Efield and Pitch Corrections

D. Rubin - for the Efield/Pitch working group

Electric field

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{m^2}{p^2} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

$$C_e \sim -2 \frac{\Delta p}{p} \langle \frac{\vec{\beta} \times \vec{E}}{Bc} \rangle$$

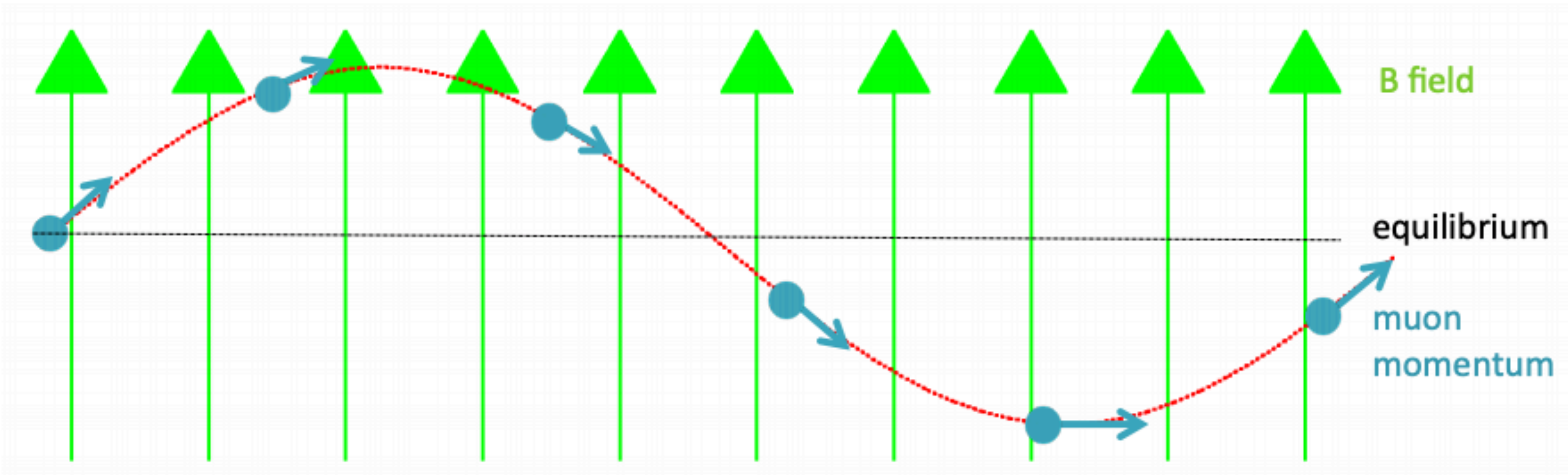
- Measure Δp/p and E-field
- As long as the quadrupole field is linear in displacement

$$\begin{aligned} \langle E_r \rangle &= n \left(\frac{v_s B}{R_0} \right) x_e \\ \frac{\Delta p}{p} &= \frac{x_e}{\eta} \\ C_e &\sim -2\beta^2 n(1-n) \frac{x_e^2}{R_0^2} \end{aligned}$$

Measurement of radial closed orbit, x_e => E-field correction

Why do we need a pitch correction?

- Muons are going up-and-down in the ring (focused by quads):

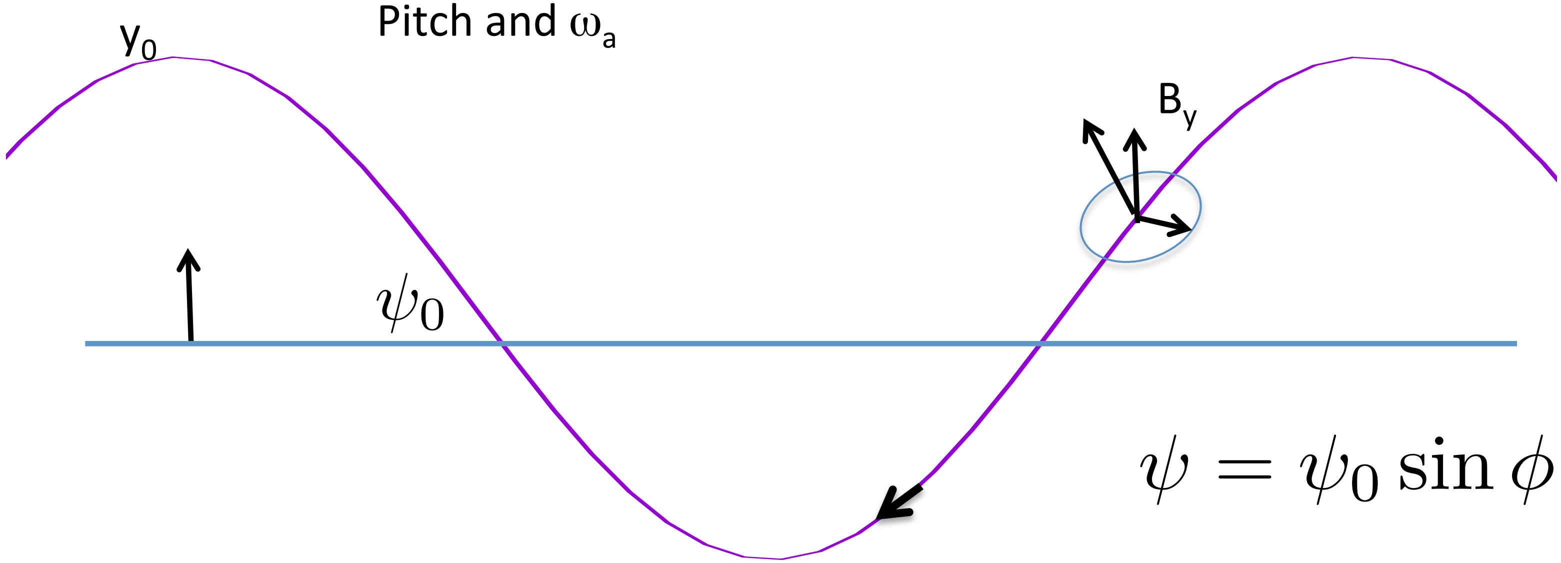


Stolen from James Mott

Precession in a single revolution is independent of vertical oscillation

But the revolution period is not.

Pitch and ω_a



$$\psi = \psi_0 \sin\phi$$

$$\omega_a = \nu\omega_c$$

We measure precession about the axis perpendicular to the direction of motion.

- The component of the magnetic field along that perpendicular axis is B cos ψ.
- The spin tune $\nu \propto \oint B_\perp dl = \oint B \cos\psi dl$
- Path length $\sim L(1 + \frac{1}{4}\psi_0^2)$
- => spin tune (ν) is independent of pitch
- But $\omega_c(\psi_0) \sim \omega_c(0)(1 - \frac{1}{4}\psi_0^2)$ $\rightarrow \omega_a(\psi_0) = \omega_a(0)(1 - \frac{1}{4}\psi_0^2)$

In the limit of continuous and perfectly aligned quads, with linear dependence of E-field on displacement, the contribution to ω_a from

Electric field

$$C_e \quad \sim \quad -2\beta^2 n(1-n)\frac{x_e^2}{R_0^2}$$ Single muon

$$\langle C_e \rangle = -2\beta^2 n(1-n)\frac{n\langle x_e^2\rangle}{R_0^2}$$ For the distribution

Pitching angle

$$C_p = -\frac{1}{4}\psi_0^2$$ Single muon

$$\langle C_p \rangle = -\frac{n\langle y^2\rangle}{2R_0^2}$$ For the distribution

In the limit of continuous and perfectly aligned quads, with linear dependence of E-field on displacement, the contribution to ω_a from

Electric field

$$C_e \quad \sim \quad -2\beta^2 n(1-n)\frac{x_e^2}{R_0^2}$$ Single muon

$$\langle C_e\rangle = -2\beta^2 n(1-n)\frac{n\langle x_e^2\rangle}{R_0^2}$$ For the distribution

Pitching angle

$$C_p = -\frac{1}{4}\psi_0^2$$ Single muon

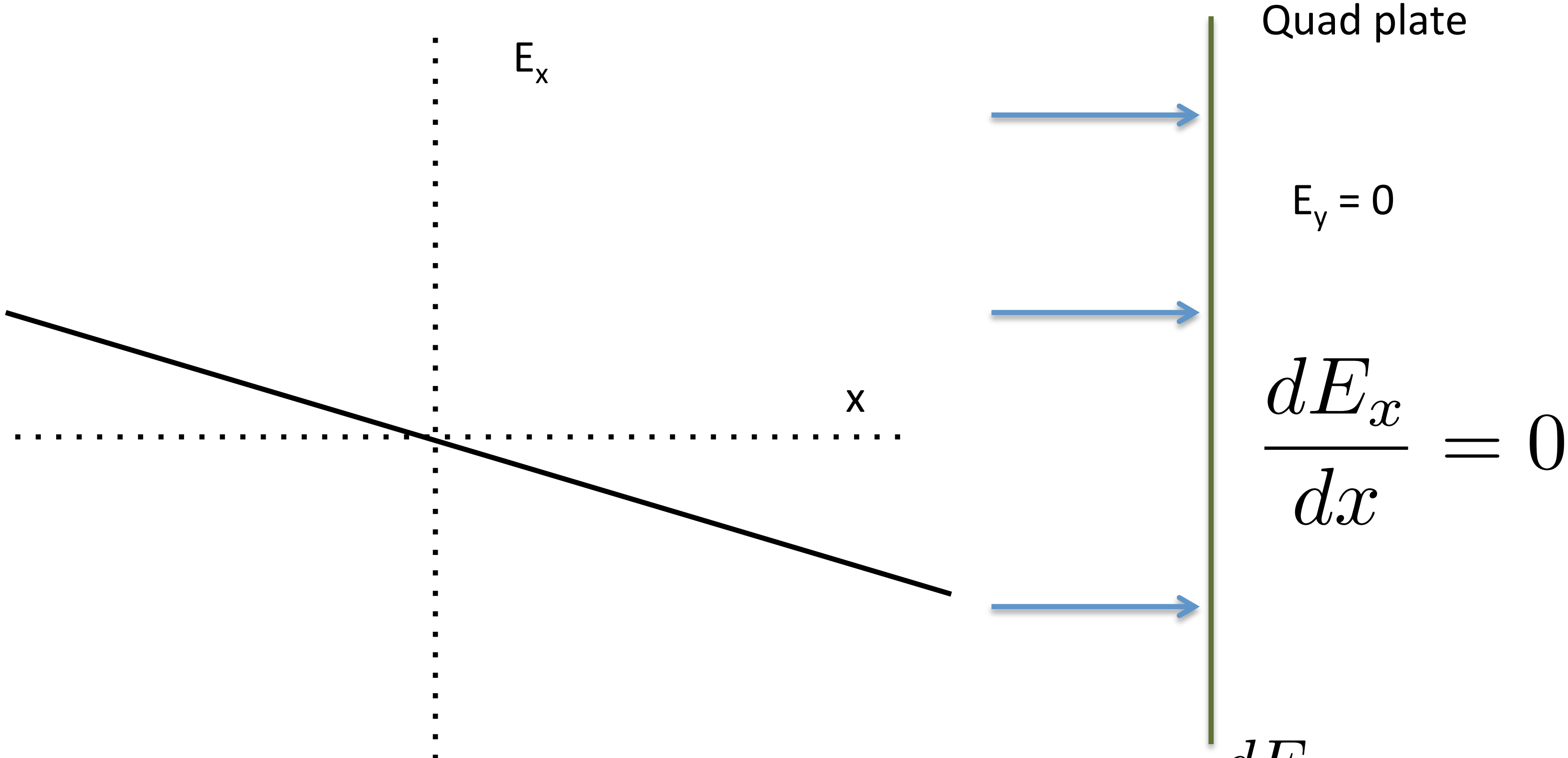
$$\langle C_p\rangle = -\frac{n\langle y^2\rangle}{2R_0^2}$$ For the distribution

Bottom line

1. *How well can we measure* $\langle x_e^2\rangle$, $\langle y^2\rangle$, *and* n?
2. *What is the effect of nonlinearity, voltage errors and misalignment?*

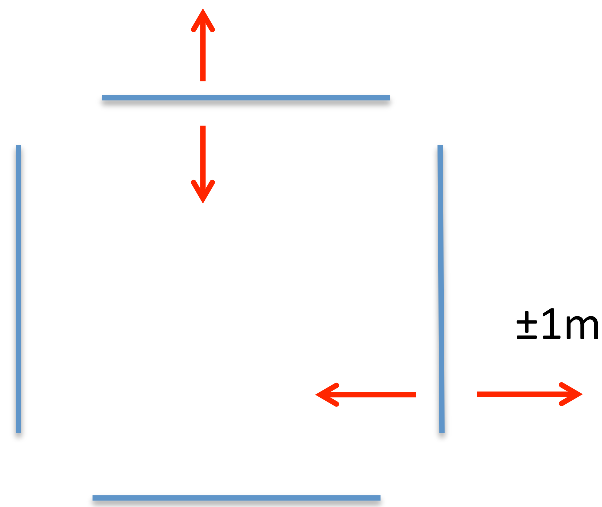
Quad Nonlinearity

- E_r is not simply linear in x
- Index n and dispersion η depend on x
- Quad curvature => quadratic dependence (sextupole-like)
- Sextupole component => amplitude dependent shift of the closed orbit



Quad index n is proportional to $\dfrac{dE_x}{dx}$

Systematically explore dependence of E-field and Pitch correction on field and alignment errors and nonlinearity with simulation



Quad plates

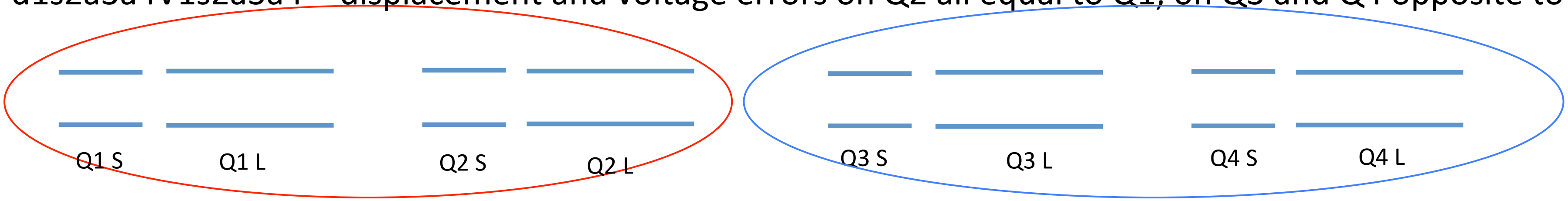
$\Delta V_p = \pm 5\% \ V_0$

There are 2^4 combinations of displacement errors. Two for each plate and 4 plates
There are 2^4 combinations of voltage errors. Two for each plate and 4 plates
=> 256 combinations of displacement and voltage errors

