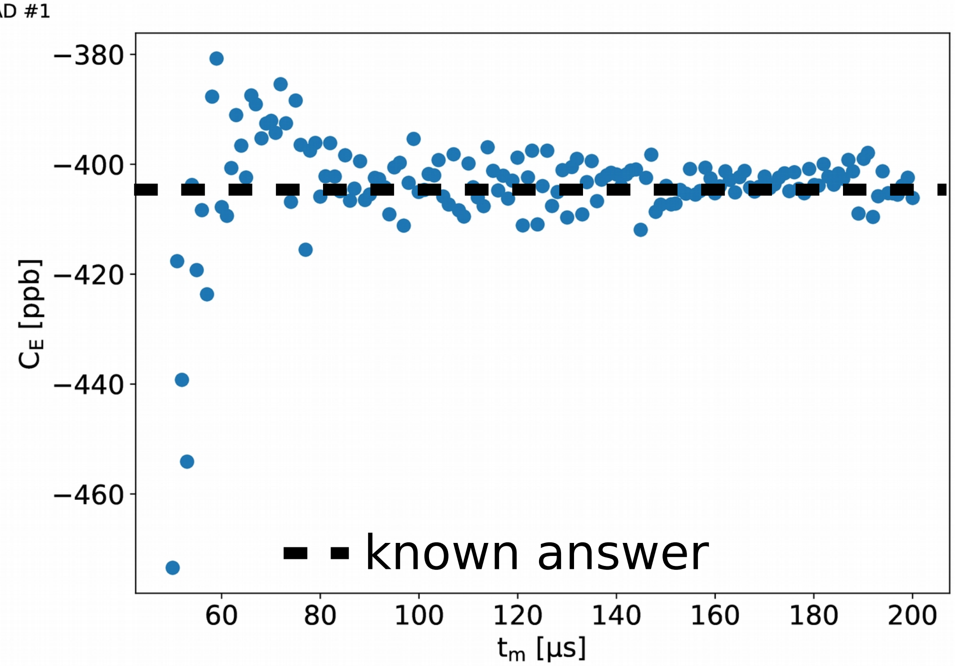
toy MC #3



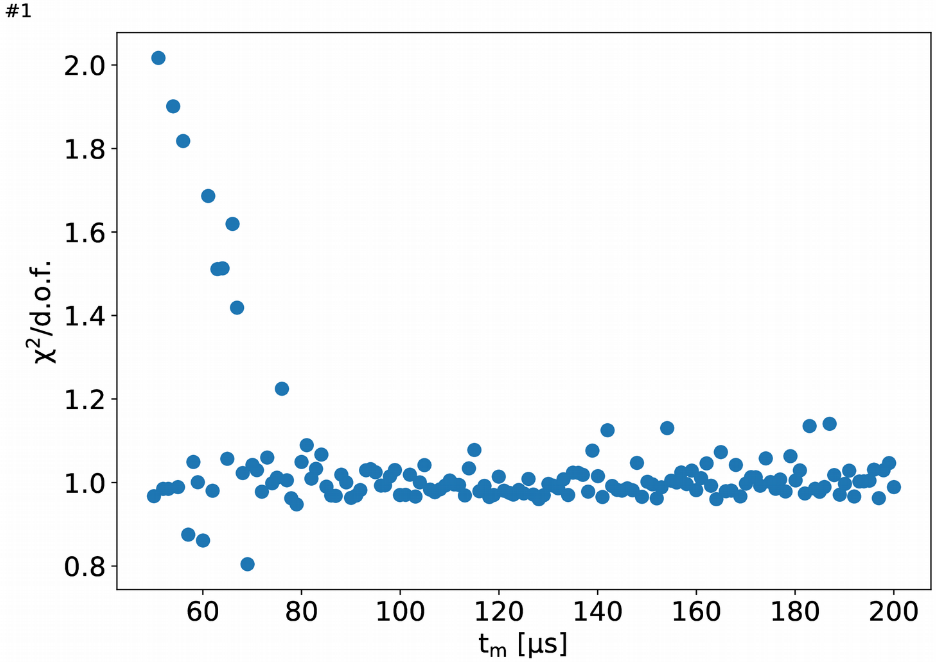
toy MC #3



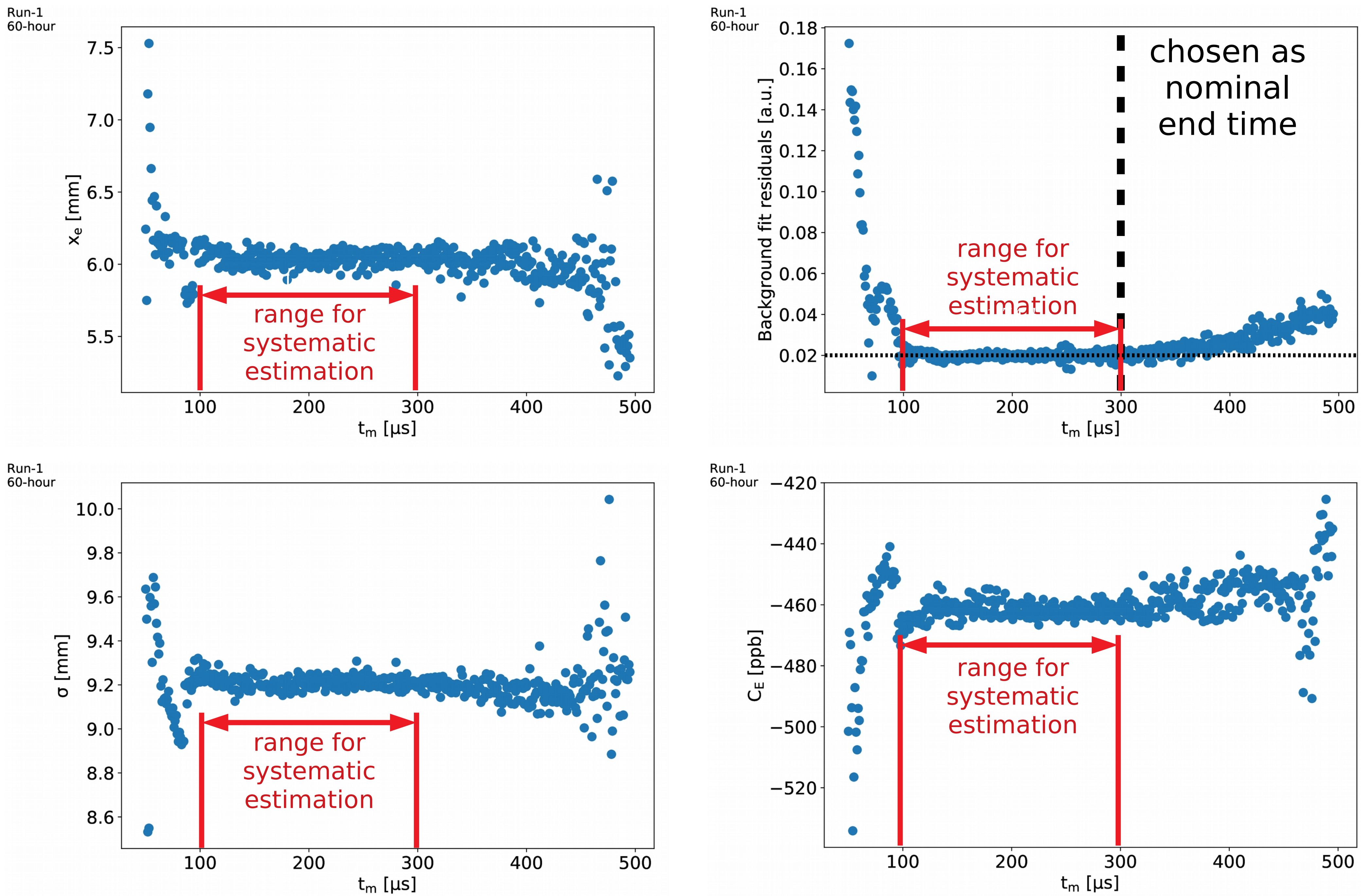
BMAD #1



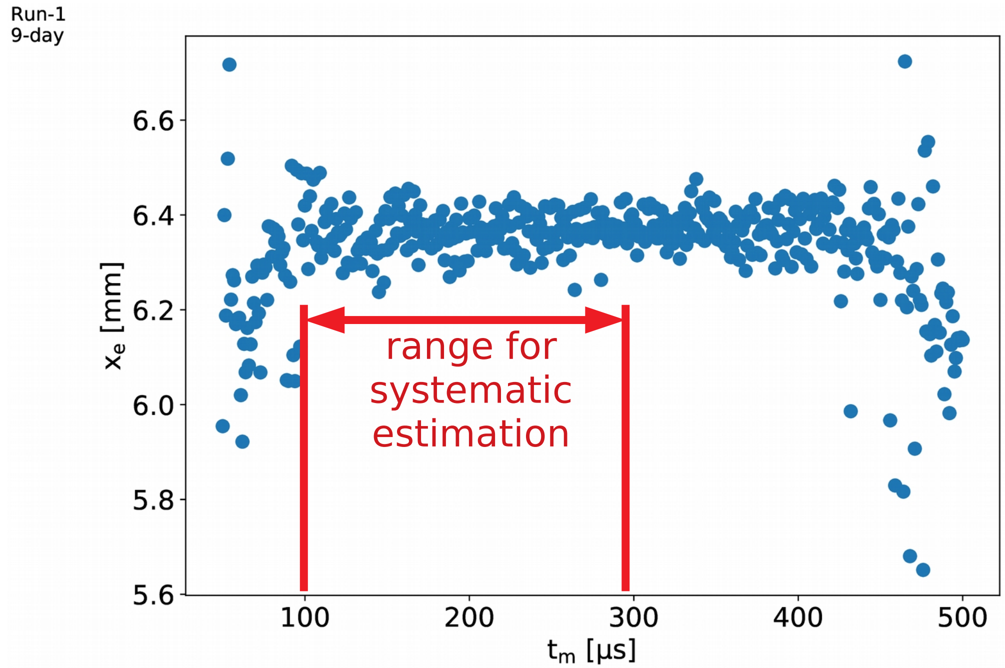
