Cornell Fast Rotation Fourier Method: Linear Frequency-Time Correlation Scan

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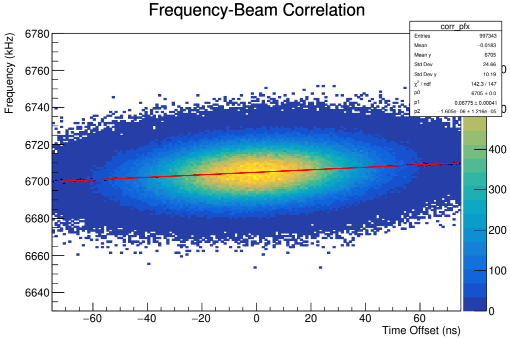
Linear Frequency-Time Correlation



Definition

$$\langle f \rangle(t) = f_m \left[1 - C(t/T_m)\right]$$

$\langle f \rangle$ = avg. frequency per time offset
t = time offset from mean of bunch
f_m, T_m = magic frequency, period
C = correlation amplitude

Interpretation

- head $(-T_m/2)$ faster by $(C/2)f_m$
- tail $(+T_m/2)$ slower by $(C/2)f_m$

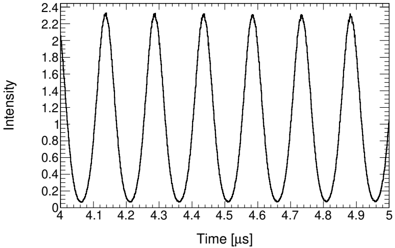
$\langle f \rangle$ changes by about Cf_m across the initial bunch, from tail to head

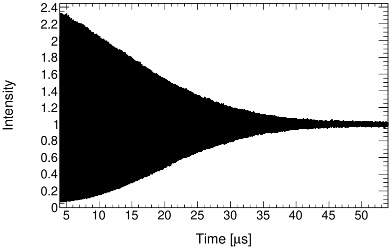
Toy Monte Carlo

Simplistic Algorithm

For each of 1,000,000 muons...

- choose random time offset t from Gaussian profile (width 25 ns);
- calculate mean frequency $\langle f \rangle$, correlated with chosen time;
- choose random frequency f from Gaussian distribution (width 0.15%);
- calculate times when muon will cross detector plane for 6,800 turns;
- bin those times into the fast rotation histogram.





Fast Rotation Fourier Analysis

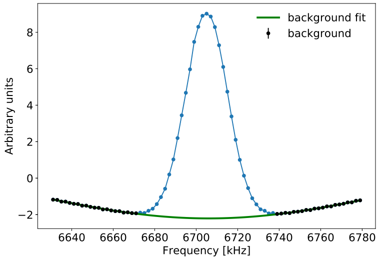
Cosine Transform

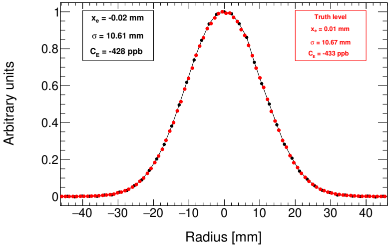
$$\hat{S}(\omega) = \int_{t_s}^{t_m} S(t)\cos[\omega(t - t_0)]\,dt.$$

$S(t)$ = fast rotation signal
t_s, t_m = start/end times for transform
t_0 = center of first turn

Background Correction

- windowing $S(t)$ on $t \in (t_s, t_m)$ introduces un-physical background
- fitting procedure uses approximate analytical form to subtract away background
- t_0 determined by minimum χ^2/ndf





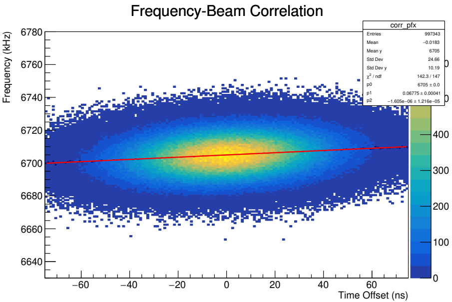
Correlation Scan

Nominal Analysis Settings

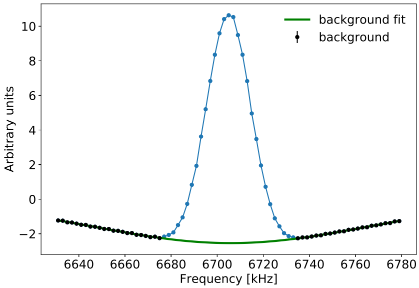
In all the slides that follow...

- $t_s = 4\ \mu\mathrm{s}$ and $t_m = 300\ \mu\mathrm{s}$
- t_0 optimization enabled
    - sinc background fit
    - 3 iterations
    - all cases reached $\chi^2/\mathrm{ndf} \approx 1$
    - all cases converged to expected t_0 (i.e. mean time on first turn)

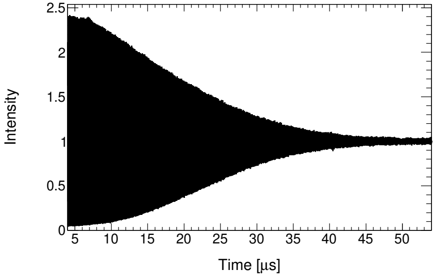
$C = -0.00150$



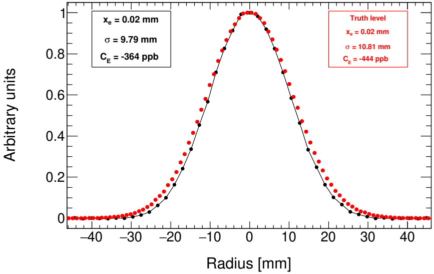
Frequency-Beam Correlation
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.19
χ² / ndf 142.3 / 147
p0 6705 ± 0.0
p1 0.06775 ± 0.00041
p2 −1.605e−06 ± 1.216e−05
Frequency (kHz)
Time Offset (ns)



background fit
background
Arbitrary units
Frequency [kHz]