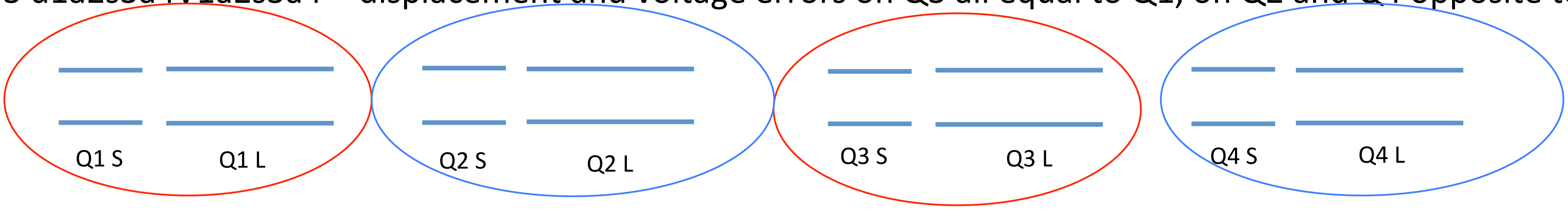
For each quad

#1 d1s2s3s4V1s2s3s4 - displacement and voltage errors for all inner/bottom/top/bottom plates the same

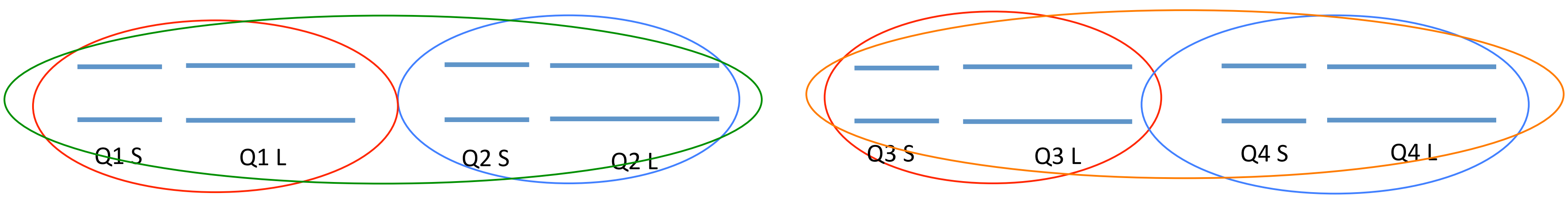
#2 d1s2a3a4V1s2a3a4 - displacement and voltage errors on Q2 all equal to Q1, on Q3 and Q4 opposite to Q1



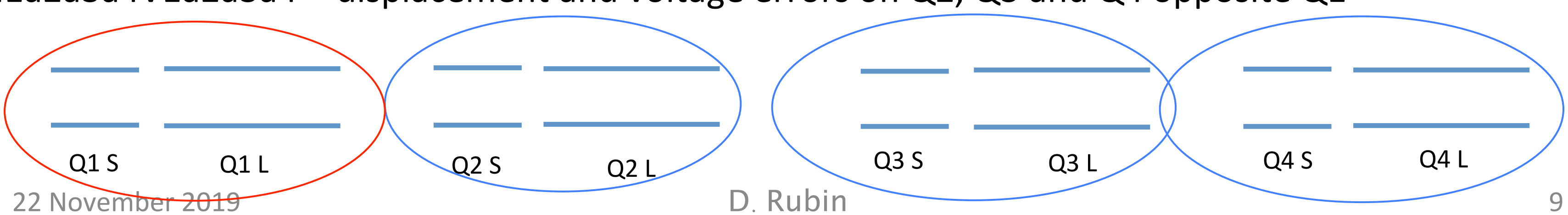
#3 d1a2s3a4V1a2s3a4 - displacement and voltage errors on Q3 all equal to Q1, on Q2 and Q4 opposite to Q1



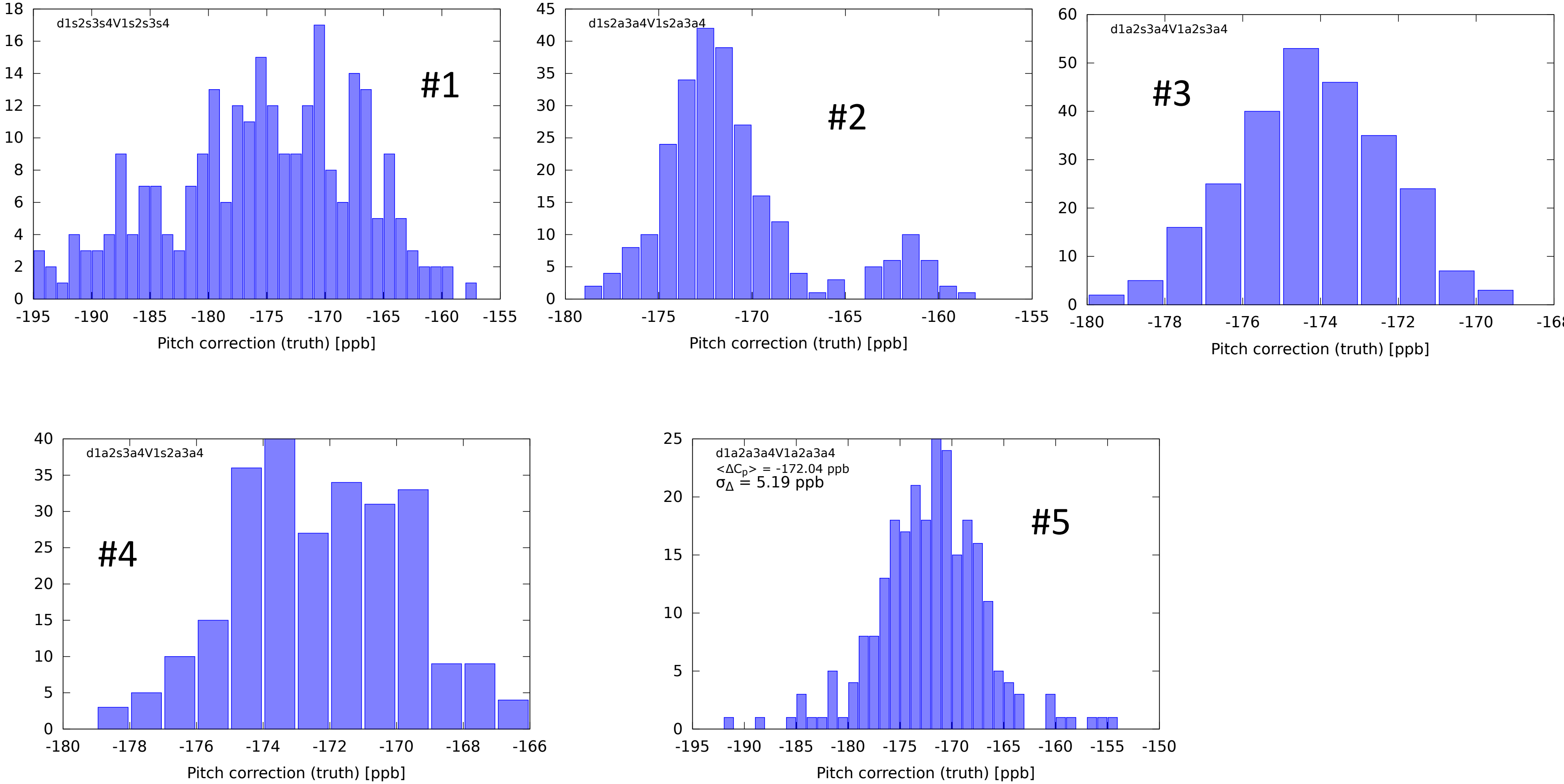
#4 d1a2s3a4V1s2a3a4 v- disp errors Q3 = Q1, Q2 & Q4 opposite Q1. volt errors Q2=Q1, Q3 & Q4 opposite Q1



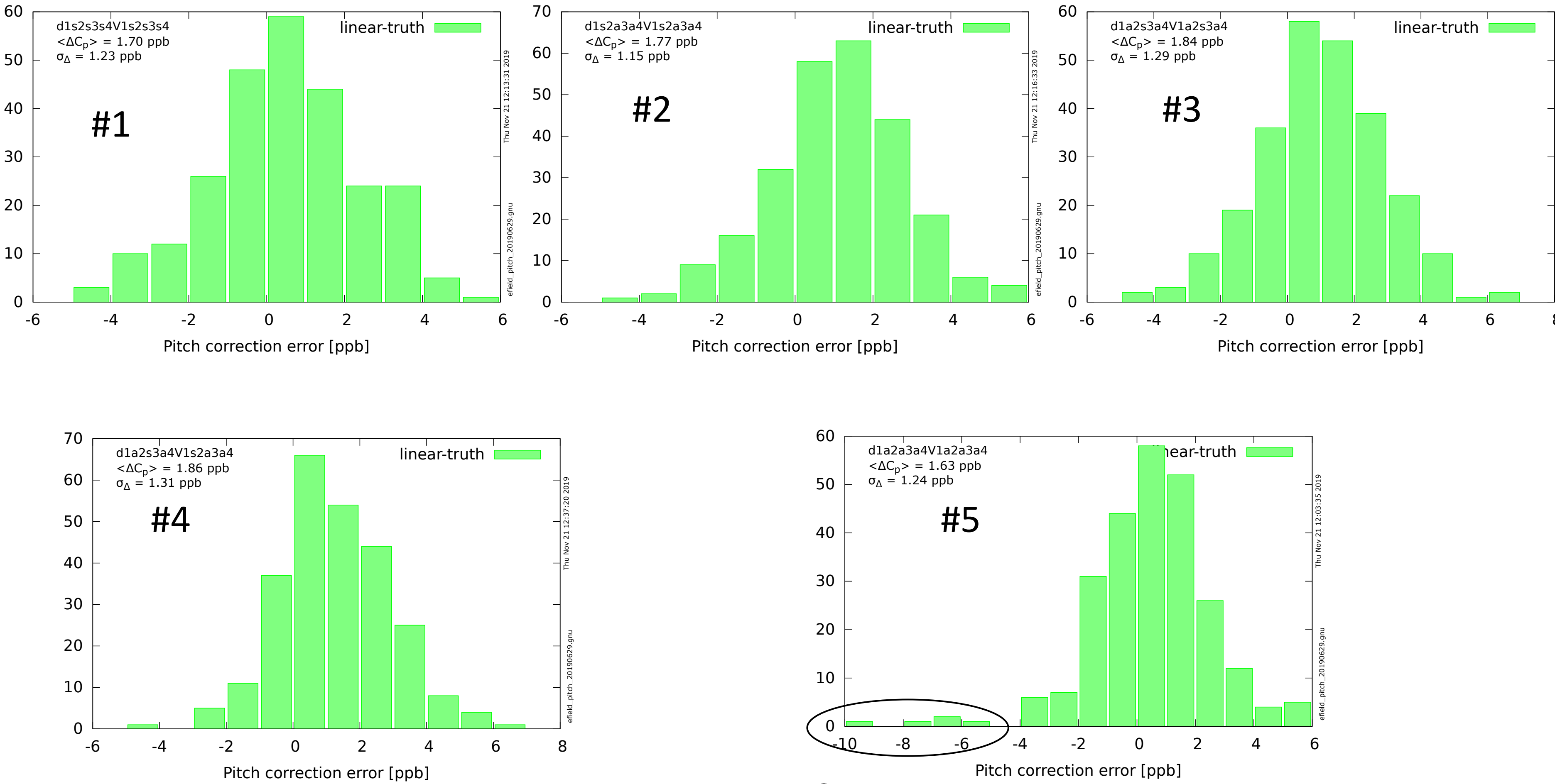
#5 d1a2a3a4V1a2a3a4 - displacement and voltage errors on Q2, Q3 and Q4 opposite Q1



Pitch correction - truth



Pitch correction –measured-truth



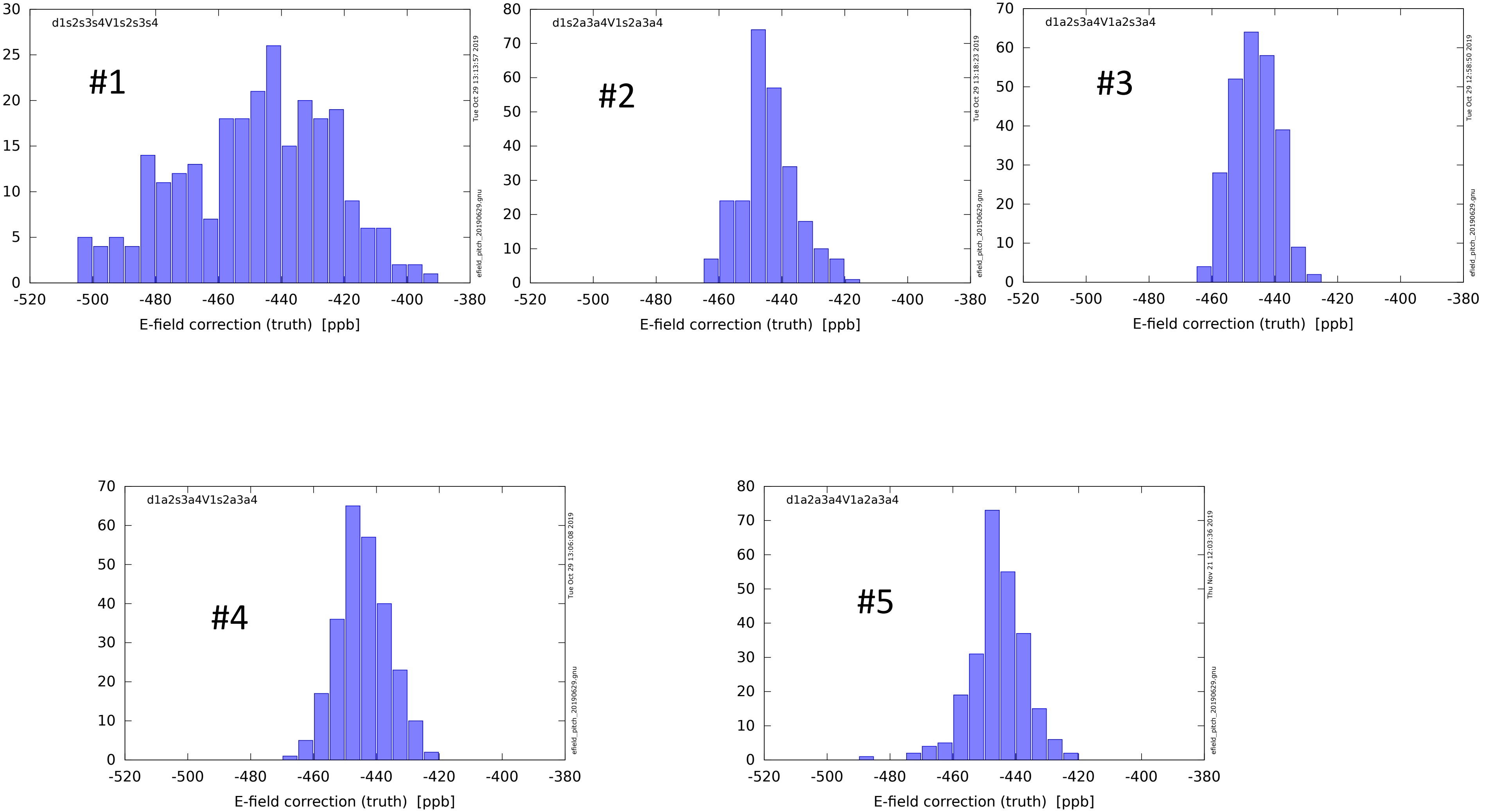
$$C_p(\text{meas}) = -\frac{n_y \langle y^2 \rangle}{2R_0^2}$$