BMAD #1

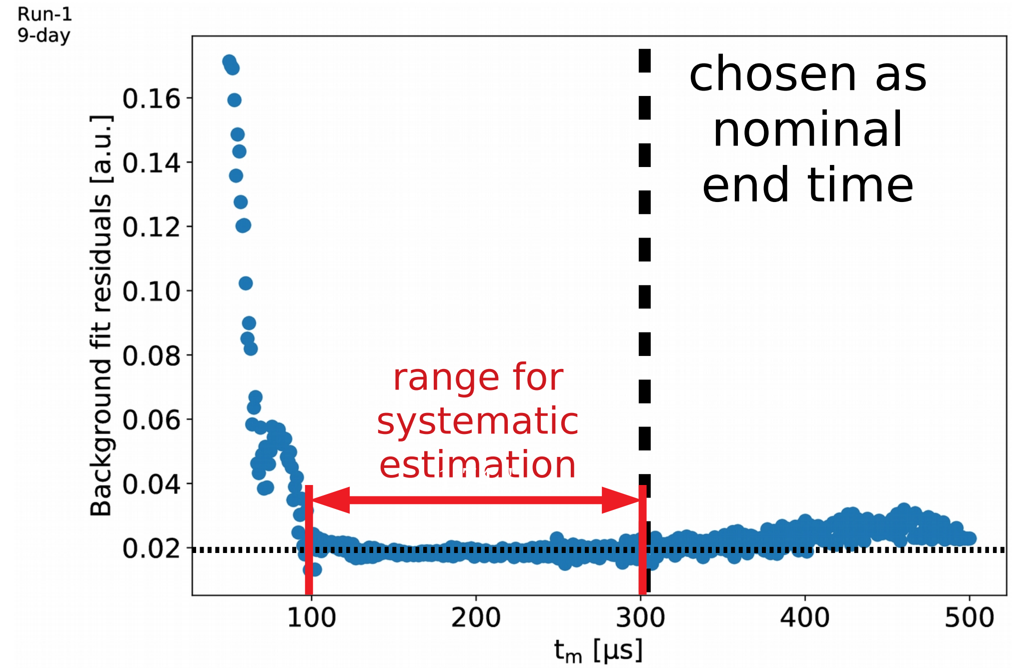


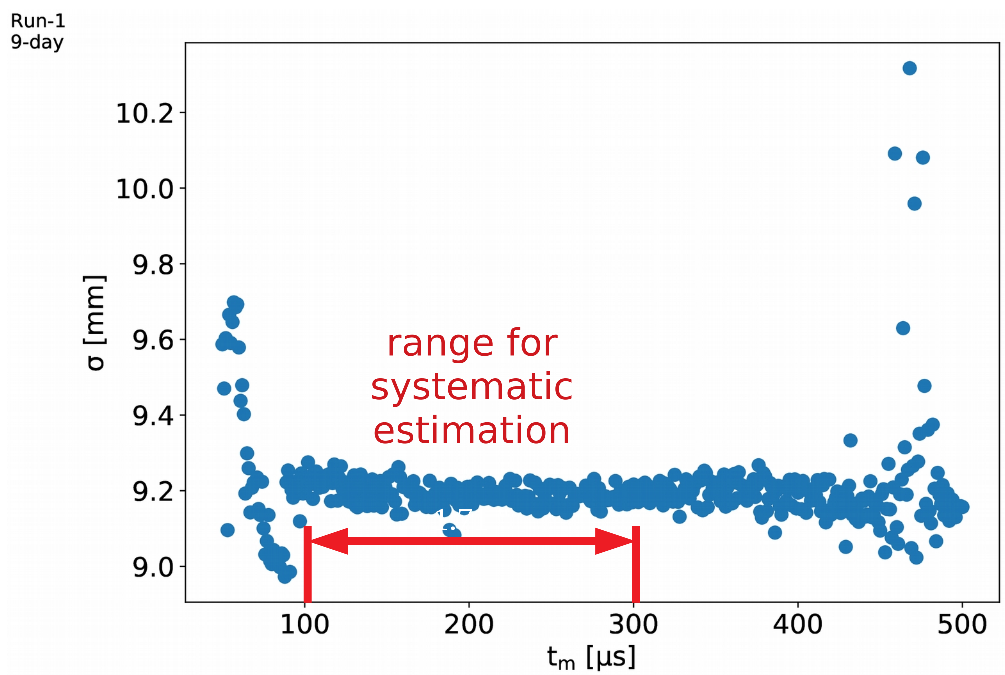
60–hour data set: Fourier end time scan

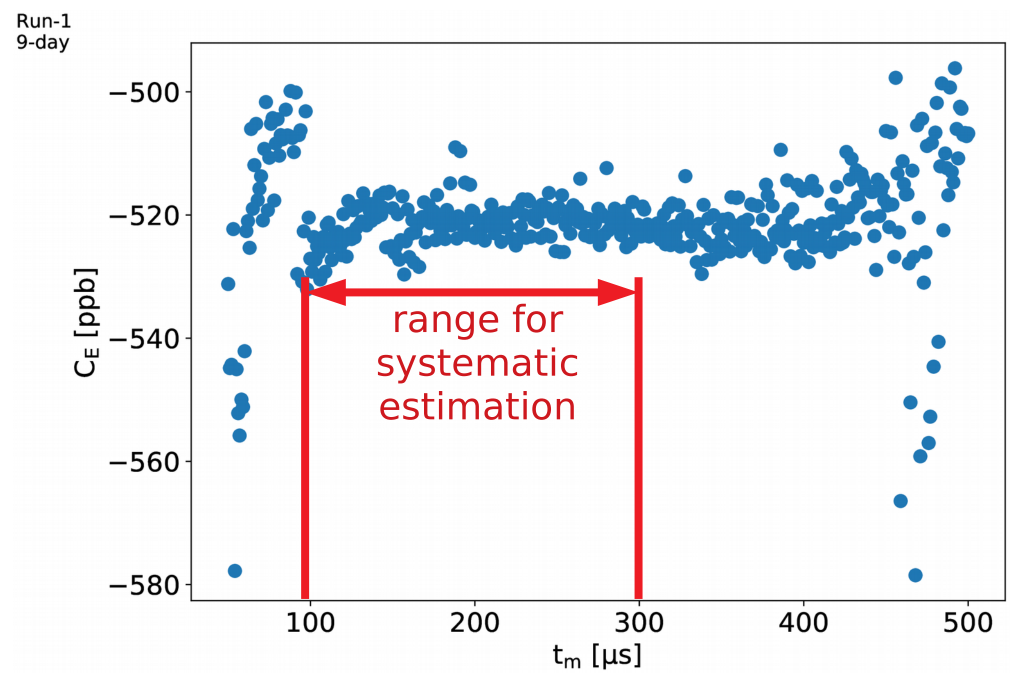


9–day data set: Fourier end time scan