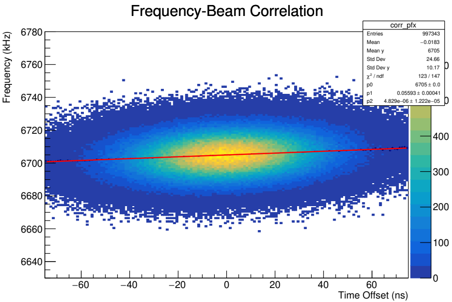


Intensity
Time [μs]

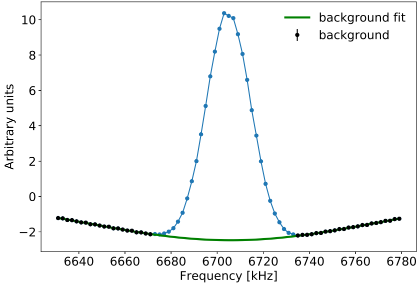


x_e = 0.02 mm
σ = 9.79 mm
C_E = -364 ppb
Truth level
x_e = 0.02 mm
σ = 10.81 mm
C_E = -444 ppb
Arbitrary units
Radius [mm]

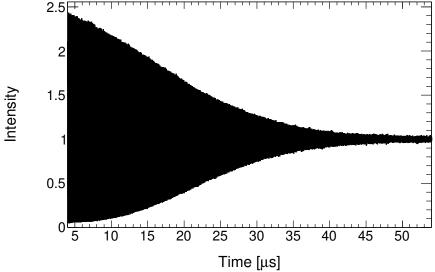
$C = -0.00125$



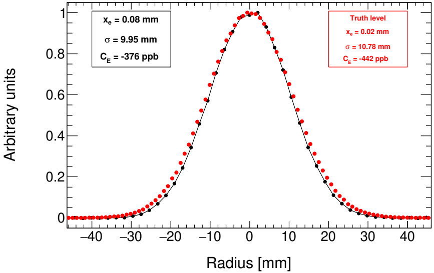
Frequency-Beam Correlation
Frequency (kHz)
Time Offset (ns)
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.17
χ² / ndf 123 / 147
p0 6705 ± 0.0
p1 0.05593 ± 0.00041
p2 4.829e−06 ± 1.222e−05



background fit
background
Arbitrary units
Frequency [kHz]

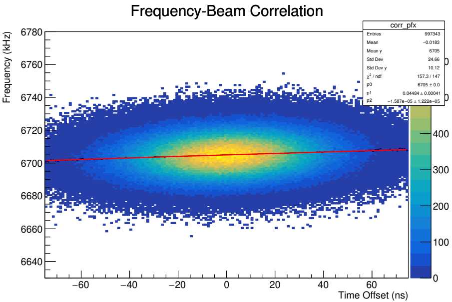


Intensity
Time [µs]

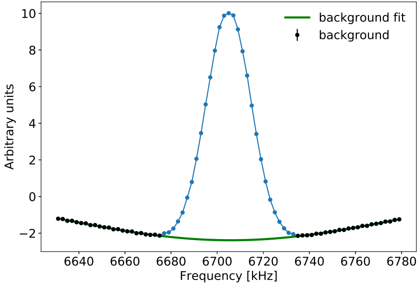


x_e = 0.08 mm
σ = 9.95 mm
C_E = -376 ppb
Truth level
x_e = 0.02 mm
σ = 10.78 mm
C_E = -442 ppb
Arbitrary units
Radius [mm]

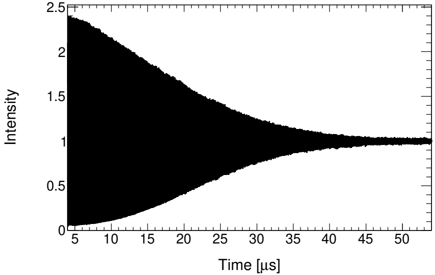
$C = -0.00100$



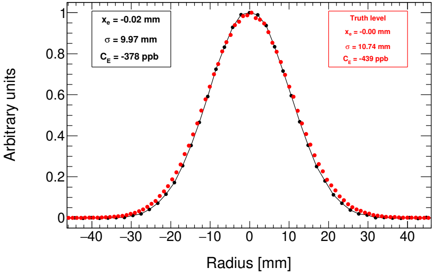
Frequency-Beam Correlation
Frequency (kHz)
Time Offset (ns)
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.12
χ^2 / ndf 157.3 / 147
p0 6705 ± 0.0
p1 0.04484 ± 0.00041
p2 −1.587e−05 ± 1.222e−05



background fit
background
Arbitrary units
Frequency [kHz]

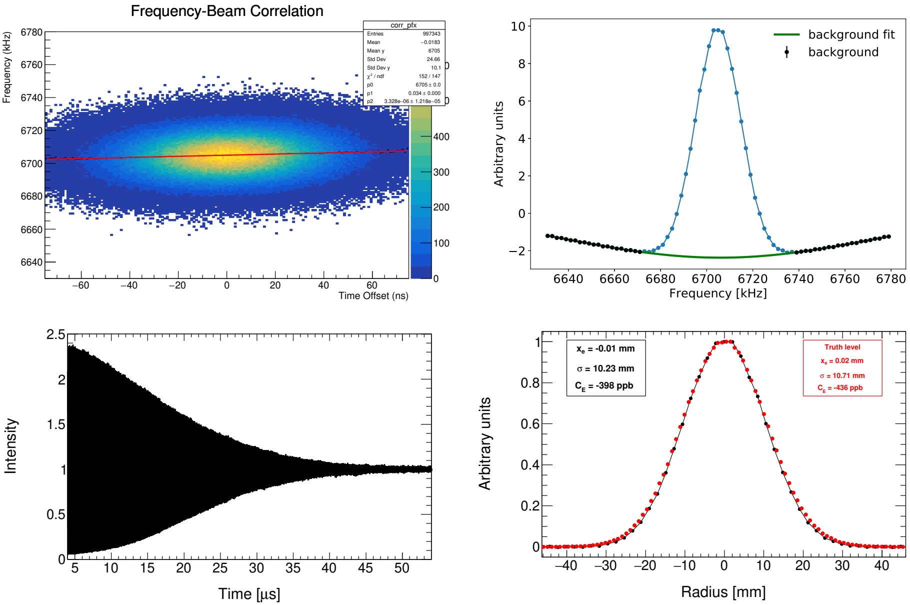


Intensity
Time [µs]