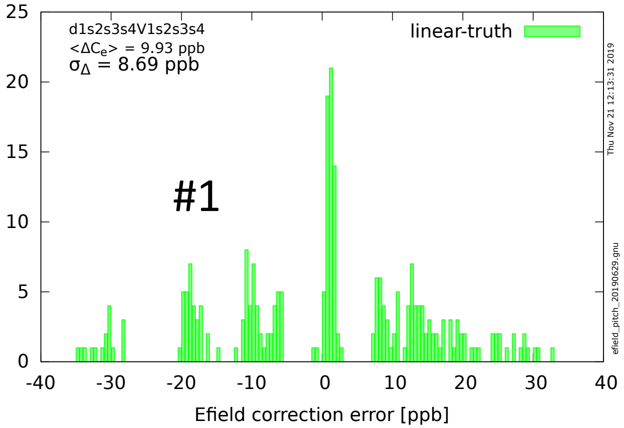
$$|C_p(\text{meas}) - C_p(\text{truth})| < 10 \text{ ppb}$$

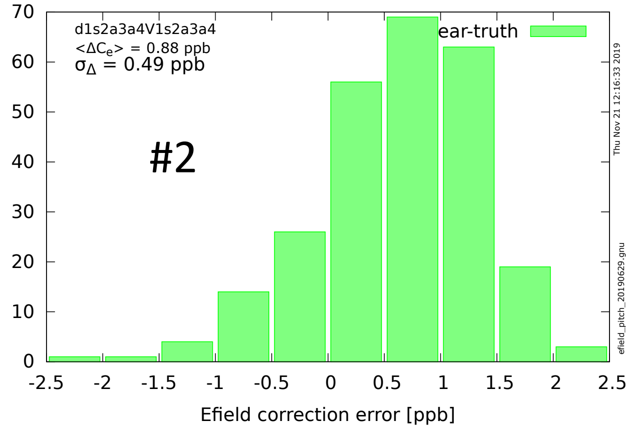
$$\sigma_\Delta \leq 1.3 \text{ ppb}$$

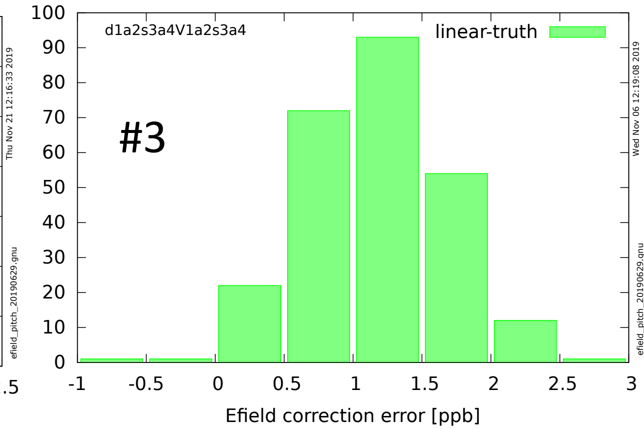
(1280 configurations)

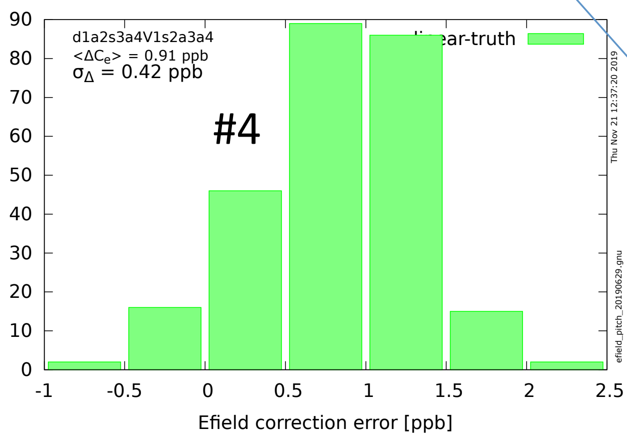
Efield correction - truth

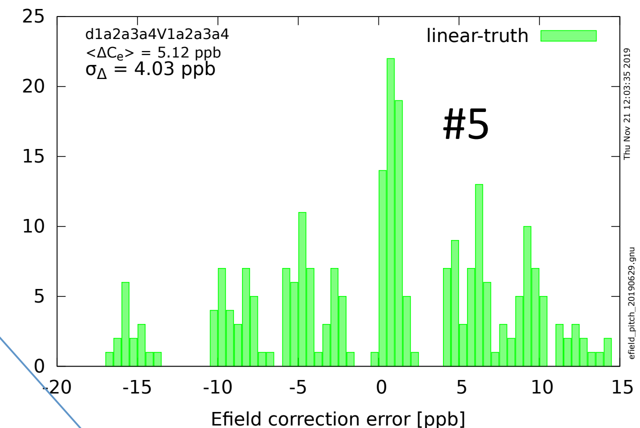










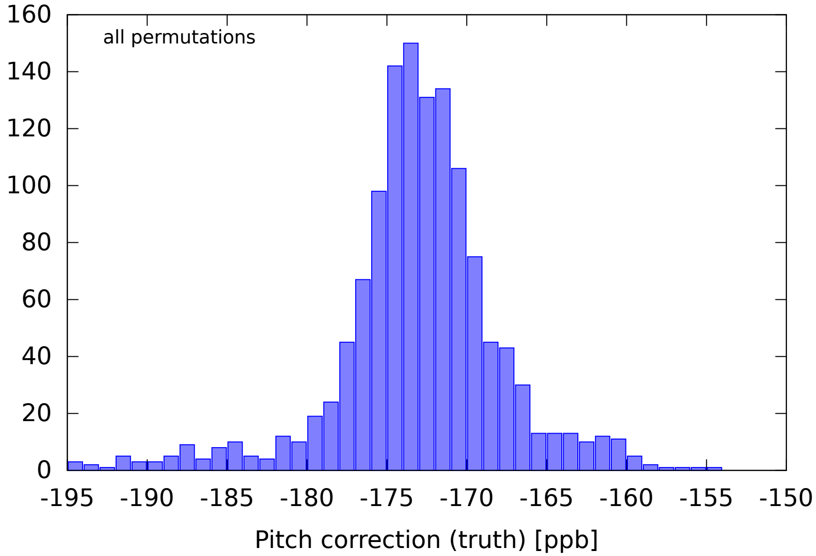


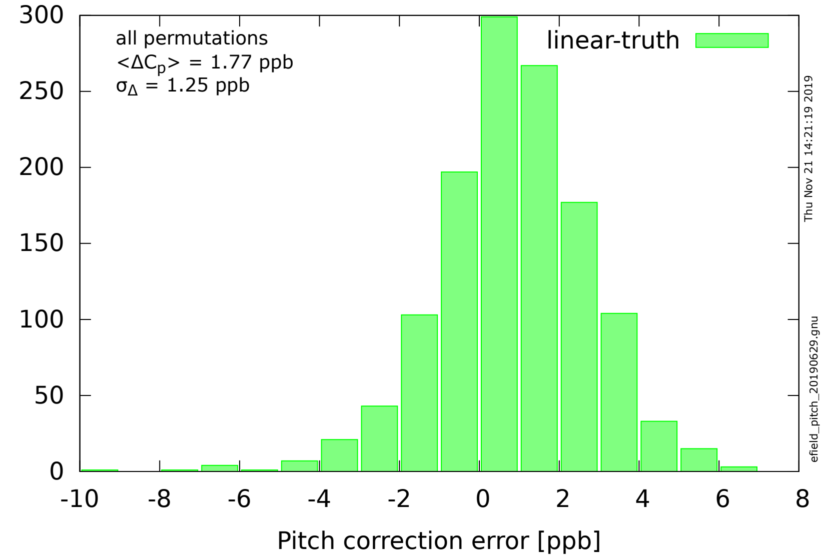
$$C_e(\mathrm{meas}) = -2\beta^2 n_x(1-n_x)\frac{\langle x_e^2\rangle}{R_0^2}$$

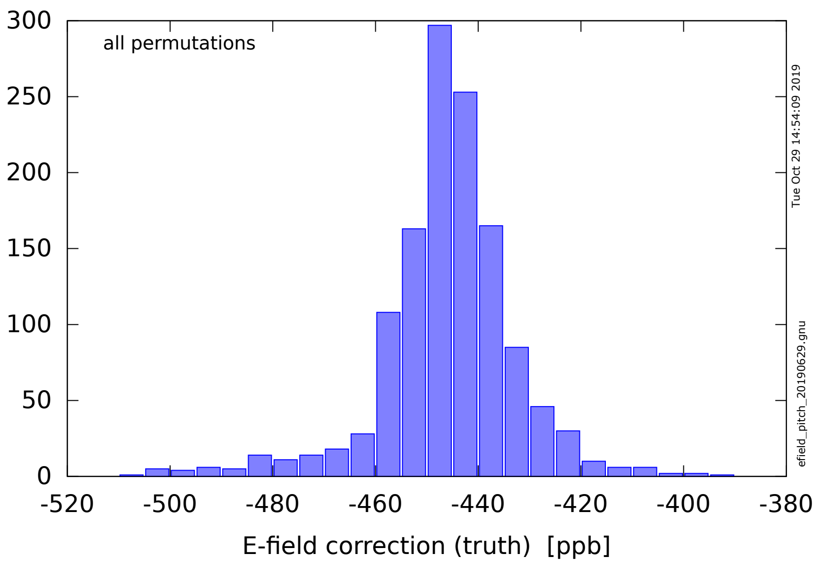
$$|C_e(\mathrm{meas}) - C_e(\mathrm{truth})| < 35\ \mathrm{ppb}$$

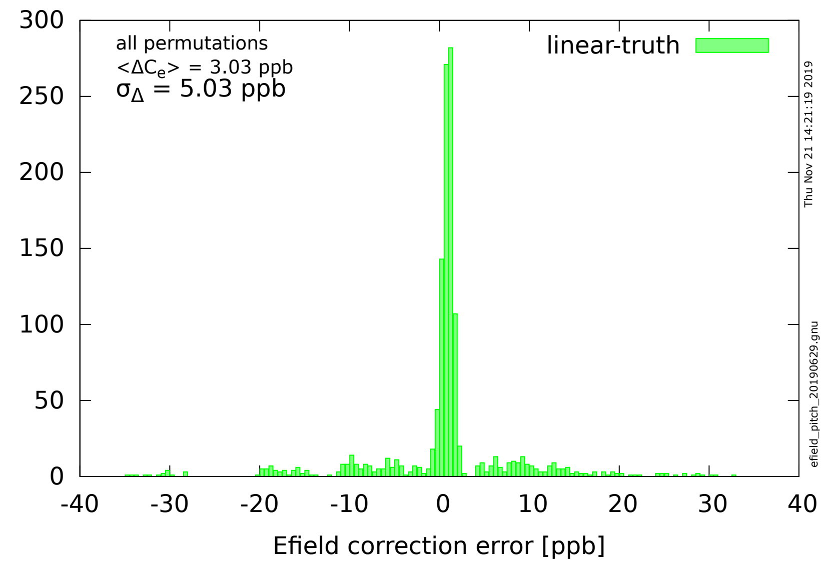
$$\sigma_\Delta \leq 8.69\ \mathrm{ppb}$$

All 1280 configurations





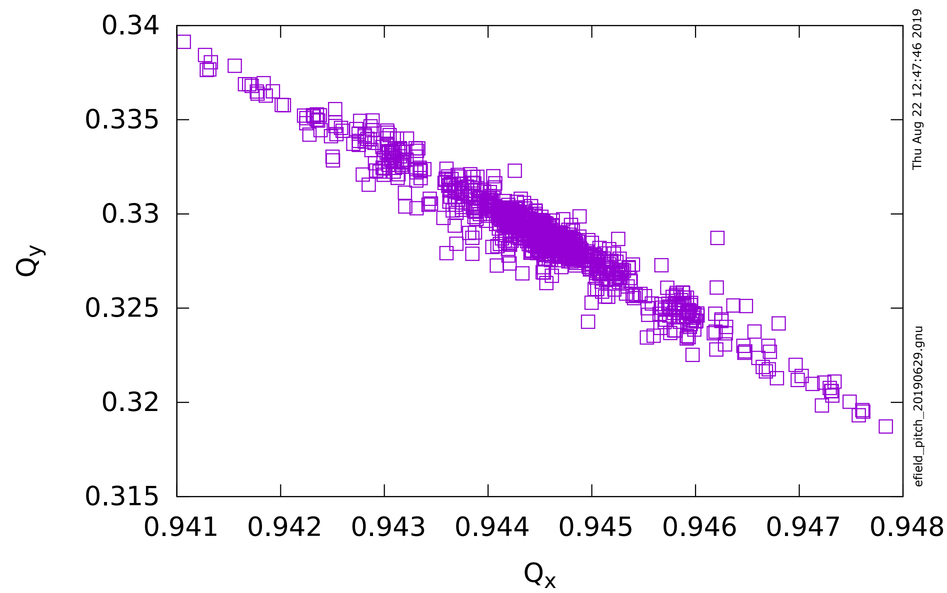




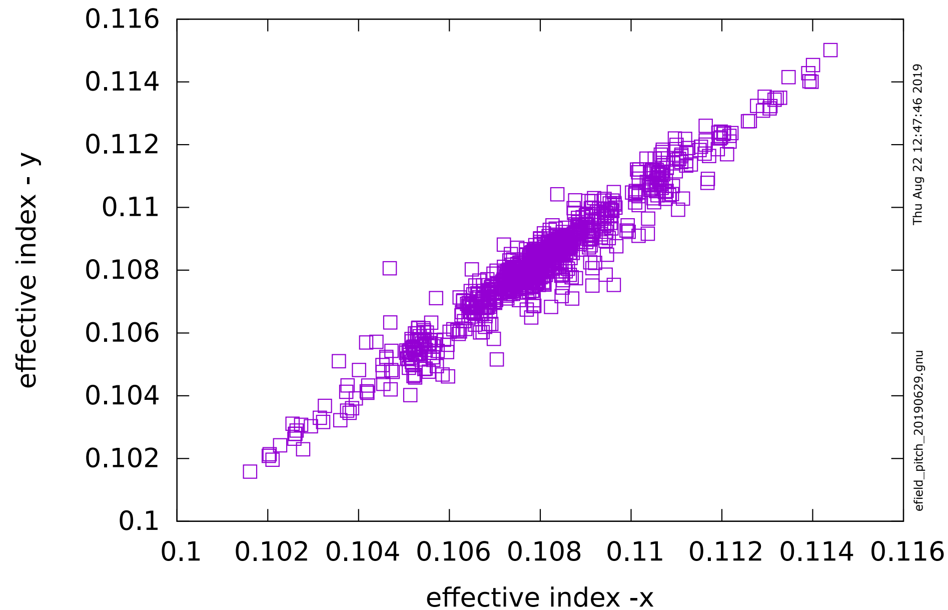
truth

measured - truth

The betatron frequencies of the horizontal and vertical motion of the centroids => Qx, Qy



$$n_y = Q_y^2$$
$$n_x = 1 - Q_x^2$$



Systematic uncertainty due to quad misalignment/voltage errors for 1280 configurations