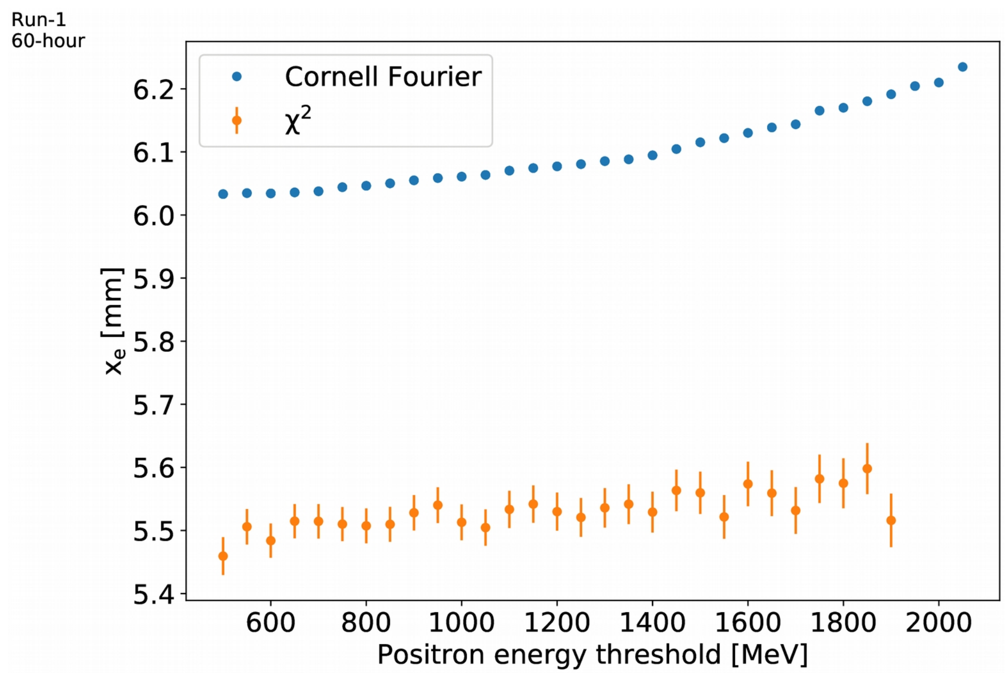






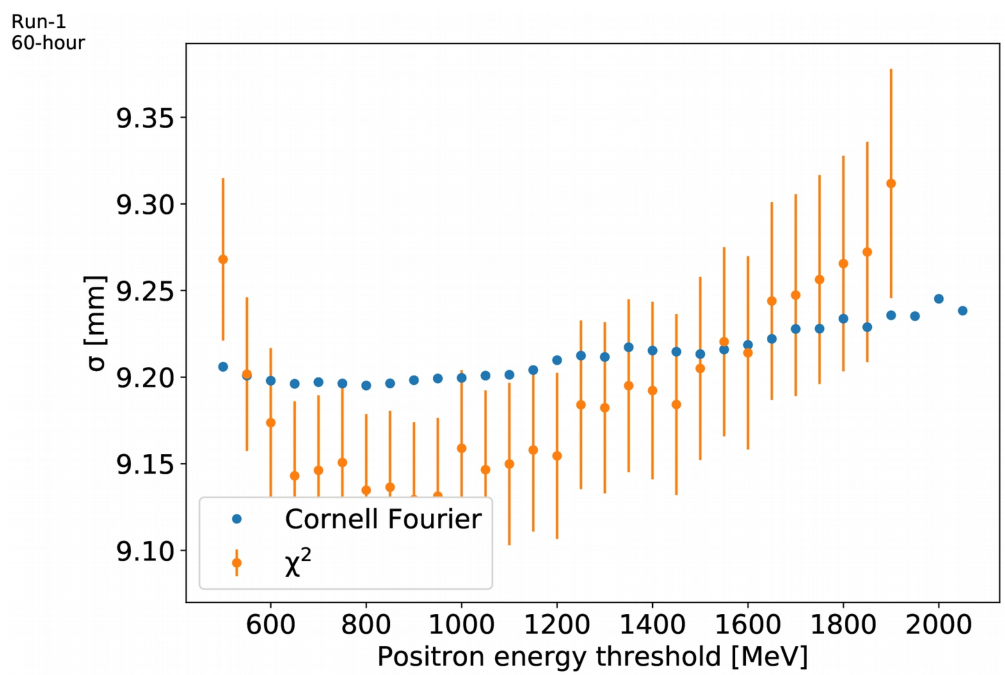


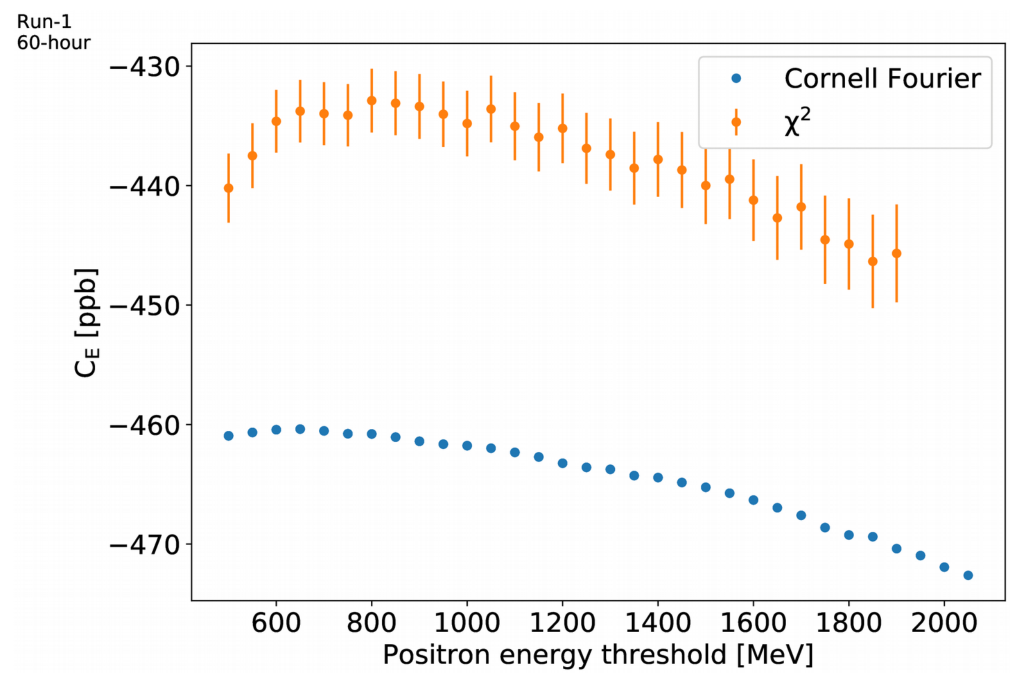
60–hour data set: positron energy threshold scan