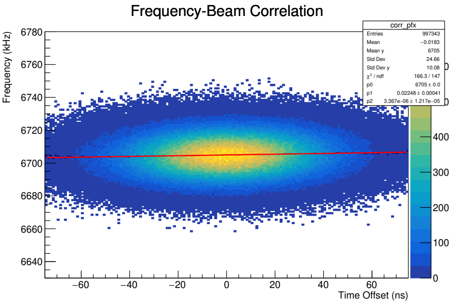
x_e = -0.02 mm
σ = 9.97 mm
C_E = -378 ppb
Truth level
x_e = -0.00 mm
σ = 10.74 mm
C_E = -439 ppb
Arbitrary units
Radius [mm]

$C = -0.00075$

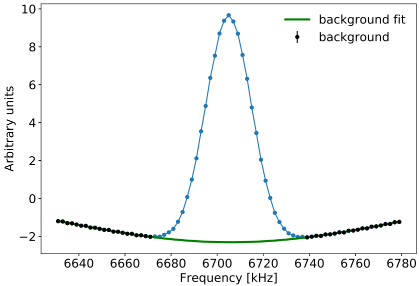


Frequency-Beam Correlation
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.1
χ² / ndf 152 / 147
p0 6705 ± 0.0
p1 0.034 ± 0.000
p2 3.328e−06 ± 1.218e−05
Frequency (kHz)
Time Offset (ns)
background fit
background
Arbitrary units
Frequency [kHz]
Intensity
Time [µs]
x_e = -0.01 mm
σ = 10.23 mm
C_E = -398 ppb
Truth level
x_e = 0.02 mm
σ = 10.71 mm
C_E = -436 ppb
Arbitrary units
Radius [mm]

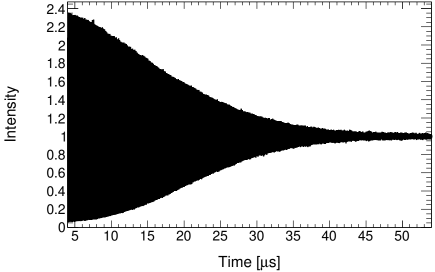
$C = -0.00050$



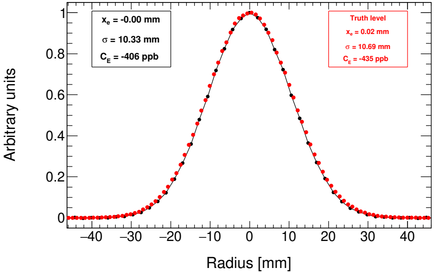
Frequency-Beam Correlation
Frequency (kHz)
Time Offset (ns)
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.08
χ² / ndf 166.3 / 147
p0 6705 ± 0.0
p1 0.02248 ± 0.00041
p2 3.367e−06 ± 1.217e−05



background fit
background
Arbitrary units
Frequency [kHz]

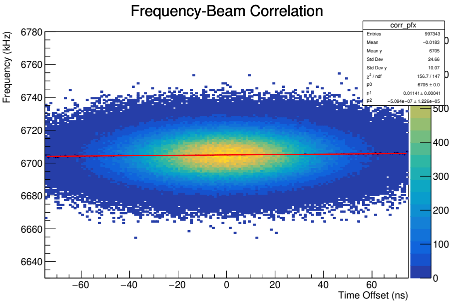


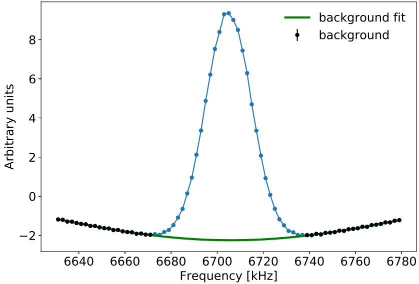
Intensity
Time [μs]

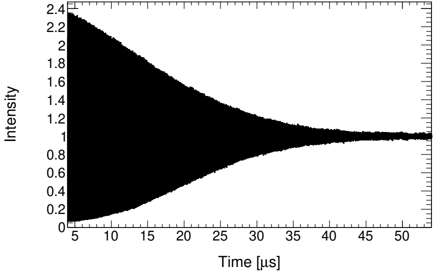


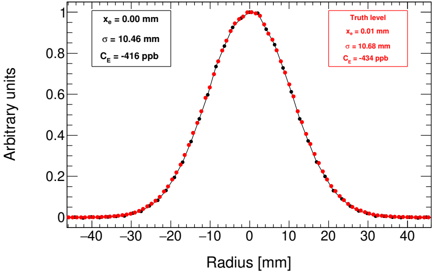
x_e = -0.00 mm
σ = 10.33 mm
C_E = -406 ppb
Truth level
x_e = 0.02 mm
σ = 10.69 mm
C_E = -435 ppb
Arbitrary units
Radius [mm]

$C = -0.00025$

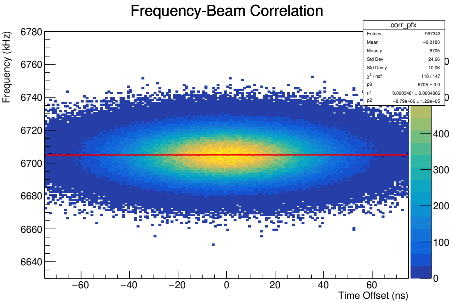

Frequency-Beam Correlation
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.07
χ2 / ndf 156.7 / 147
p0 6705 ± 0.0
p1 0.01141 ± 0.00041
p2 −5.094e−07 ± 1.226e−05
Frequency (kHz)
Time Offset (ns)

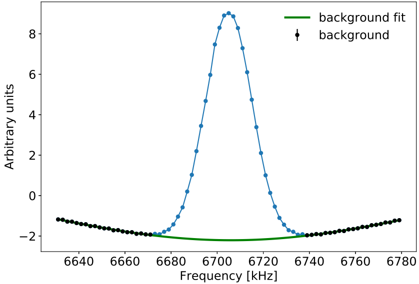

background fit
background
Arbitrary units
Frequency [kHz]