Efield

$$C_e(\text{meas}) = -2\beta^2 n_x(1-n_x)\frac{\langle x_e^2\rangle}{R_0^2}$$

$$|C_e(\text{meas}) - C_e(\text{truth})| < 35 \text{ ppb}$$

$$\sigma_\Delta \leq 8.69 \text{ ppb}$$

Where $n_x = 1 - Q_x^2$ and $x_e = \dfrac{\beta c}{2\pi f_{cyc}} - R_0$

Pitch

$$C_p(\text{meas}) = -\frac{n_y\langle y^2\rangle}{2R_0^2}$$

$$|C_p(\text{meas}) - C_p(\text{truth})| < 10 \text{ ppb}$$

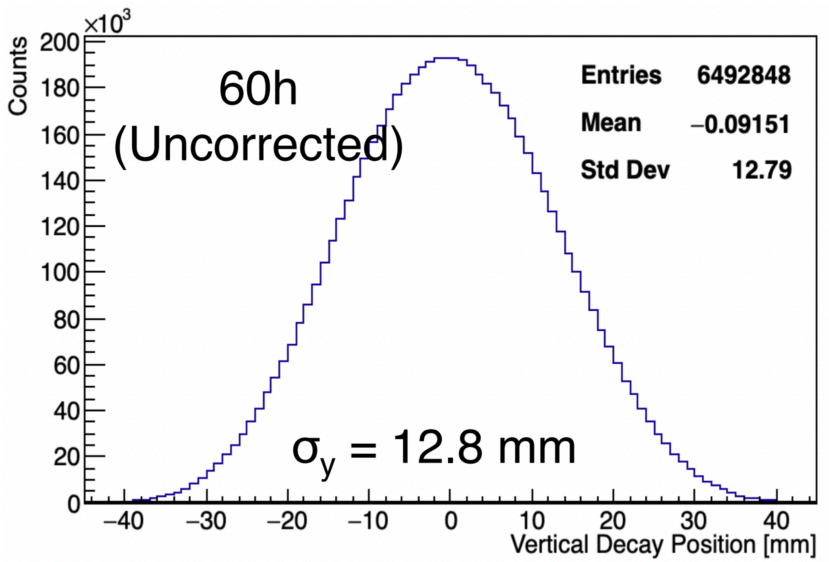
$$\sigma_\Delta \leq 1.3 \text{ ppb}$$

Where $n_y = Q_y^2$

Pitch Measurement

- Measure vertical position of decay muon by reconstructing trajectory of positron in straw tracker
  - All muons => <y^2>
  - <y(t)> => vertical tune and $n = Q_y^2$

$$\langle C_p \rangle = -\frac{n\langle y^2 \rangle}{2R_0^2}$$



For us (60h):
σ_y ~ 12.5 mm, y^2 ~ 150 mm^2
C_p ~ 160 ppb

J. Mott

Tracker systematics

- Alignment
  - External Alignment
  - Internal Alignment
  - Detector curvature
  - Straw angle alignment
- Tracking algorithm
  - Time to distance
  - t_0 offset
  - Material Density
  - Track finding
  - Measurement resolution
- Cross-talk
- Lost muons

- Tracking resolution
- Beam spot resolution
- Tracker Acceptance
- Calo acceptance

A small fraction of the muons that contribute to measurement of ω_a generate tracker hits

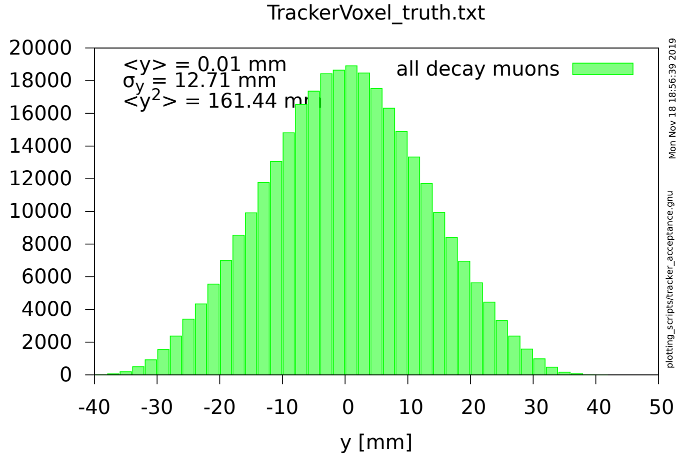
J. Mott

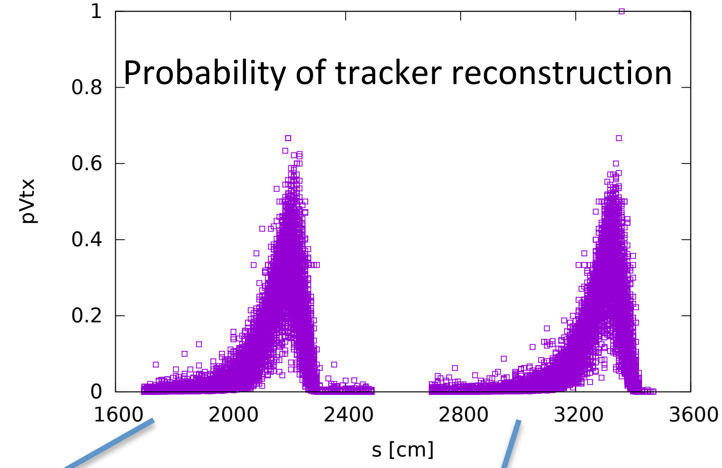
Uncertainties Summary:

Systematic	C_p systematic [ppb]
Tracking	8.6
Vertex Resolution	3
Total	**9.1**

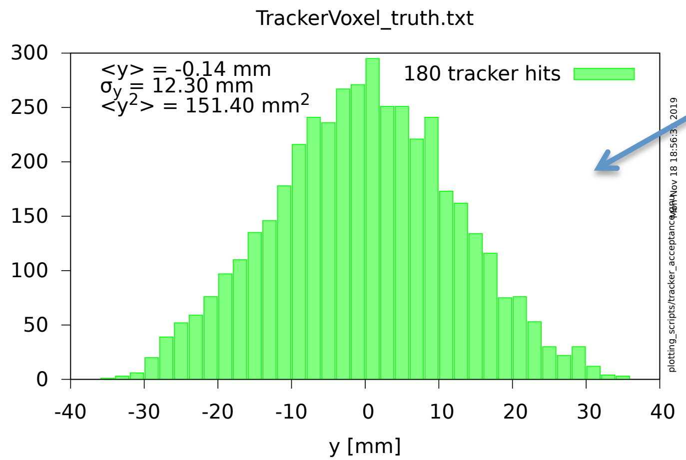
- Uncertainty excluding acceptance is ~10 ppb
- Acceptance correction itself will be less than 20 ppb with small error
- Simulation/Model uncertainty is also 10 ppb
- Total correction expected to be **~160 ± 15 ppb**.
- *Misalignment/voltage error ~ 2 ppb*

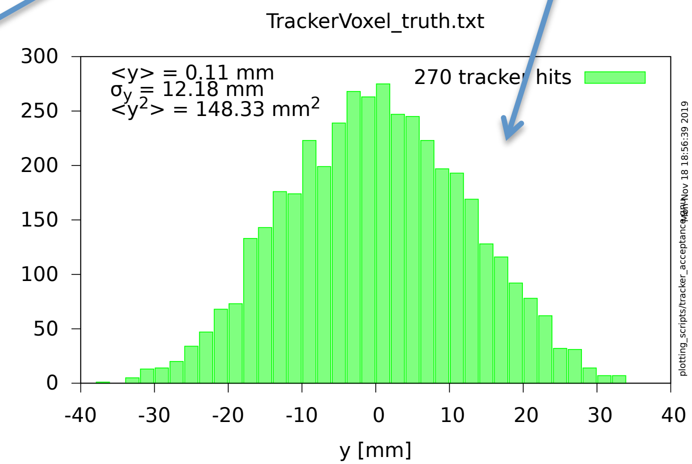
J. Mott





simulation



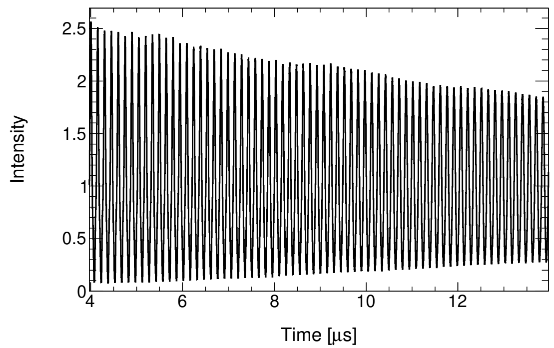


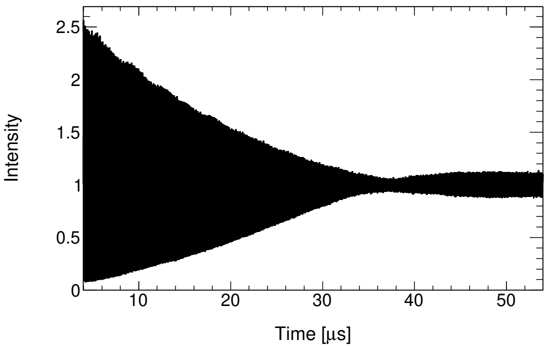
=> Acceptance correction < 20 ppb

Efield correction measurement

Extract momentum distribution (or equivalently equilibrium radial distribution) from *fast rotation signal*

$$\langle C_e \rangle = -2\beta^2 n(1-n)\frac{n\langle x_e^2\rangle}{R_0^2}$$



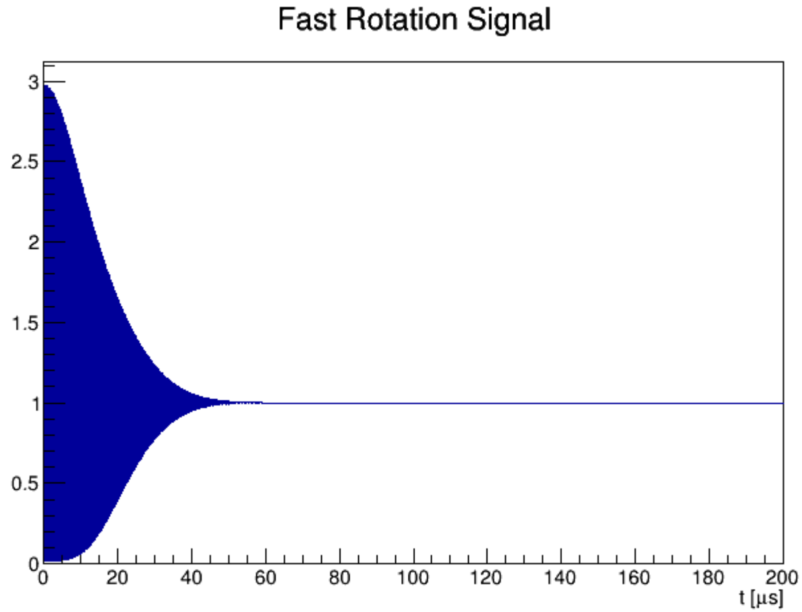


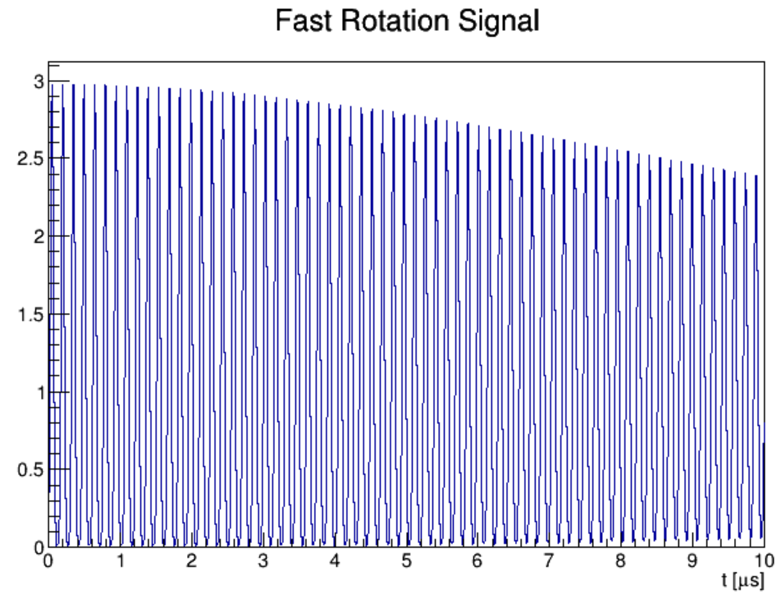
Two methods

- Fourier method
- CERN III (χ^2) method

The fast rotation signal is comprised of all calo hits above threshold

No acceptance correction required





At $t = 0$, beam is maximally bunched. As $t \to \infty$, muons distributed uniformly around ring

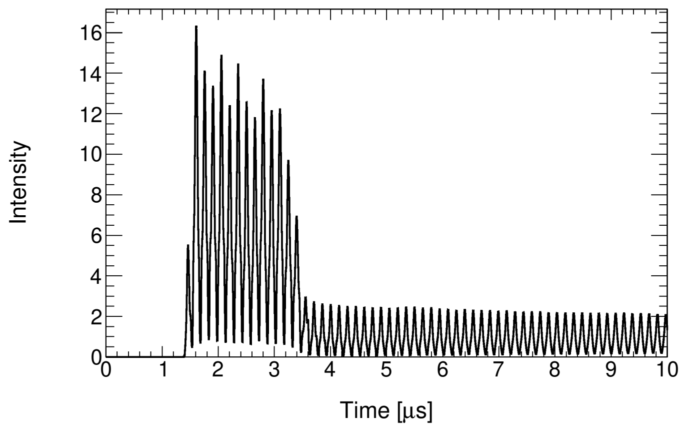
S(t) is symmetric about $t = 0$

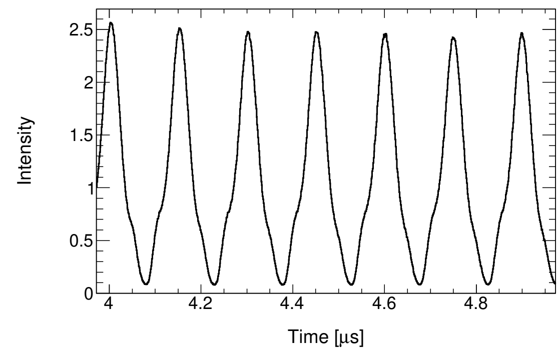
$$\hat{S}(\omega) = \int_{t_s}^{t_m} S(t)\cos[\omega(t - t_0)]dt$$