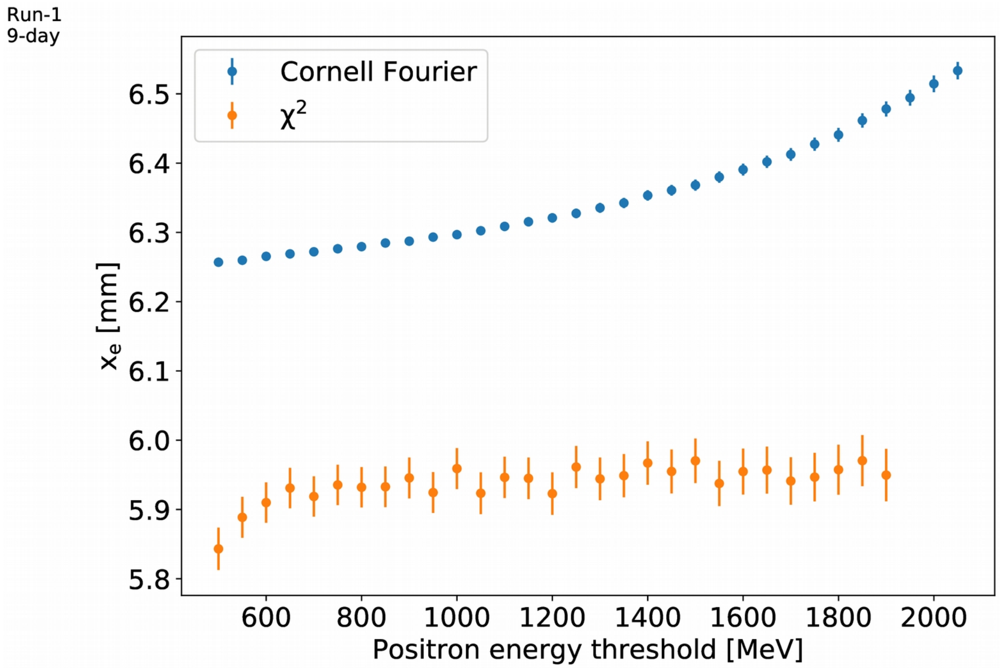
error bar = statistical uncertainty

Similar trends between the two methods



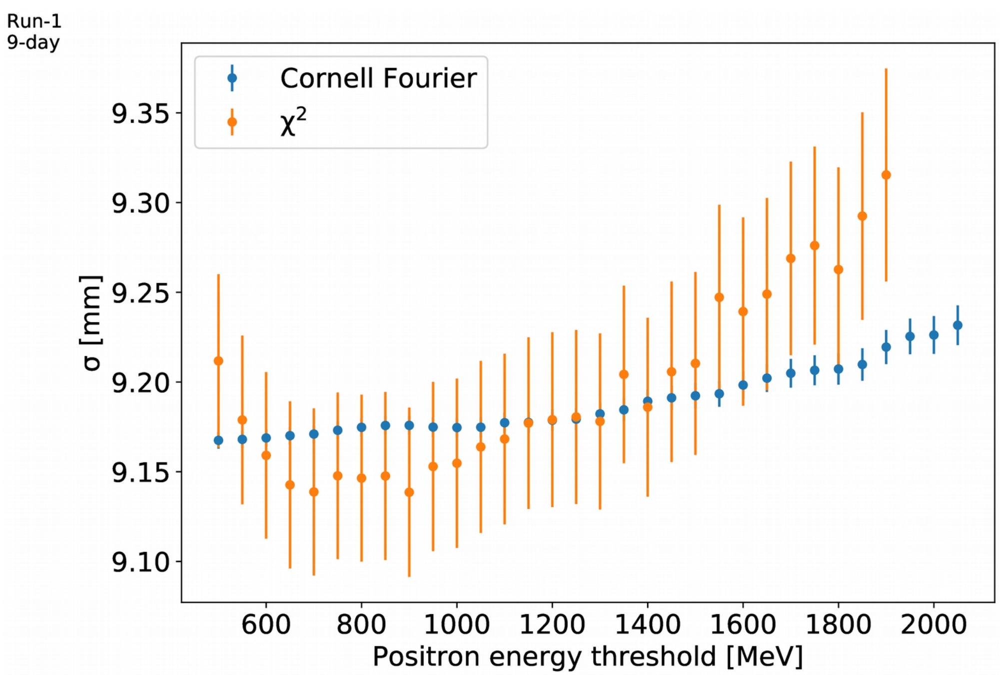


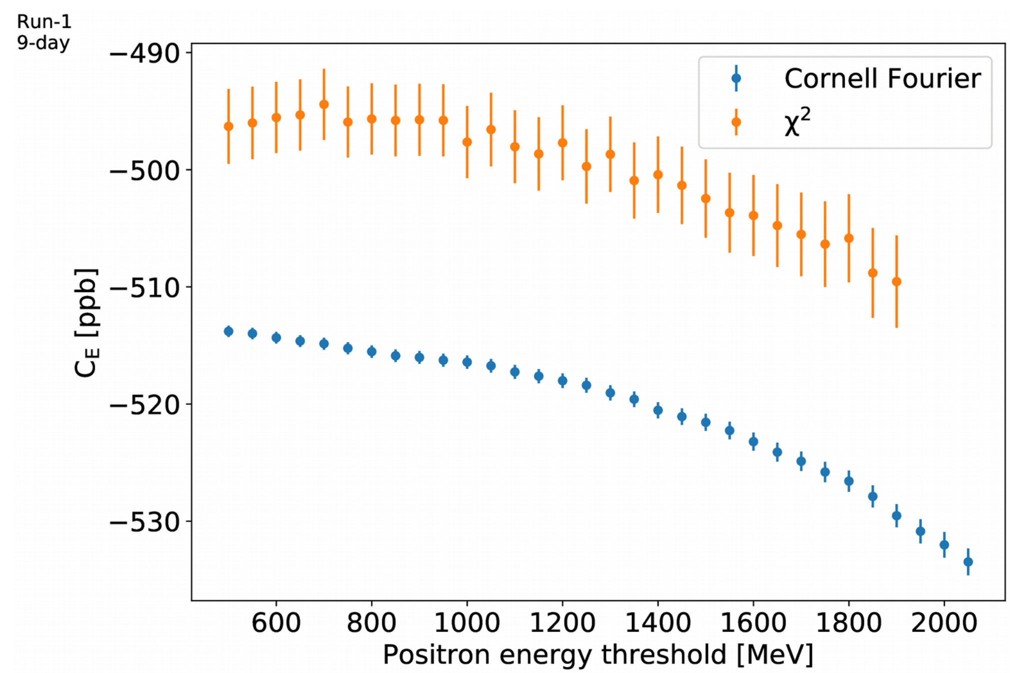
9–day data set: positron energy threshold scan