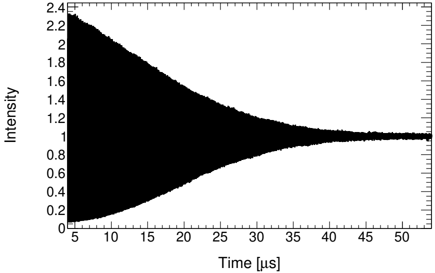

Intensity
Time [µs]

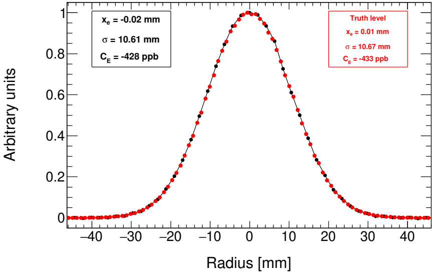

x_e = 0.00 mm
σ = 10.46 mm
C_E = -416 ppb
Truth level
x_e = 0.01 mm
σ = 10.68 mm
C_E = -434 ppb
Arbitrary units
Radius [mm]

$C = +0.00000$

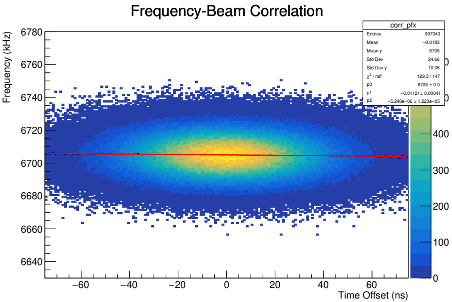

Frequency-Beam Correlation
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.06
χ^2 / ndf 118 / 147
p0 6705 ± 0.0
p1 0.0003481 ± 0.0004086
p2 −8.79e−06 ± 1.22e−05
Frequency (kHz)
Time Offset (ns)

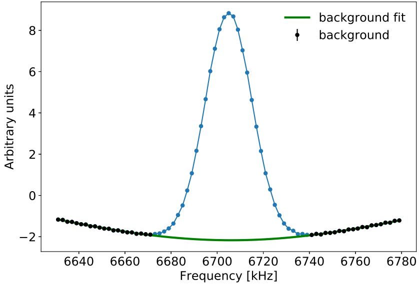

background fit
background
Arbitrary units
Frequency [kHz]

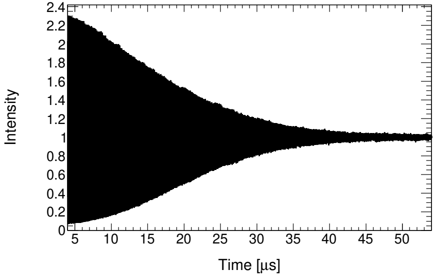

Intensity
Time [µs]

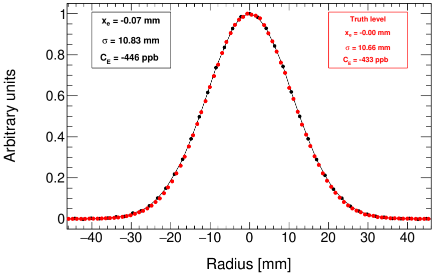

x_e = -0.02 mm
σ = 10.61 mm
C_E = -428 ppb
Truth level
x_e = 0.01 mm
σ = 10.67 mm
C_E = -433 ppb
Arbitrary units
Radius [mm]

$C = +0.00025$

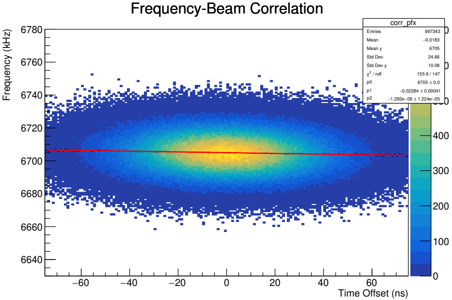

Frequency-Beam Correlation
Frequency (kHz)
Time Offset (ns)
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.06
χ² / ndf 129.3 / 147
p0 6705 ± 0.0
p1 −0.01121 ± 0.00041
p2 −5.248e−06 ± 1.223e−05


background fit
background
Arbitrary units
Frequency [kHz]

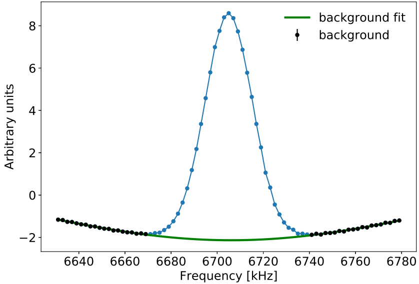

Intensity
Time [µs]


x_e = -0.07 mm
σ = 10.83 mm
C_E = -446 ppb
Truth level
x_e = -0.00 mm
σ = 10.66 mm
C_E = -433 ppb
Arbitrary units
Radius [mm]

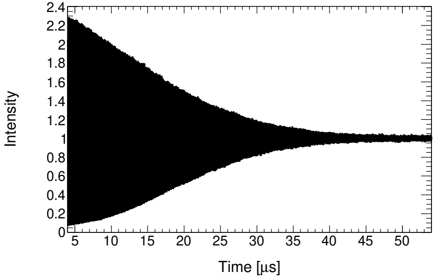
$C = +0.00050$



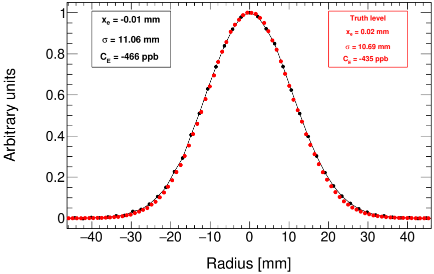
Frequency-Beam Correlation
Frequency (kHz)
Time Offset (ns)
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.08
χ2 / ndf 155.6 / 147
p0 6705 ± 0.0
p1 −0.02284 ± 0.00041
p2 −1.292e−06 ± 1.224e−05



background fit
background
Arbitrary units
Frequency [kHz]