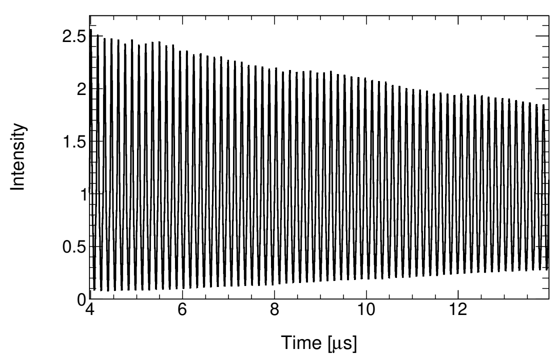
Fourier method

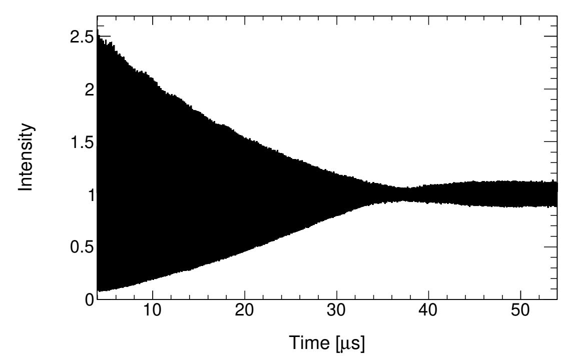
J. Fagin

First 4 μs contaminated by positrons - $t_s > 4\mu s$



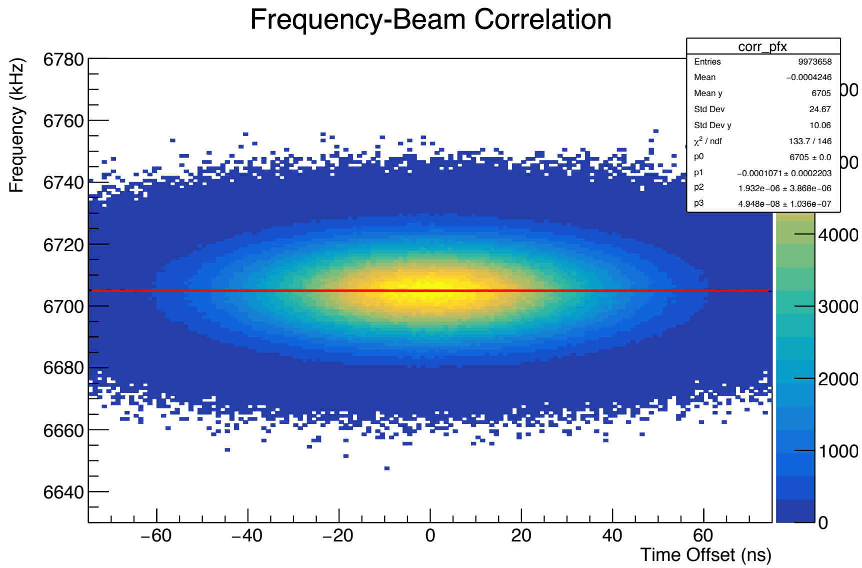


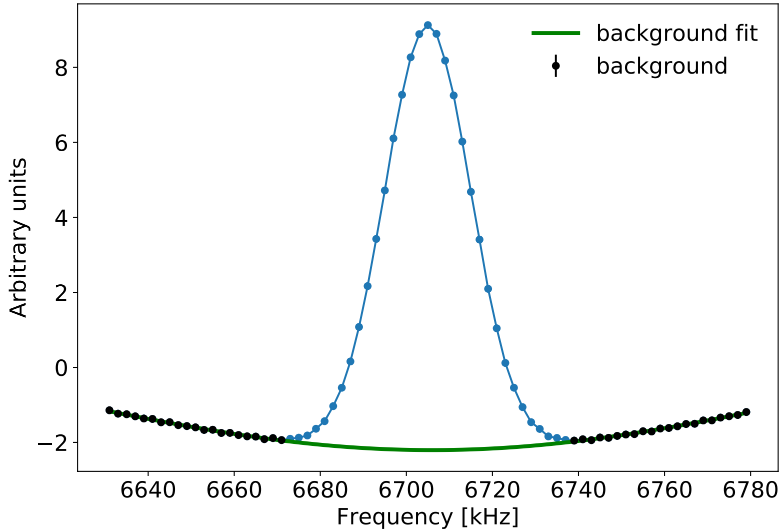


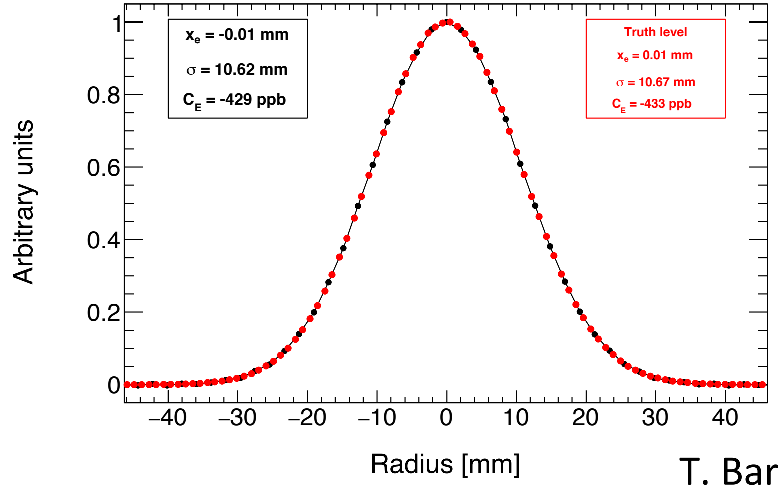


A. Chapelain

Toy Monte Carlo

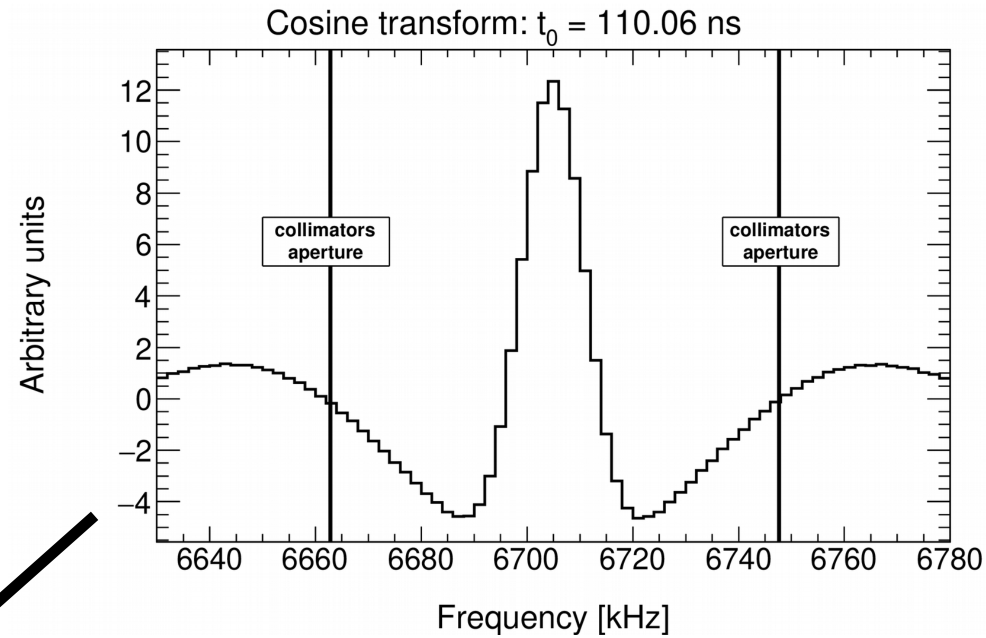


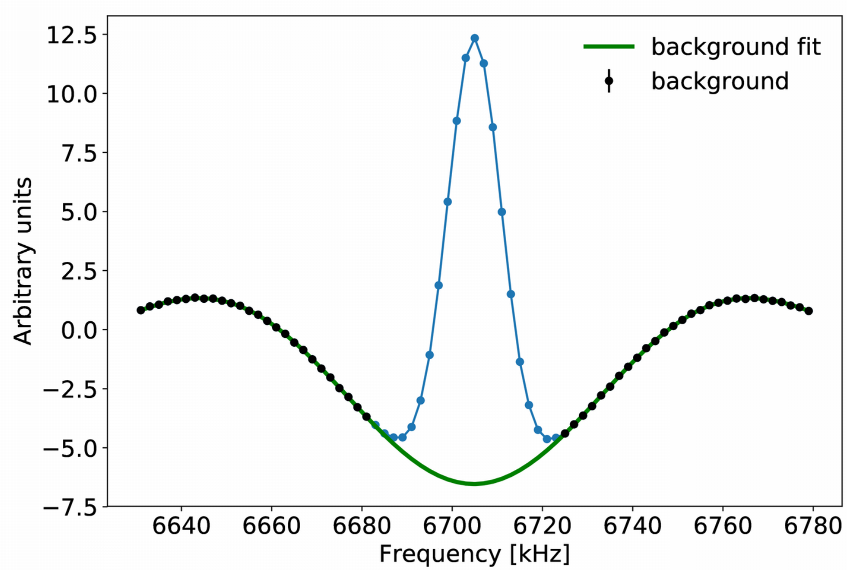


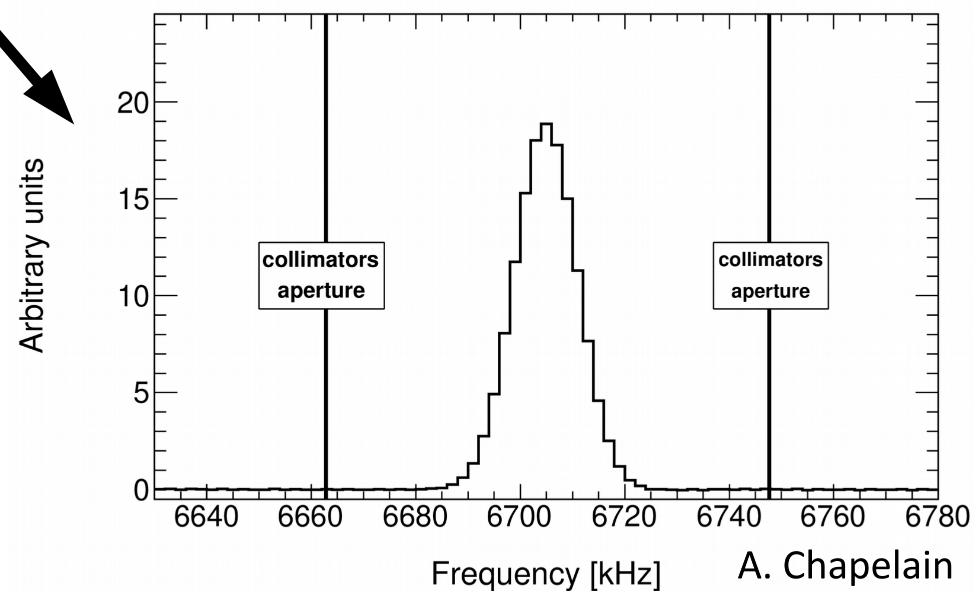


T. Barrett

Cornell Fourier implementation: correct for the so-called “background” (caused by missing early time) via a direct background fit.







“background” = size bands region outside of the main peak

A. Chapelain

The Cornell Fast Rotation Fourier analysis has been done on 3 data set:

60-hour link to the analysis note:
https://gm2-docdb.fnal.gov/cgi-bin/private/ShowDocument?docid=19150

9-day link to the analysis note:
https://gm2-docdb.fnal.gov/cgi-bin/private/ShowDocument?docid=19252

End-game link to the analysis note:
https://gm2-docdb.fnal.gov/cgi-bin/private/ShowDocument?docid=19258

The above will be amended with analysis by run number

Analysis of High Kick in progress

How do we convince ourselves that the Fourier method is giving us the right answer?

So far we only know that it works for Toy MC data

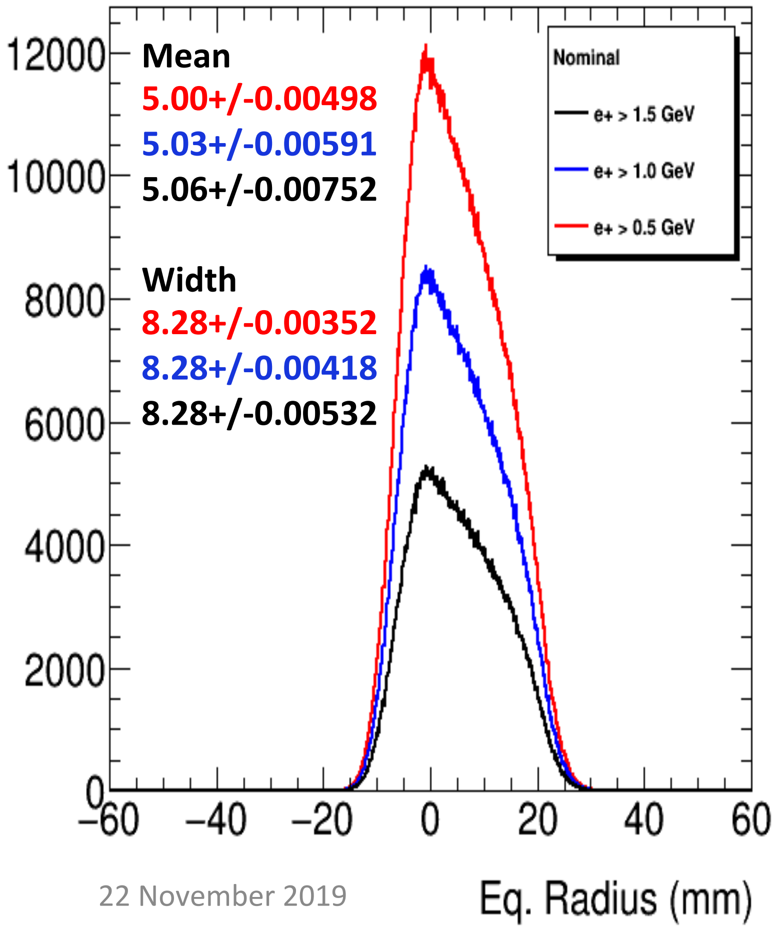
Simulation with gm2ringsim

- 10^9 muons thrown at ring
- Equilibrium radius (truth) measured at tracking planes
- Fast rotation signal is calo hits