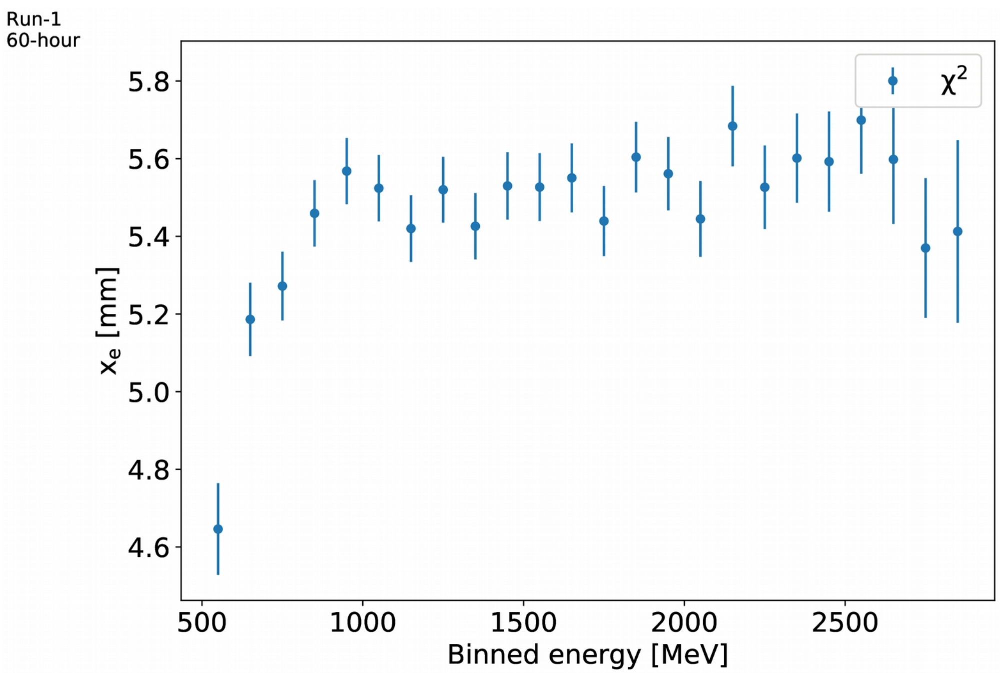
error bar = statistical uncertainty

Similar trends between the two methods



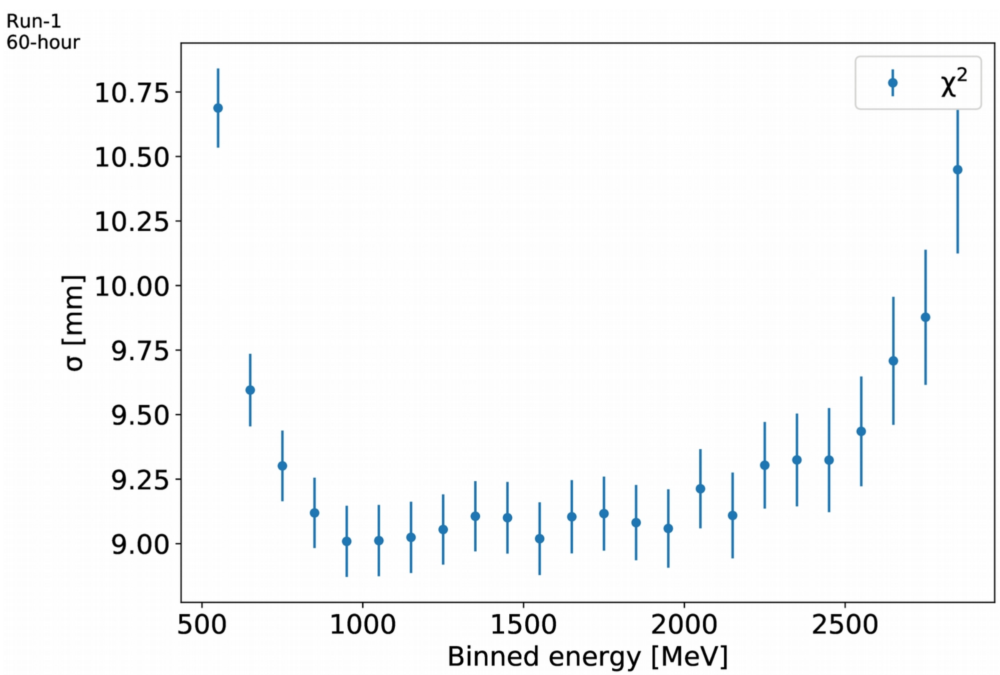


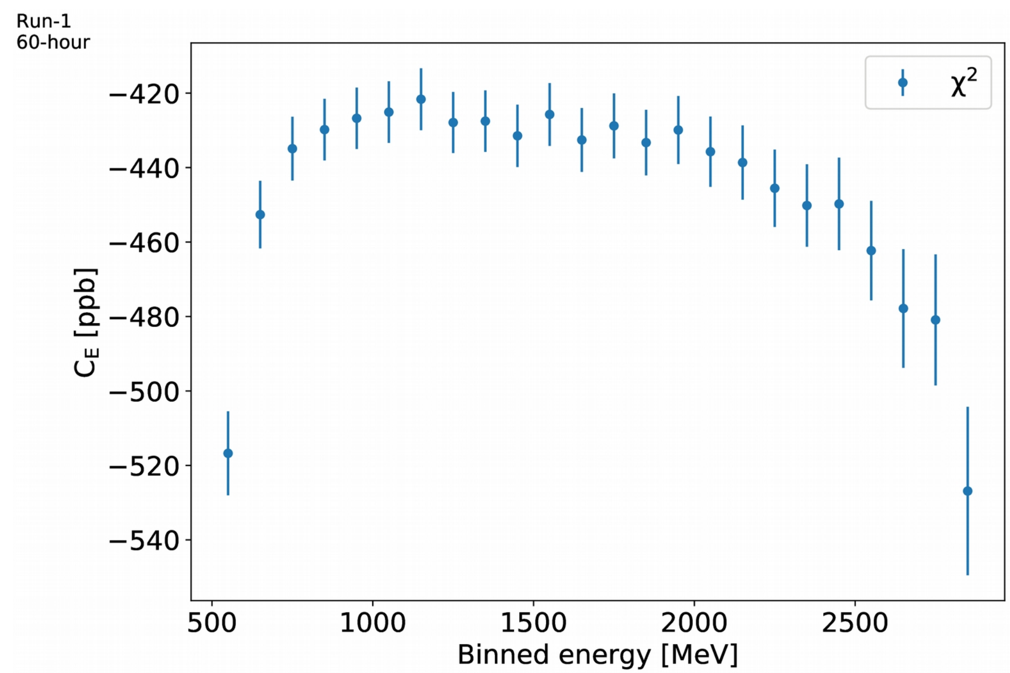
60–hour data set: energy binned analysis