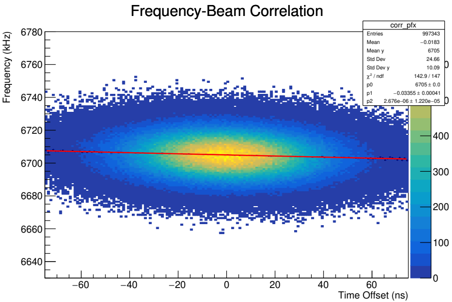


Intensity
Time [µs]

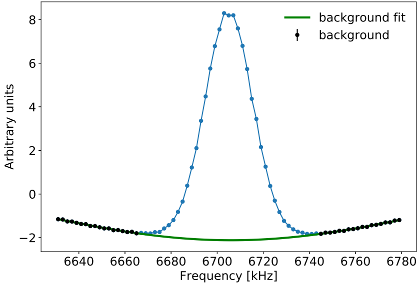


x_e = -0.01 mm
σ = 11.06 mm
C_E = -466 ppb
Truth level
x_e = 0.02 mm
σ = 10.69 mm
C_E = -435 ppb
Arbitrary units
Radius [mm]

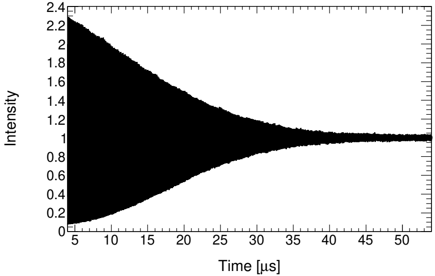
$C = +0.00075$



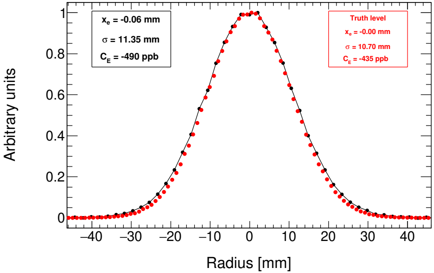
Frequency-Beam Correlation
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.09
χ² / ndf 142.9 / 147
p0 6705 ± 0.0
p1 −0.03355 ± 0.00041
p2 2.676e−06 ± 1.220e−05
Frequency (kHz)
Time Offset (ns)



background fit
background
Arbitrary units
Frequency [kHz]

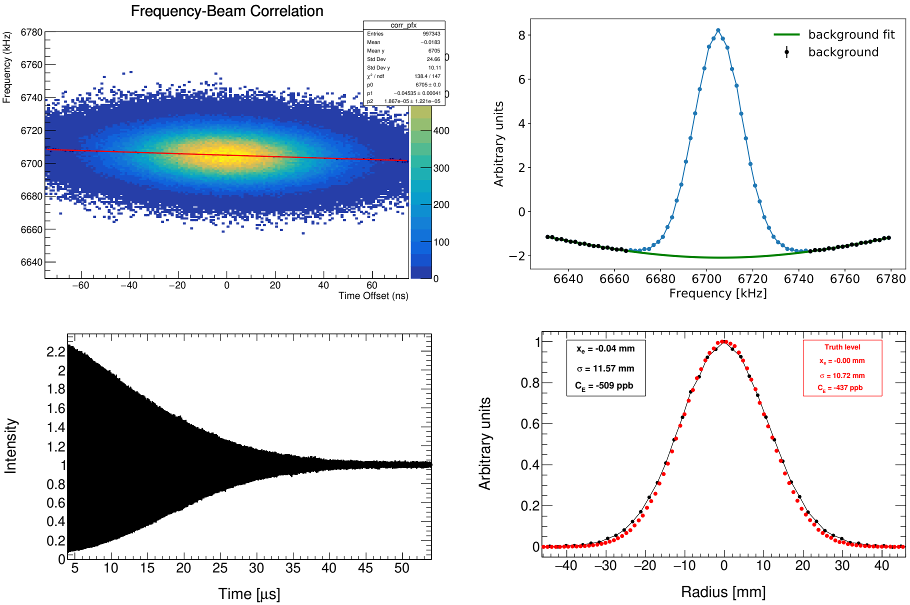


Intensity
Time [µs]



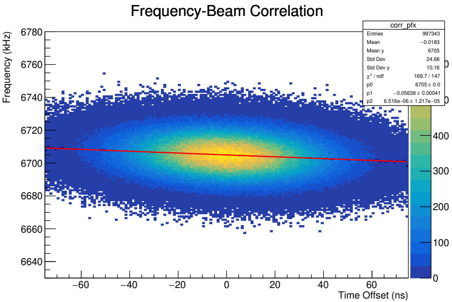
x_e = -0.06 mm
σ = 11.35 mm
C_E = -490 ppb
Truth level
x_e = -0.00 mm
σ = 10.70 mm
C_E = -435 ppb
Arbitrary units
Radius [mm]

$C = +0.00100$

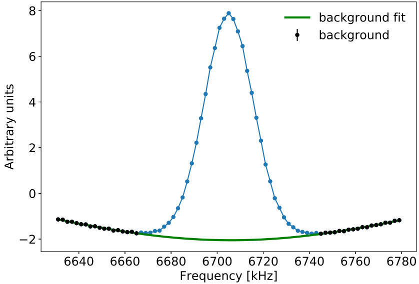


Frequency-Beam Correlation
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.11
χ^2 / ndf 138.4 / 147
p0 6705 ± 0.0
p1 −0.04535 ± 0.00041
p2 1.867e−05 ± 1.221e−05
Frequency (kHz)
Time Offset (ns)
background fit
background
Arbitrary units
Frequency [kHz]
Intensity
Time [µs]
x_e = -0.04 mm
σ = 11.57 mm
C_E = -509 ppb
Truth level
x_e = -0.00 mm
σ = 10.72 mm
C_E = -437 ppb
Arbitrary units
Radius [mm]