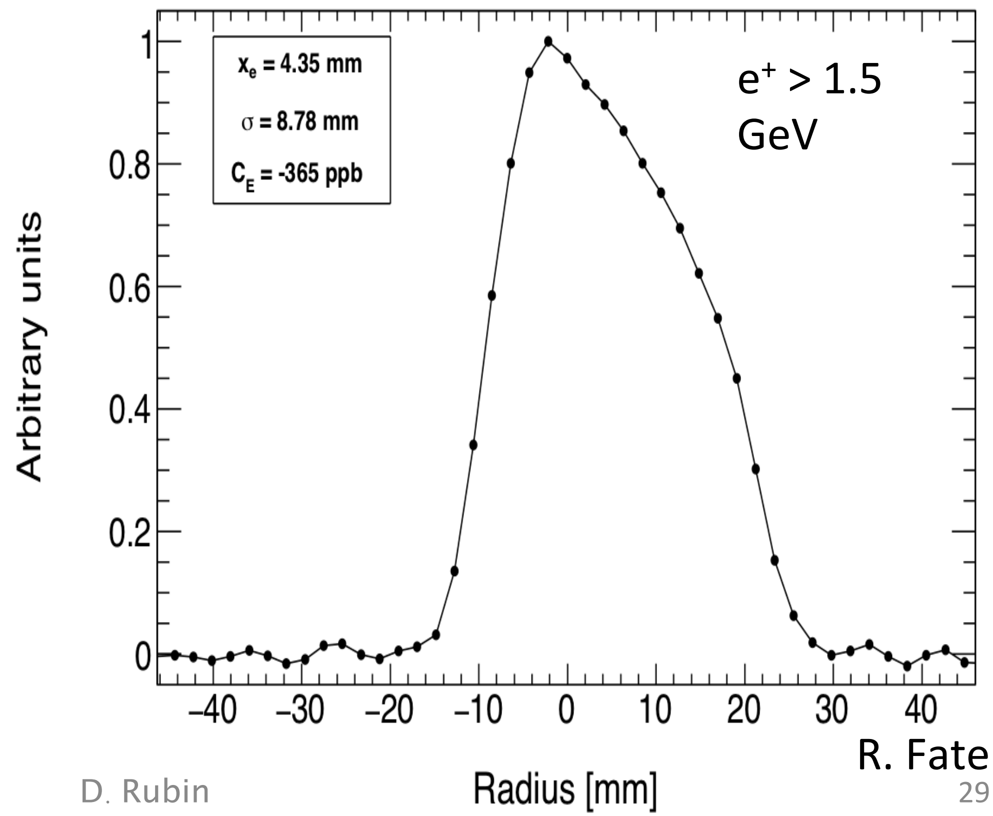
From tracking planes at t>30 μs

$$x_e = R_{magic} \frac{\Delta p}{p_{magic}(1-n)}$$

Cornell FR reconstruction from decay positrons in calorimeters (4< t < 150 **μ**s)



Means are different 5.060 → 4.35.
but shape comparison looks good.

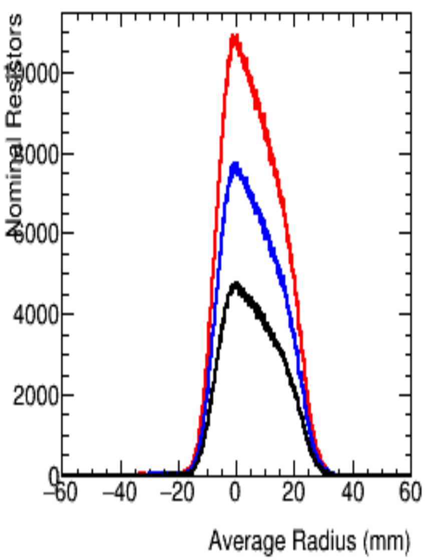


R. Fatemi

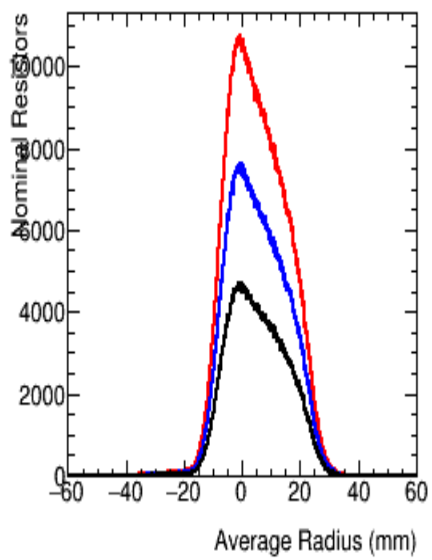
Average Radius <R> from Tracking planes for t > 30 **μ**s.

Averaged over all planes <R> ~ 5.6 mm

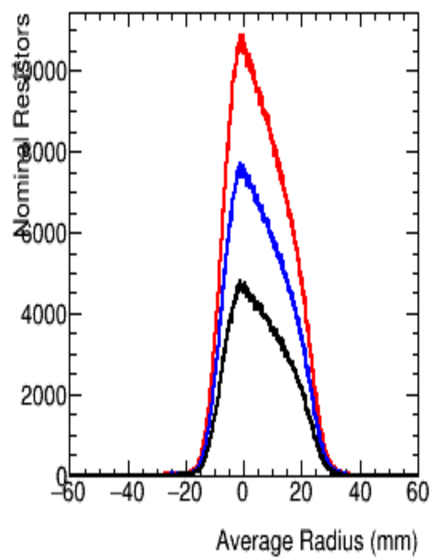
Plane 0 Mean
5.73 +/- 0.0057
5.76 +/- 0.0068
5.80 +/- 0.0086



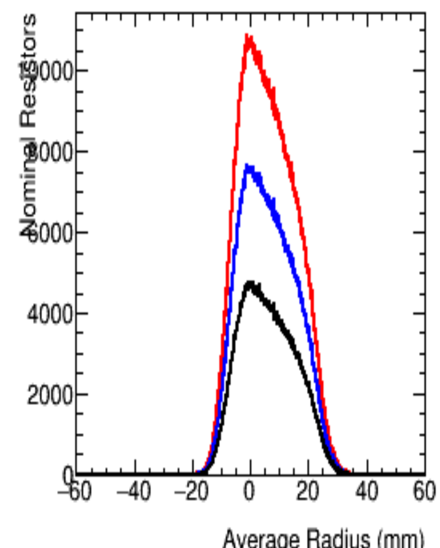
Plane 1 Mean
5.25+/-0.0059
5.29+/-0.0070
5.34+/-0.0089



Plane 2 Mean
5.57 +/- 0.0058
5.59 +/- 0.0069
5.62 +/- 0.0088



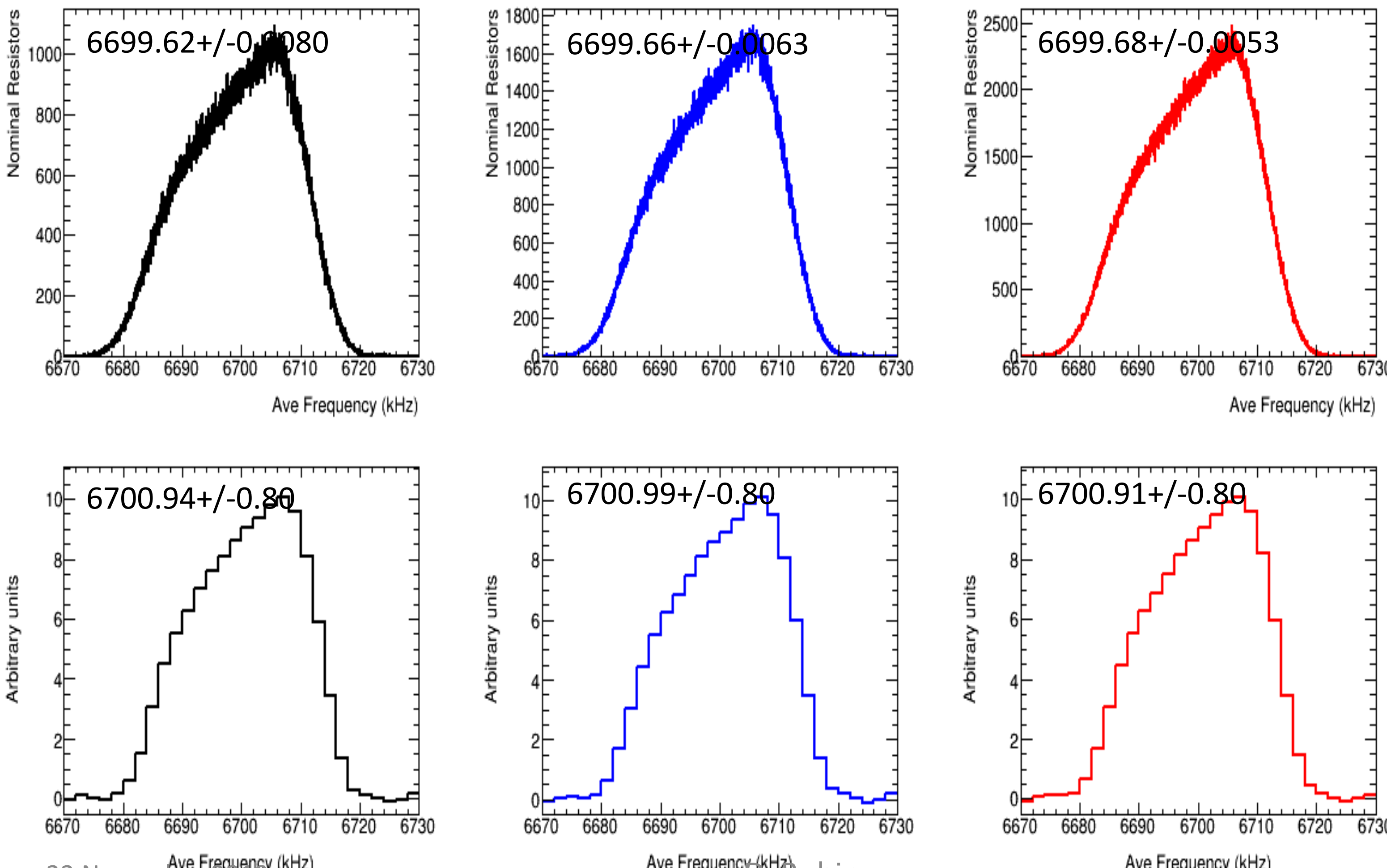
Plane 3 Mean
6.039 +/- 0.0057
6.049 +/- 0.0067
6.071 +/- 0.0085



R. Fatemi

Compare Frequency Spectrum

$e^+ > 1.5$ GeV | $e^+ > 1.0$ GeV | $e^+ > 0.5$ GeV



Tracking Plane 0 (all look the same)

Fast Rotation Extraction

R. Fatemi

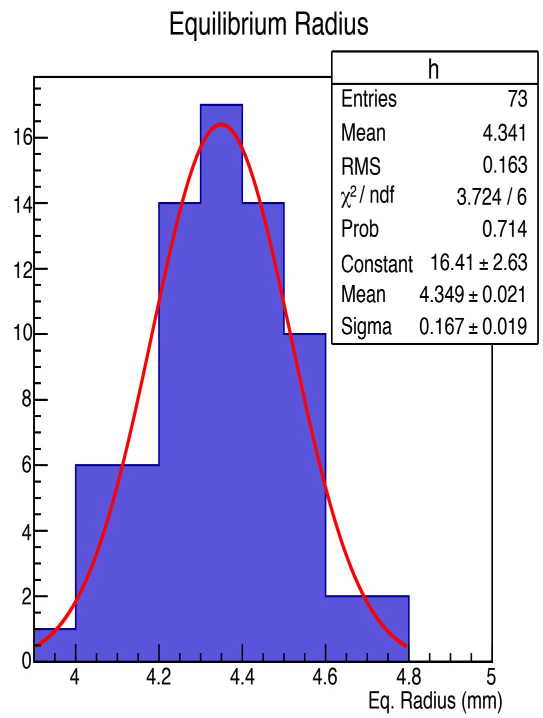
``Statistical Error" on FR extraction

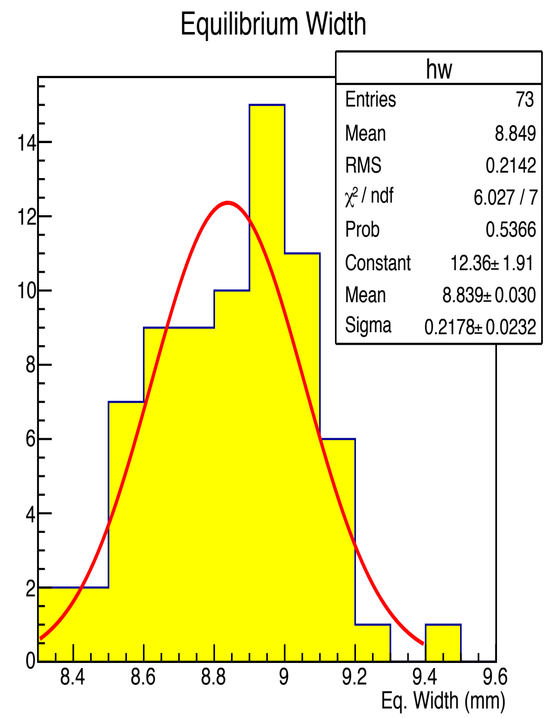
73 random variations over same input data

Average Mean = 4.35 +/- 0.021
Width of Mean = 0.17 +/- 0.019

Average Width = 8.84 +/- 0.030
Width of Width = 0.22 +/- 0.023

Difference between truth and FR reconstruction is significant.





R. Fatemi

Maximal Error?