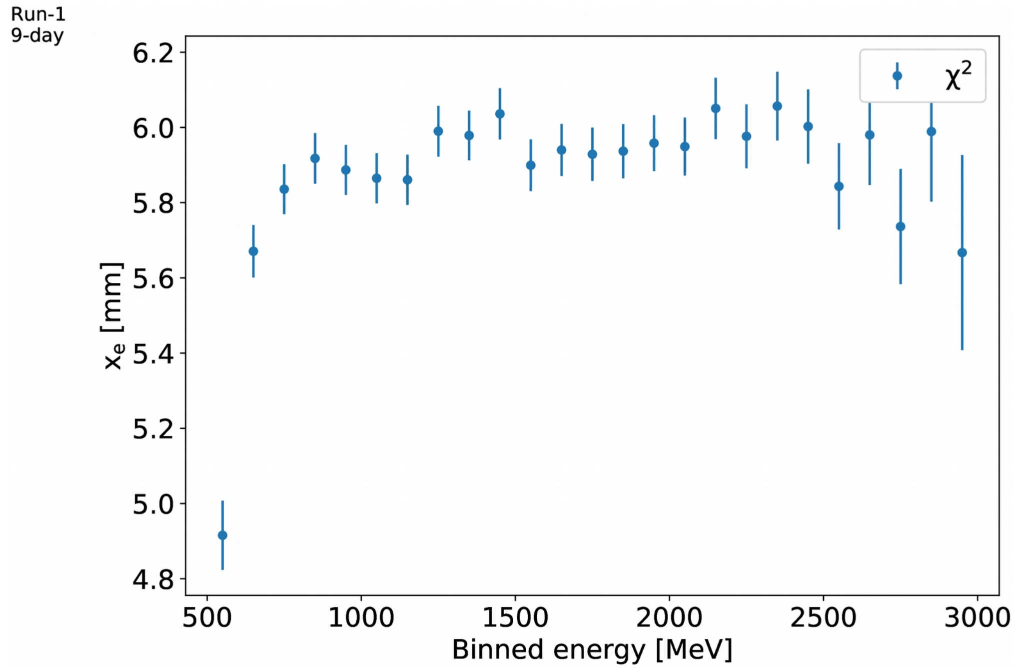
error bar = statistical uncertainty

Similar dependency to energy threshold scan



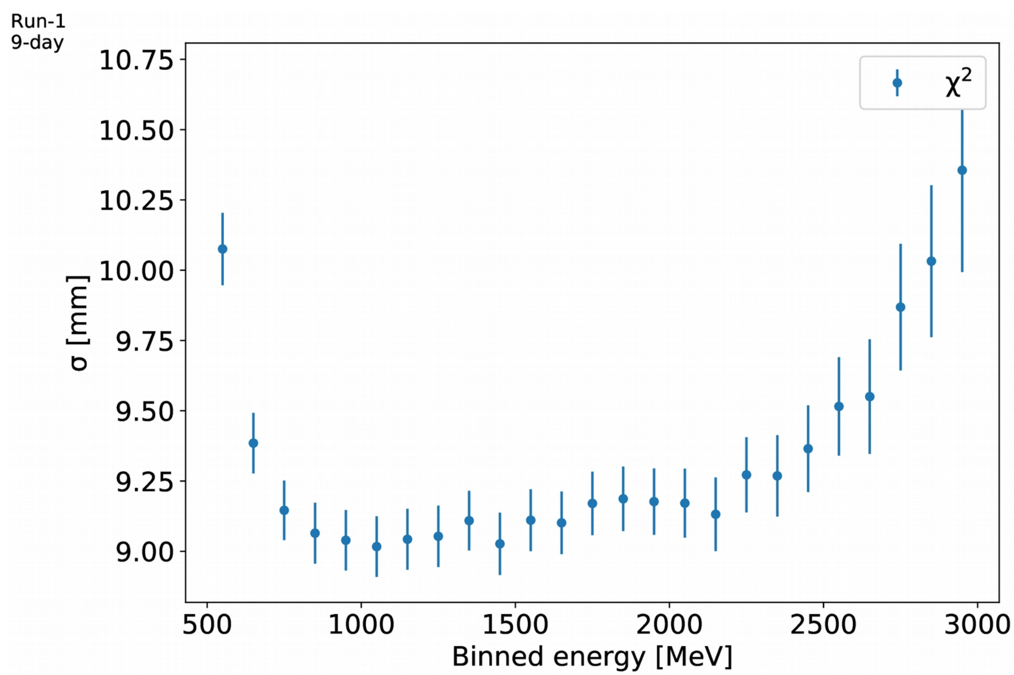


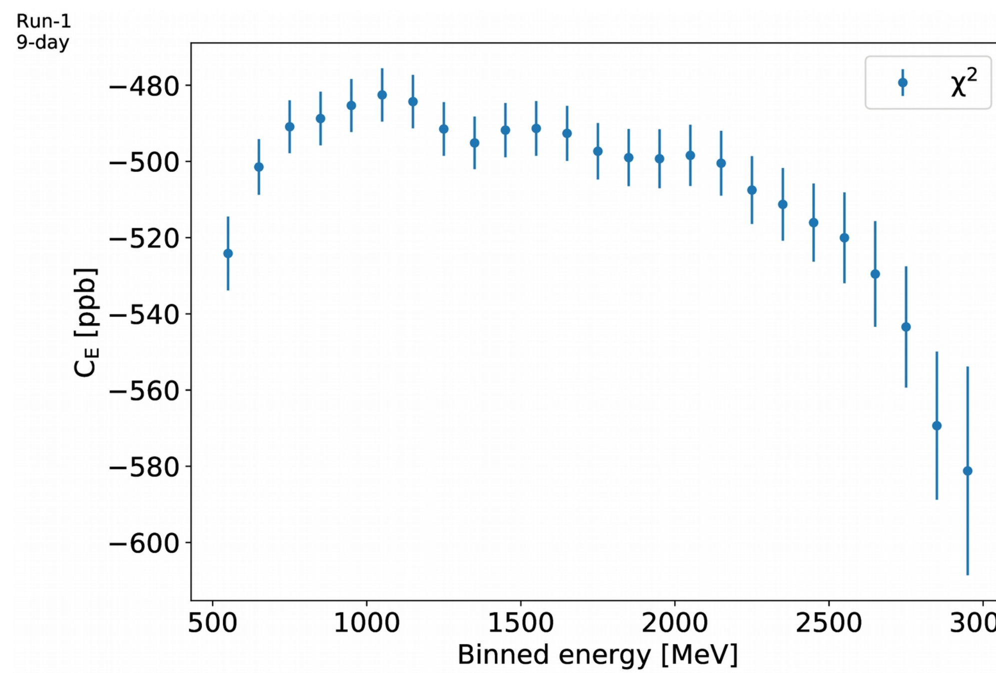
9–day data set: energy binned analysis



error bar = statistical uncertainty

Similar dependency to energy threshold scan





60–hour data set: Fourier method systematics

Unknown correlation between the various systematics, if assuming:

no correlation

	x_e [mm]	σ [mm]	C_E [ppb]
t_0	0.08	0.05	7
t_s	0.02	0.10	6
t_m	0.15	0.10	6
$t_0 \pm T_c$	0.01	0.05	4
positron $E_{threshold}$	0.10	0.03	6
wiggle fit	0.01	0.01	1
frequency resolution	0.03	0.05	4
background fit	0.05	0.06	5
background tails	0.03	0.03	3
sytematics (in quadrature)	0.23	0.18	15
statistics	0.01	0.01	1
total (sum in quadrature)	0.23	0.18	15

100% correlation

	x_e [mm]	σ [mm]	C_E [ppb]
t_0	0.08	0.05	7
t_s	0.02	0.10	6
t_m	0.15	0.10	6
$t_0 \pm T_c$	0.01	0.05	4
positron $E_{threshold}$	0.10	0.03	6
wiggle fit	0.01	0.01	1
frequency resolution	0.03	0.05	4
background fit	0.05	0.06	5
background tails	0.03	0.03	3
sytematics (linear sum)	0.48	0.48	42
statistics	0.01	0.01	1
total (sum in quadrature)	0.48	0.48	42

Will take the average of the two scenarios:

	x_e [mm]	σ [mm]	C_E [ppb]
sytematics	0.36	0.33	29
statistics	0.01	0.01	1
total (sum in quadrature)	0.36	0.33	29

9–day data set: Fourier method systematics

Unknown correlation between the various systematics, if assuming:

no correlation

	x_e [mm]	σ [mm]	C_E [ppb]
t_0	0.13	0.06	9
t_s	0.04	0.05	3
t_m	0.15	0.10	12
$t_0 \pm T_c$	0.03	0.05	3
positron $E_{threshold}$	0.15	0.03	10
wiggle fit	0.02	0.01	2
frequency resolution	0.03	0.05	4
background fit	0.06	0.10	8
background tails	0.03	0.07	6
sytematics (in quadrature)	0.25	0.19	22
statistics	0.01	0.01	1
total (sum in quadrature)	0.25	0.19	22

100% correlation

	x_e [mm]	σ [mm]	C_E [ppb]
t_0	0.13	0.06	9
t_s	0.04	0.05	3
t_m	0.15	0.10	12
$t_0 \pm T_c$	0.03	0.05	3
positron $E_{threshold}$	0.15	0.03	10
wiggle fit	0.02	0.01	2
frequency resolution	0.03	0.05	4
background fit	0.06	0.10	8
background tails	0.03	0.07	6
sytematics (linear sum)	0.61	0.52	57
statistics	0.01	0.01	1
total (sum in quadrature)	0.61	0.52	57

Will take the average of the two scenarios:

	x_e [mm]	σ [mm]	C_E [ppb]
sytematics	0.43	0.35	39
statistics	0.01	0.01	1
total (sum in quadrature)	0.43	0.35	39