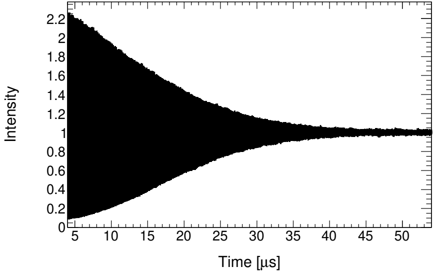
$C = +0.00125$



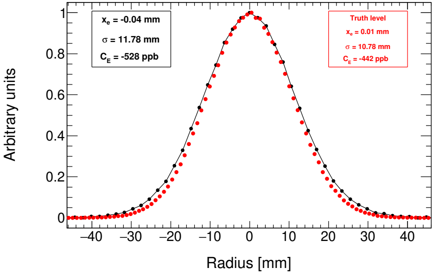
Frequency-Beam Correlation
Frequency (kHz)
6780
6760
6740
6720
6700
6680
6660
6640
-60
-40
-20
0
20
40
60
Time Offset (ns)
400
300
200
100
0
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.16
χ^2 / ndf 169.7 / 147
p0 6705 ± 0.0
p1 −0.05638 ± 0.00041
p2 6.516e−06 ± 1.217e−05



background fit
background
Arbitrary units
8
6
4
2
0
−2
6640
6660
6680
6700
6720
6740
6760
6780
Frequency [kHz]

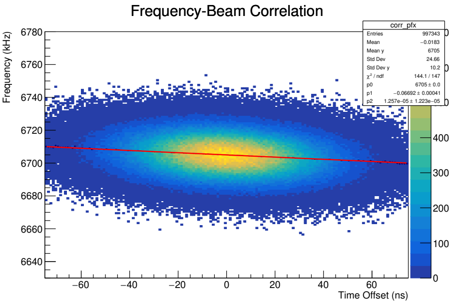


Intensity
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
5
10
15
20
25
30
35
40
45
50
Time [µs]

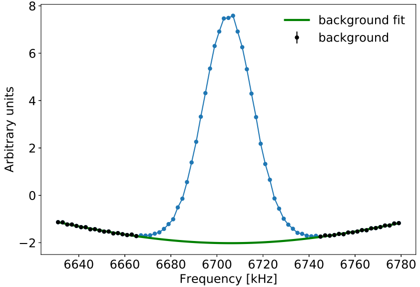


x_e = -0.04 mm
σ = 11.78 mm
C_E = -528 ppb
Truth level
x_e = 0.01 mm
σ = 10.78 mm
C_E = -442 ppb
Arbitrary units
1
0.8
0.6
0.4
0.2
0
−40
−30
−20
−10
0
10
20
30
40
Radius [mm]

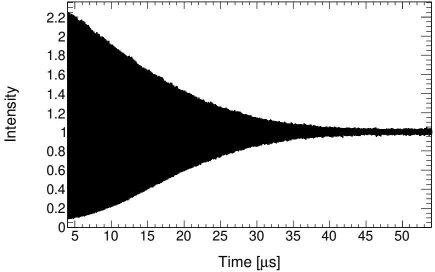
$C = +0.00150$



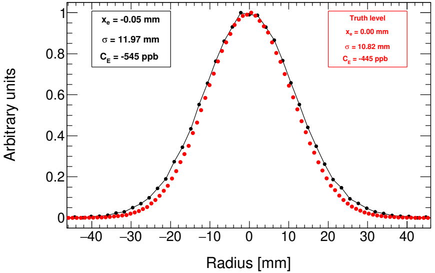
Frequency-Beam Correlation
Frequency (kHz)
Time Offset (ns)
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
Std Dev y 10.2
χ^2 / ndf 144.1 / 147
p0 6705 ± 0.0
p1 −0.06692 ± 0.00041
p2 1.257e−05 ± 1.223e−05



background fit
background
Arbitrary units
Frequency [kHz]



Intensity
Time [µs]



x_e = -0.05 mm
σ = 11.97 mm
C_E = -545 ppb
Truth level
x_e = 0.00 mm
σ = 10.82 mm
C_E = -445 ppb
Arbitrary units
Radius [mm]

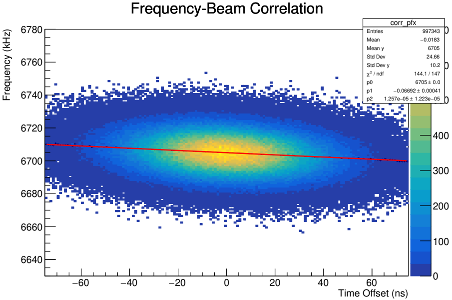
Correlation Scan

Early Start Time

In the slide that follows...

- $t_s \approx t_0$ and $t_m = 300\ \mu\mathrm{s}$
- t_0 optimization enabled
    - quadratic background fit
    - 3 iterations
    - reached $\chi^2/\mathrm{ndf} \approx 1$
    - converged to expected t_0 (i.e. mean time on first turn)

$C = +0.00150$



Frequency-Beam Correlation
corr_pfx
Entries 997343
Mean −0.0183
Mean y 6705
Std Dev 24.66
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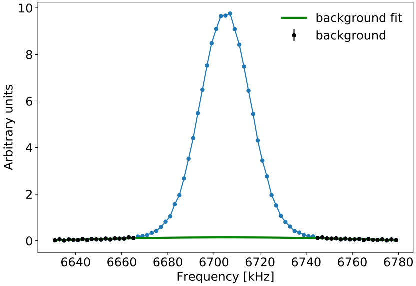
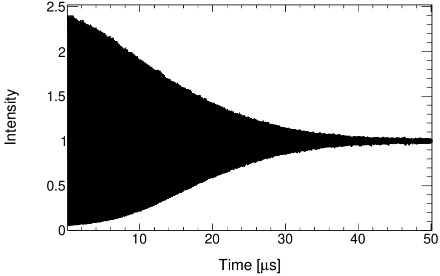
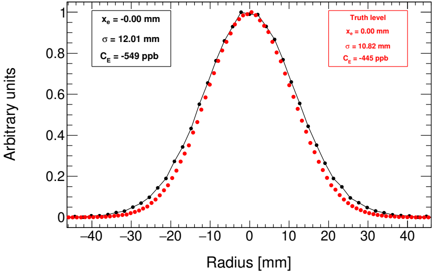
χ² / ndf 144.1 / 147
p0 6705 ± 0.0
p1 −0.06692 ± 0.00041
p2 1.257e−05 ± 1.223e−05
Frequency (kHz)
Time Offset (ns)



background fit
background
Arbitrary units
Frequency [kHz]



Intensity
Time [µs]



x_e = -0.00 mm
σ = 12.01 mm
C_E = -549 ppb
Truth level
x_e = 0.00 mm
σ = 10.82 mm
C_E = -445 ppb
Arbitrary units
Radius [mm]

Conclusion

Summary

- linear correlation in simulated fast rotation signal leads to incorrect E-field correction from Fourier analysis method
  - not due to start time t_s, background fit, t_0, etc.
  - variation of about ± 100 ppb across this scan
- currently do not have any good indicators for the amount of correlation in the fast rotation signal
  - effect could be in real data, and we might not know

Next Steps

- identifying indicators for the presence of correlation in the signal
- exploring modifications to the Fourier method

Acknowledgment

This material is based upon work supported by the National Science Foundation Graduate Research Fellowship under Grant No. DGE-1650441.