How does the deviation between the fast rotation analysis and the "truth" from the tracking planes bias the E-field correction? (n = 0.108)

FR ➔ 365 ppb
- Mean 4.35
- Width 8.78

$$C_E = -\frac{2n(n-1)\beta^2}{R^2_{magic}}\langle x_e^2 \rangle$$

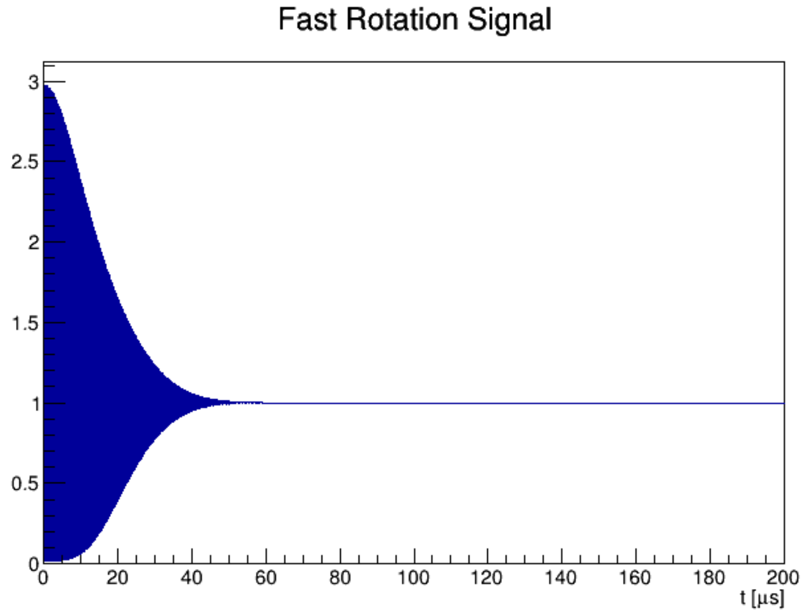
TRUTH ➔ 358 ppb
- Mean 5.06
- Width 8.28

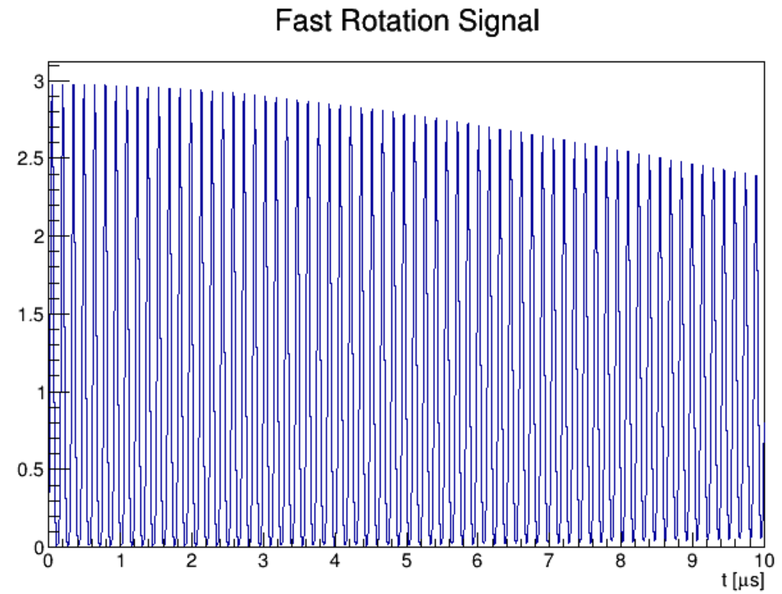
Deviation is 7 ppb

Fortuitous?

$$\langle x_e^2 \rangle = \langle x_e \rangle^2 + \sigma_e^2$$

R. Fatemi





At $t = 0$, beam is maximally bunched. As $t \to \infty$, muons distributed uniformly around ring

S(t) is symmetric about $t = 0$

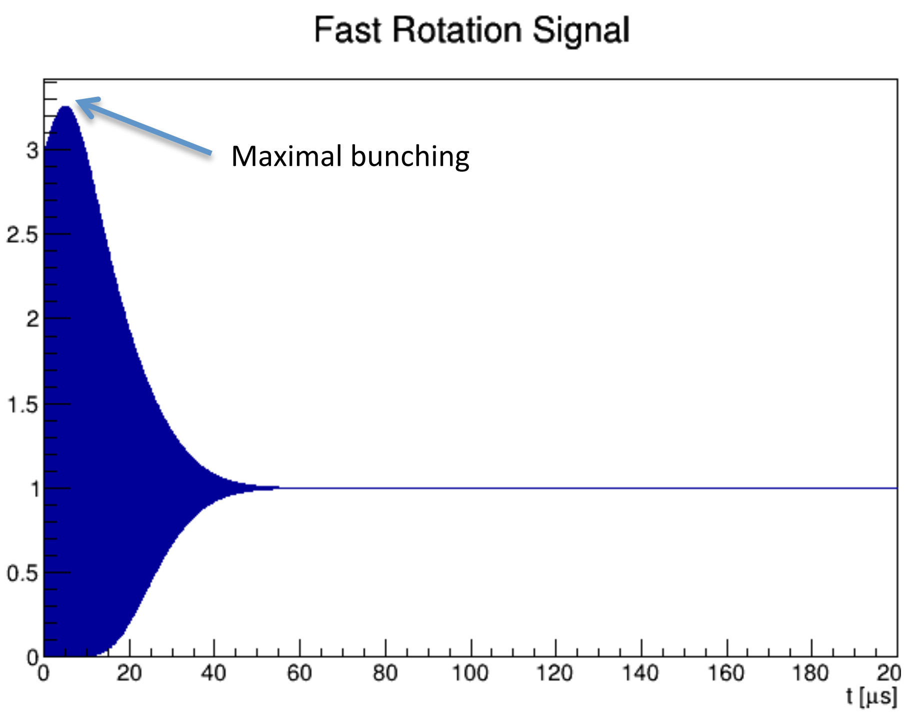
$$\hat{S}(\omega) = \int_{t_s}^{t_m} S(t)\cos[\omega(t - t_0)]dt$$

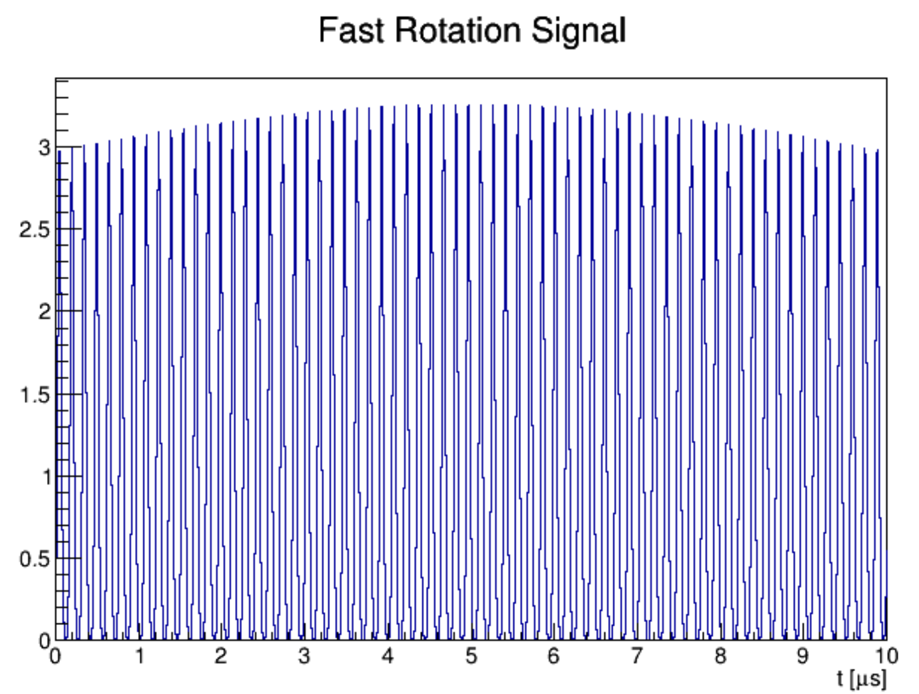
Fourier method assumes symmetry about t_0

Time-momentum correlation in injected distribution breaks symmetry

Suppose for example, that the average momentum at the head of the bunch is high – and the average momentum at the tail is low.

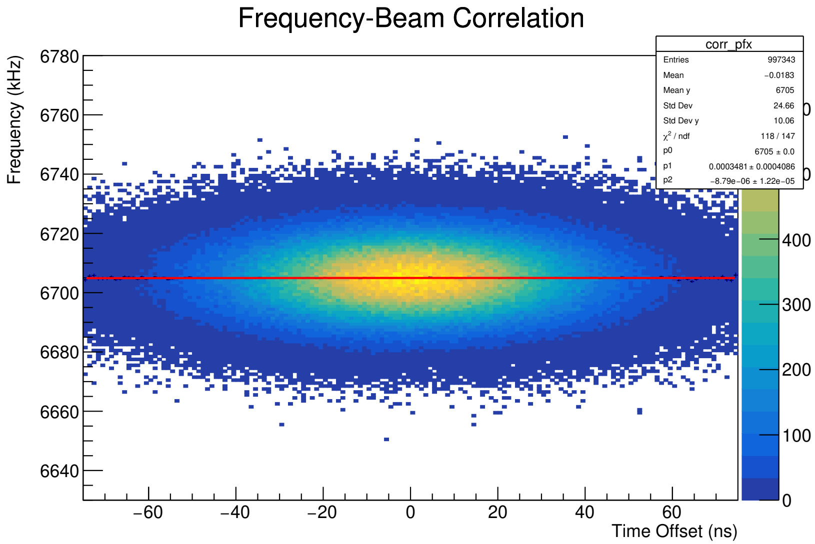
Then as the tail catches up with the head the distribution becomes more bunches
Extension to negative times will not be symmetric about 0