Results comparison with uncertainty

Fourier:

- statistical
- systematic: conservative estimation because we can afford it for Run-1

χ^2:

- statistical

		x_e [mm]	σ [mm]	C_E [ppb]
60–hour	Fourier	6.12 ± 0.36	9.21 ± 0.33	-465 ± 29
	χ^2	5.57 ± 0.03	9.20 ± 0.05	-440 ± 3
9–day	Fourier	6.35 ± 0.43	9.19 ± 0.35	-520 ± 39
	χ^2	5.95 ± 0.03	9.24 ± 0.05	-503 ± 3

Conclusion/Outlook

Cornell Fourier

Analysis improved and matured over the past year with various performance studies conducted in simulation (toy MC, BMAD)

First analysis round of 60–hour and 9–day completed with conservative systematic uncertainties estimation → will move on to End game and other data set

Future work using simulation (toy MC, BMAD, gm2ringsim)

- study impact from early in-fill time effects: scraping, gain, pile-up
- study impact from momentum-time correlation of injected beam
- exploration to improve further the method → hope to reach later start time
- continue performance assessment and systematic exploration

Cornell Fourier: Alea iacta est*

Antoine will start a new non g-2 position this coming September: “Oh boy, it has been 5 yippee ki-yay years with you all. A thousand thanks!”

Looking for someone to take **responsibility/owner-ship** of the Cornell Fourier method

Various notes will be available:

- theory of the Fourier method
- detailed explanation of the Cornell Fourier method implementation
- performance studies with toy MC and with BMAD
- analysis code user’s guide
- analysis of Run-1 data set

*Crossing the Rubicon

χ^2 method

This FRA analyses (like the FT analysis) really needs someone to take it over as their dedicated project!

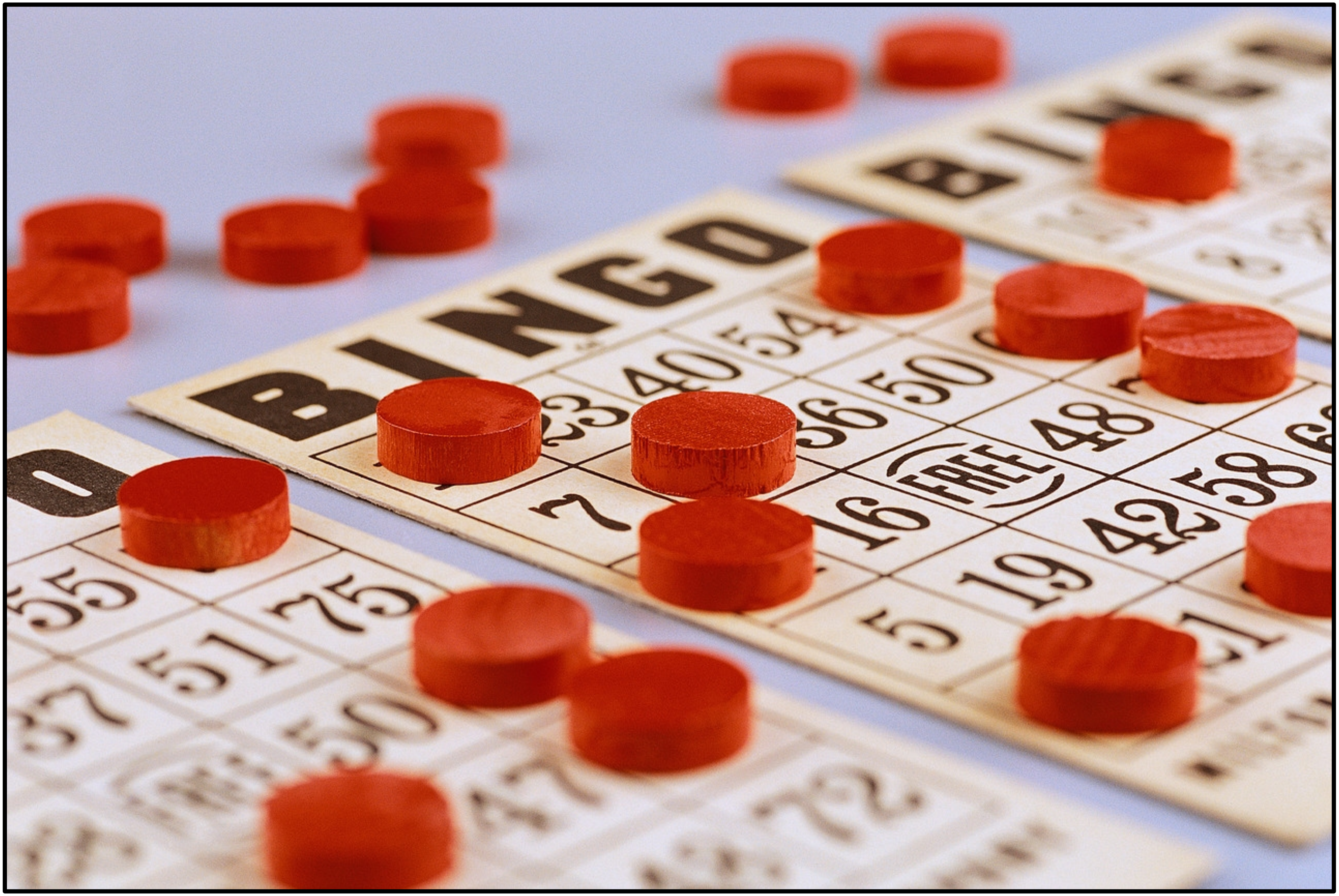
Updates/work to be done (not for Run-1)

- More robust determination of statistical errors.
  - Currently only bin-by-bin stat errors.
  - In reality, needs to take into account correlations between radial bins.
  - And correlations between injection pulse time bins.
- Upgrade to global 2D fit of injection pulse and radial distribution.

Systematic studies to be perform

- Systematic errors on choice of injection pulse.
- Systematic errors on choice of t_0.
- Systematic error on starting fit at 4 μs.
- Systematic error on ending fit at 100 μs.
- How sensitive are we to early beam effects, scraping etc.
- Systematic uncertainty evaluation on energy threshold choice.
- Dependence on effects from pileup.

Tentative final numbers?



Tentative final numbers?

Taking Cornell Fourier results as reference, and in quadrature:

Tentative final numbers?

Taking Cornell Fourier results as reference and in quadrature:

- add conservative systematic uncertainties from χ^2 and χ^2/Fourier comparison
  - differences from nominal and scans (t_s, energy threshold)

Tentative final numbers?

Taking Cornell Fourier results as reference and in quadrature:

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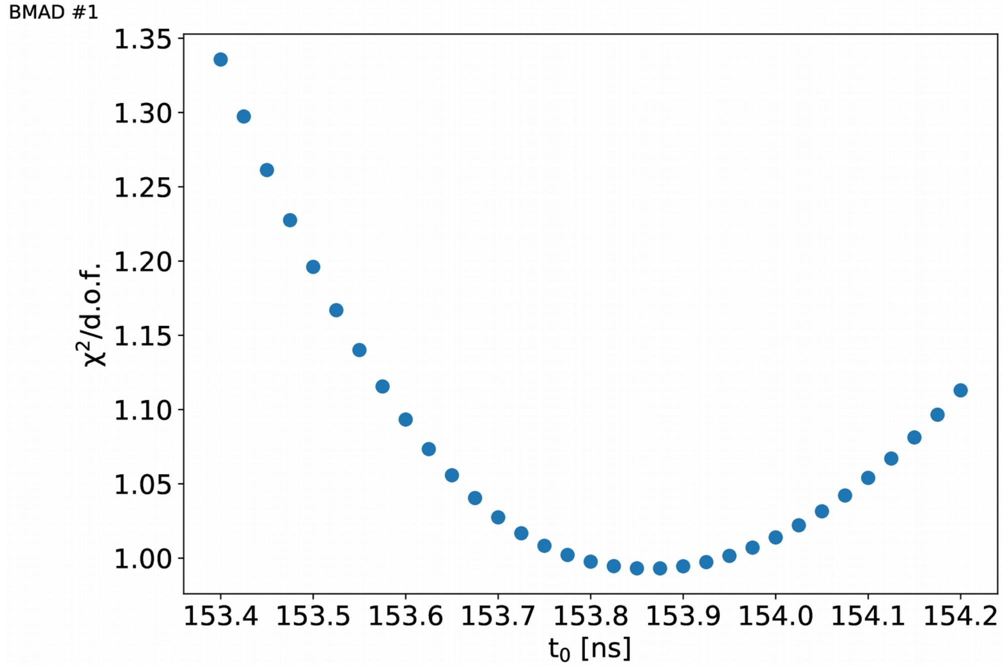
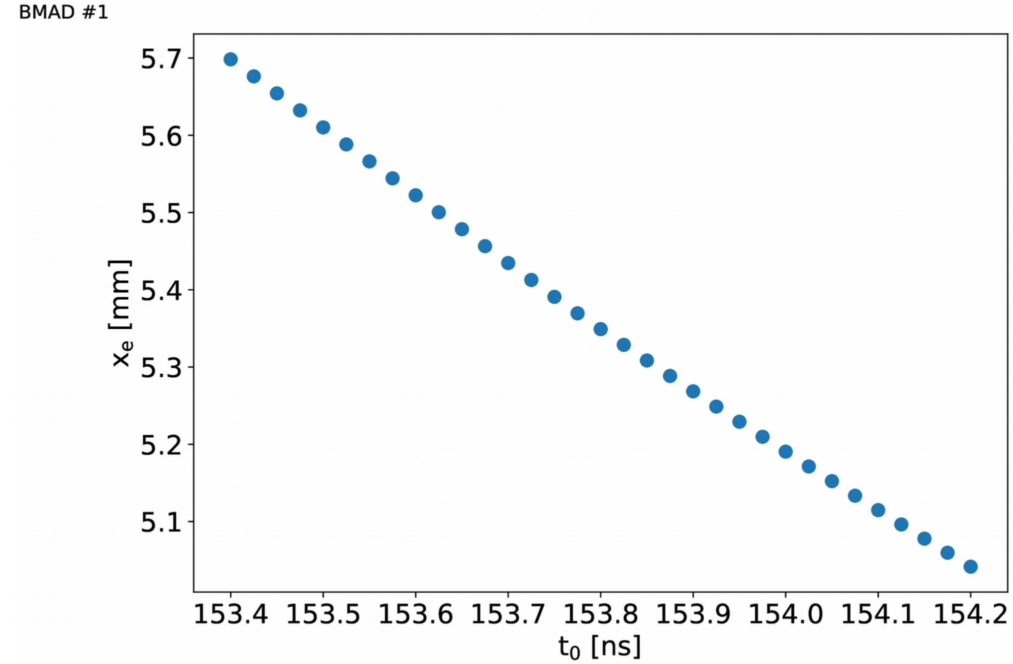
60–hour: $C_E = -465 \pm 63$ ppb

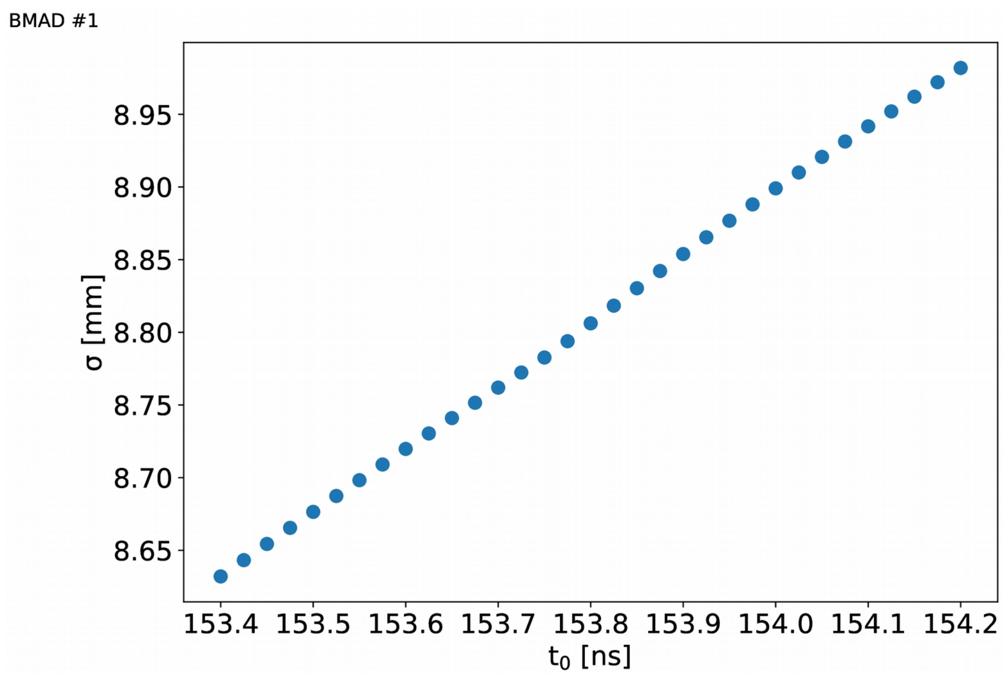
9–day: $C_E = -520 \pm 67$ ppb

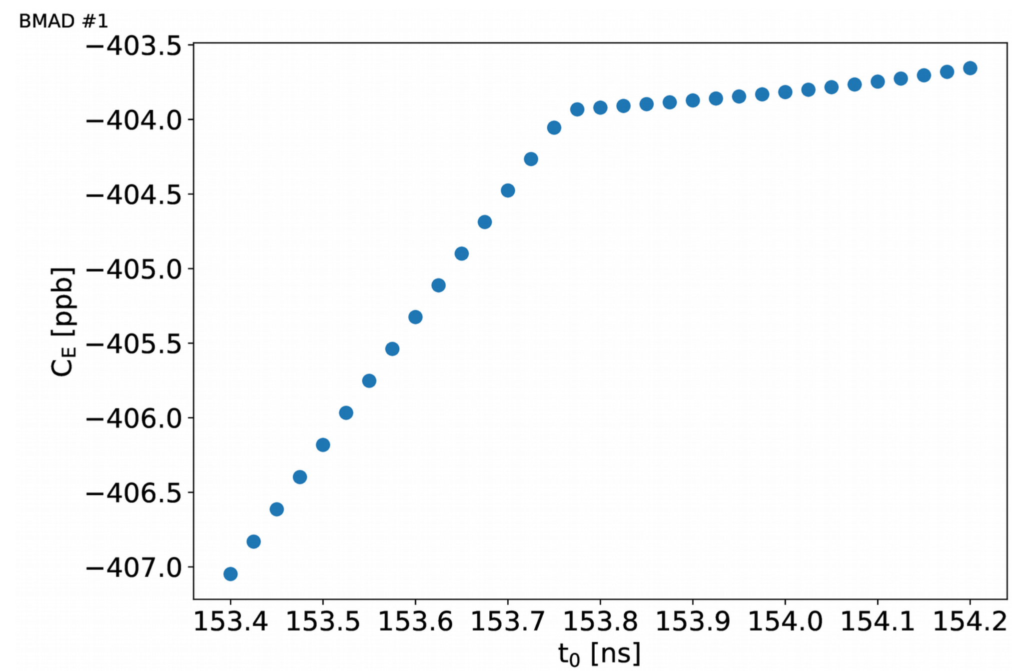
Additional materials

Fourier t_0 scan with simulation









60-hour:

20 ppb from chi2 start time scan

13 ppb from chi2/Fourier central values difference

9-day:

20 ppb from chi2 start time scan

9 ppb from chi2/Fourier central values difference