Monte Carlo

$$S(t) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \xi(t') \frac{\rho(\frac{t-t'}{nT+t_0} - 1)}{nT+t_0} dt' \longrightarrow S'(t) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \xi(t') \frac{\rho(\frac{t-t'}{nT+t_0} - 1 + r\frac{t'-t_\xi}{T})}{nT+t_0} dt'$$

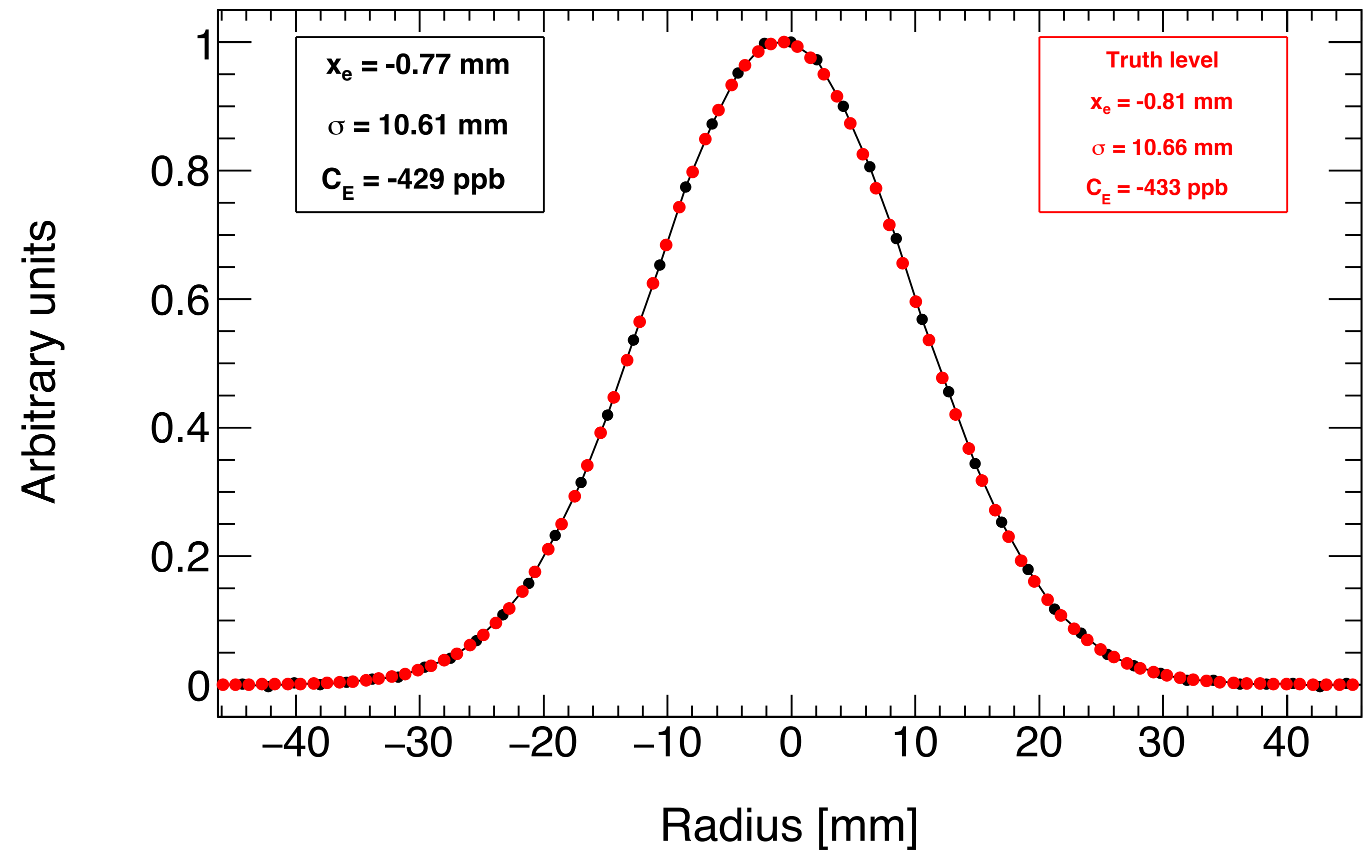
Change in fast rotation signal

$$\rho(\Delta) \longrightarrow \rho'(\Delta) = \int_{-\infty}^{\infty} \xi(t') \rho\Big(\Delta + r\frac{t'-t_\xi}{T}\Big) dt'$$

Change in "true frequency distribution"

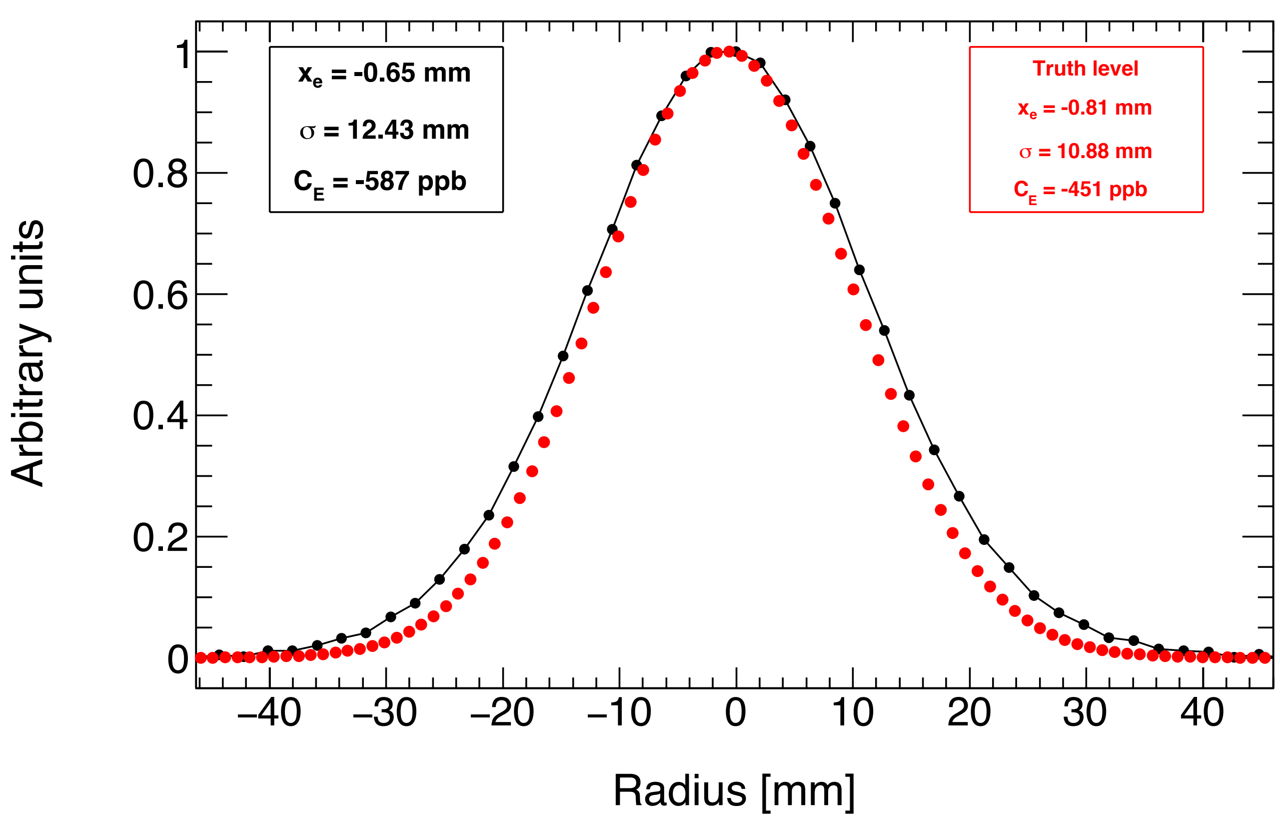
- Monte Carlo works by direct numerical integration for a given frequency distribution rho and longitudinal beam profile xi.
- The derivation of the Monte Carlo method and user-guide to will be posted to docdb along the code and config file so anyone can use it.

Exact Same Problem As Tyler

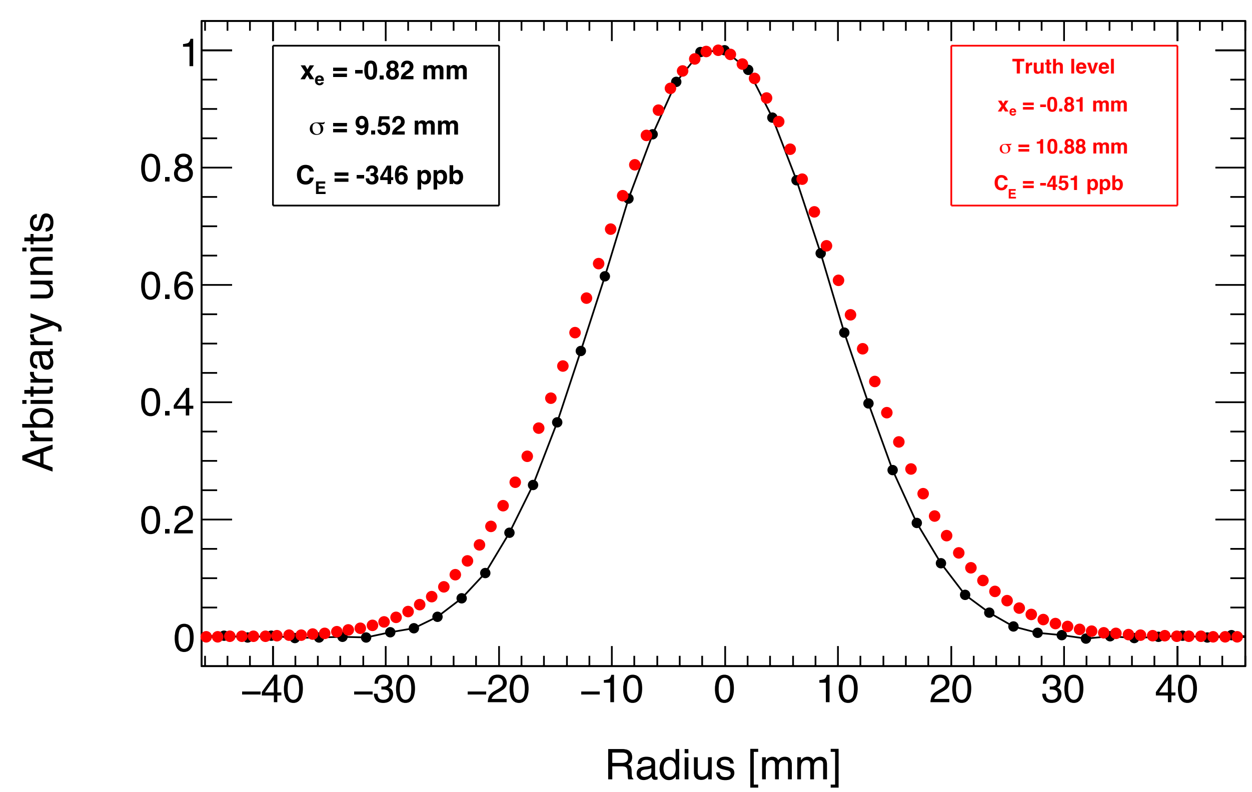


No correlation

correlation +0.15%



correlation -0.15%



Correlation Scan