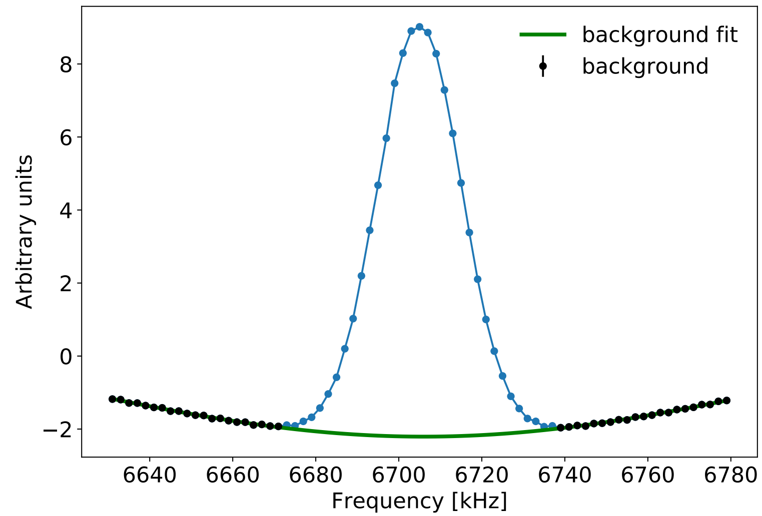


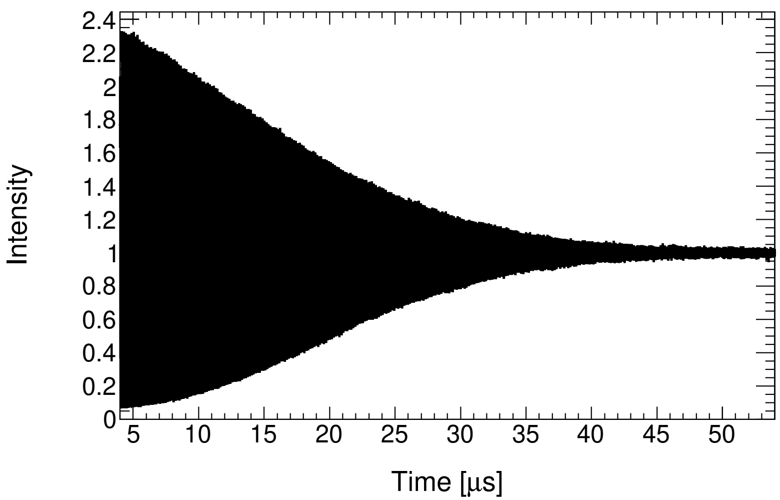


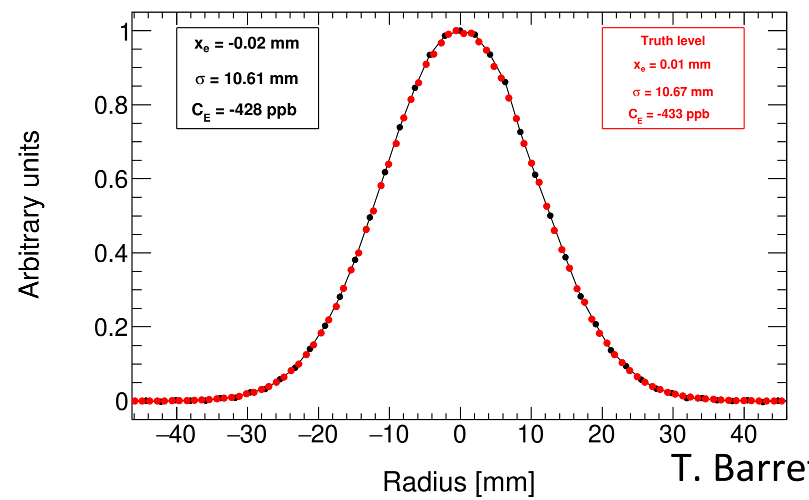
J.Fagin

$C = +0.00000$



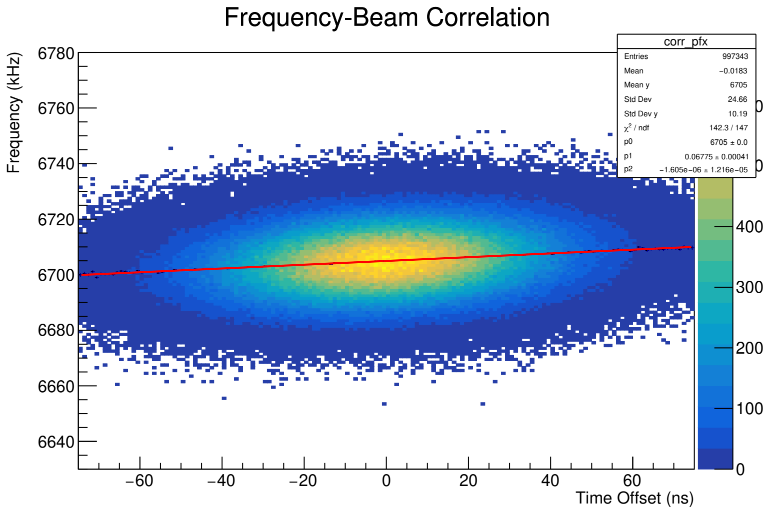


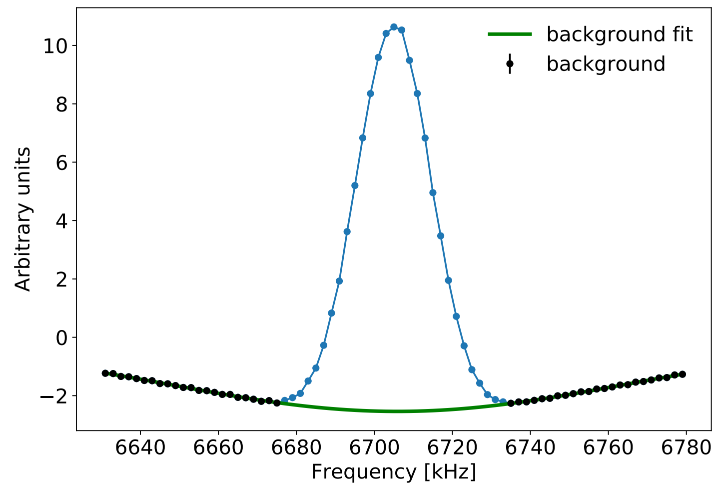


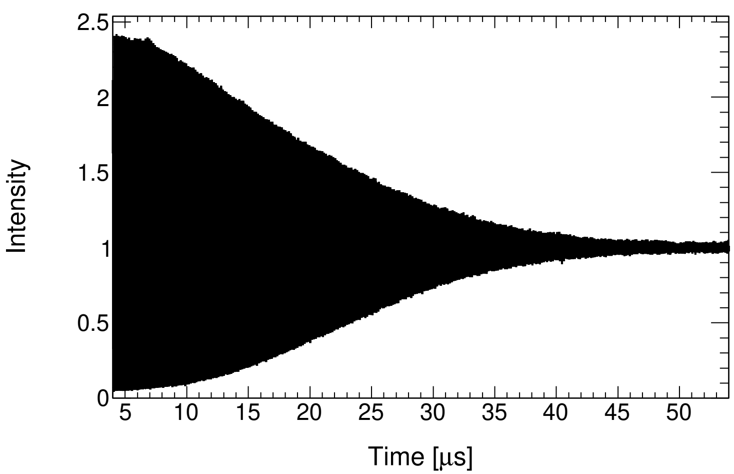


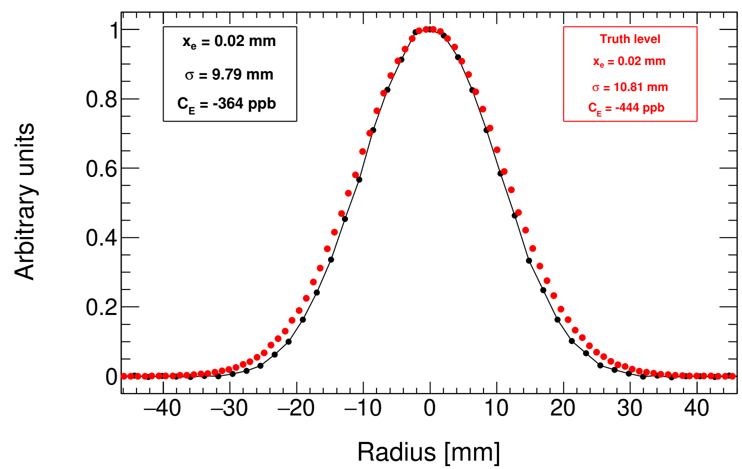
T. Barrett

$C = -0.00150$





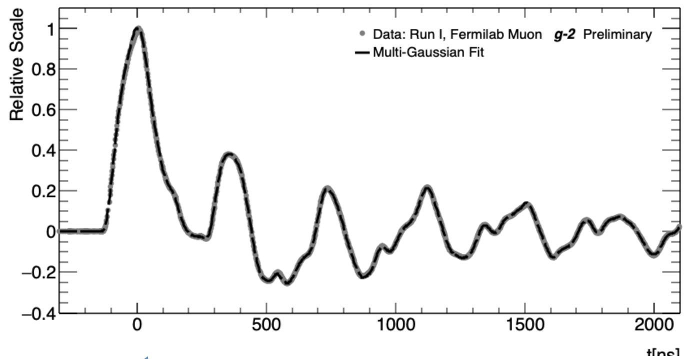




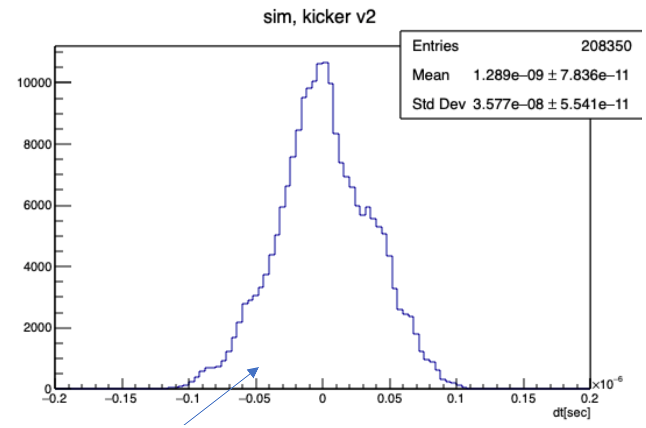
T. Barrett

Evidently the Fourier method is not robust to time-momentum correlation in the distribution

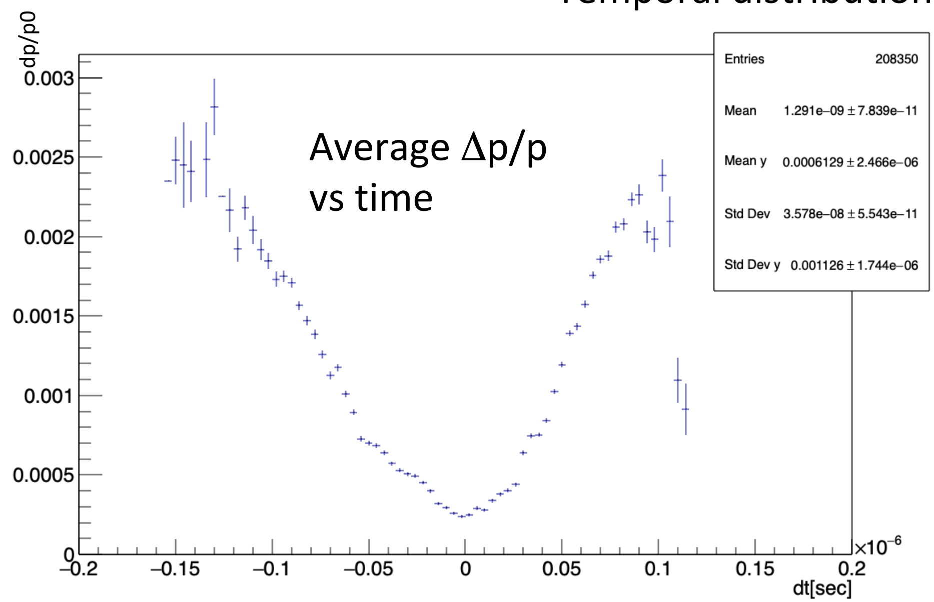
Correlation is introduced by the kicker



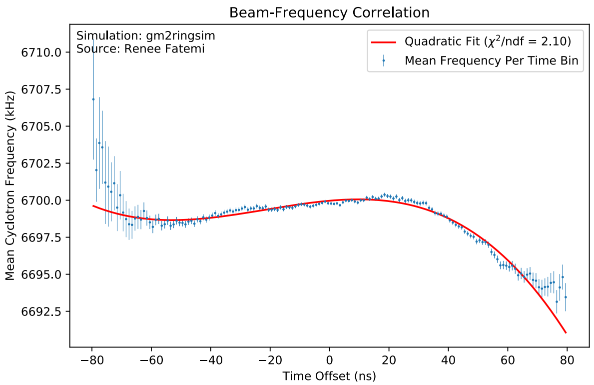
Kicker pulse



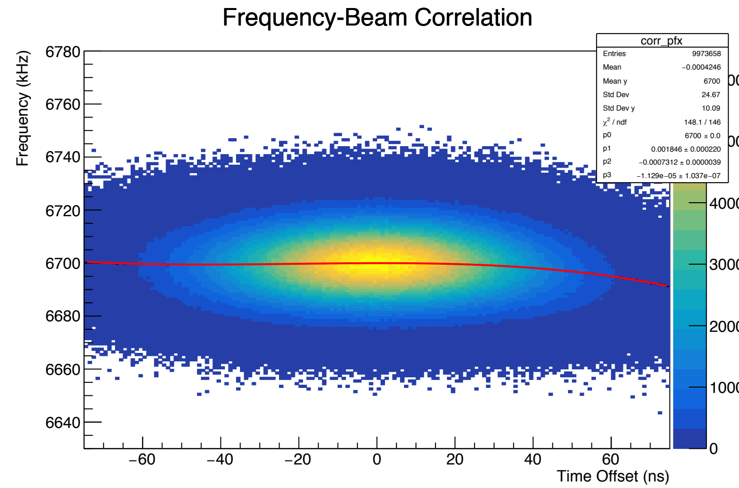
Temporal distribution of injected pulse



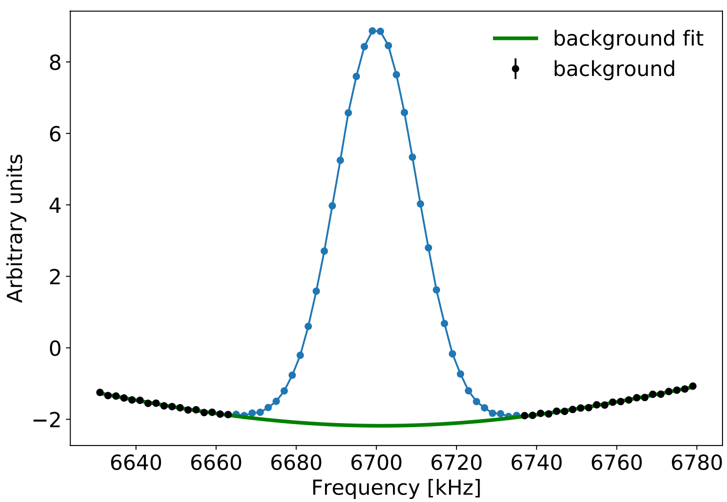
D. Tarazona



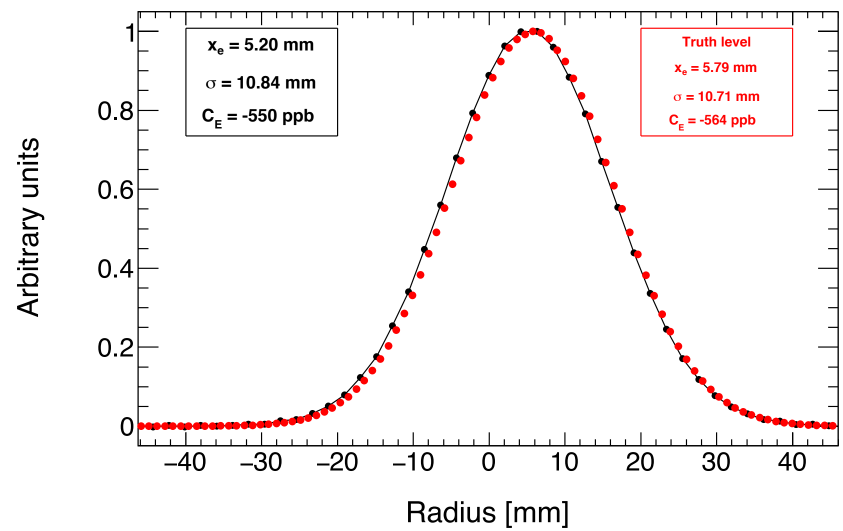
Beam-Frequency Correlation
Simulation: gm2ringsim
Source: Renee Fatemi
Quadratic Fit (χ^2/ndf = 2.10)
Mean Frequency Per Time Bin
Mean Cyclotron Frequency (kHz)
Time Offset (ns)



Frequency-Beam Correlation
corr_pfx
Entries 9973658
Mean −0.0004246
Mean y 6700
Std Dev 24.67
Std Dev y 10.09
χ^2 / ndf 148.1 / 146
p0 6700 ± 0.0
p1 0.001846 ± 0.000220
p2 −0.0007312 ± 0.0000039
p3 −1.129e−05 ± 1.037e−07
Frequency (kHz)
Time Offset (ns)

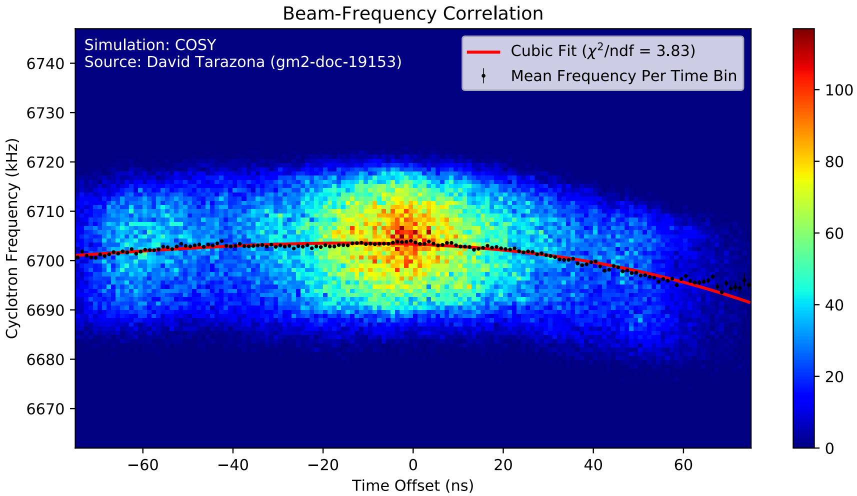


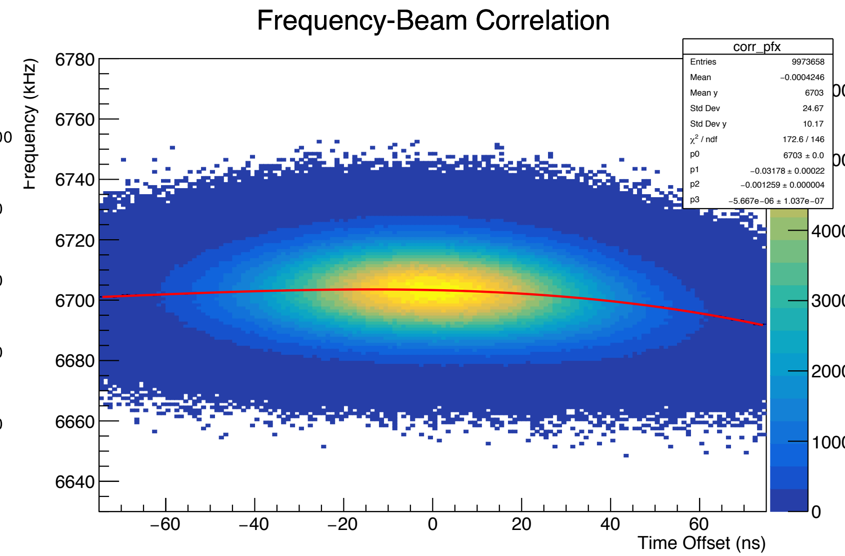
background fit
background
Arbitrary units
Frequency [kHz]

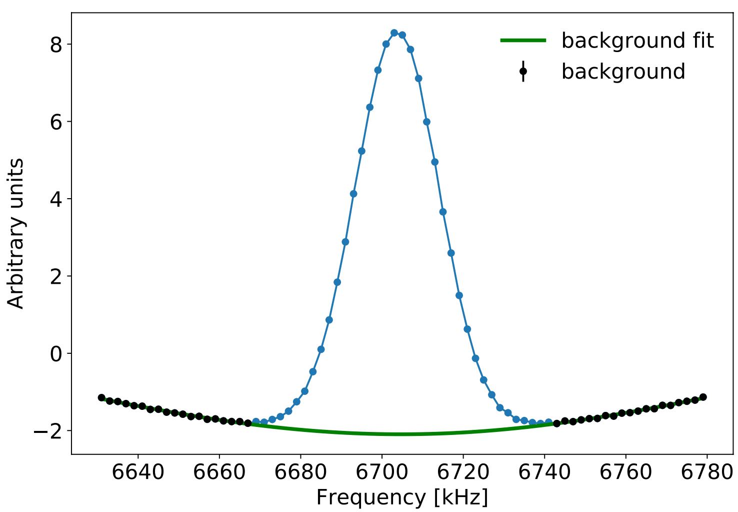


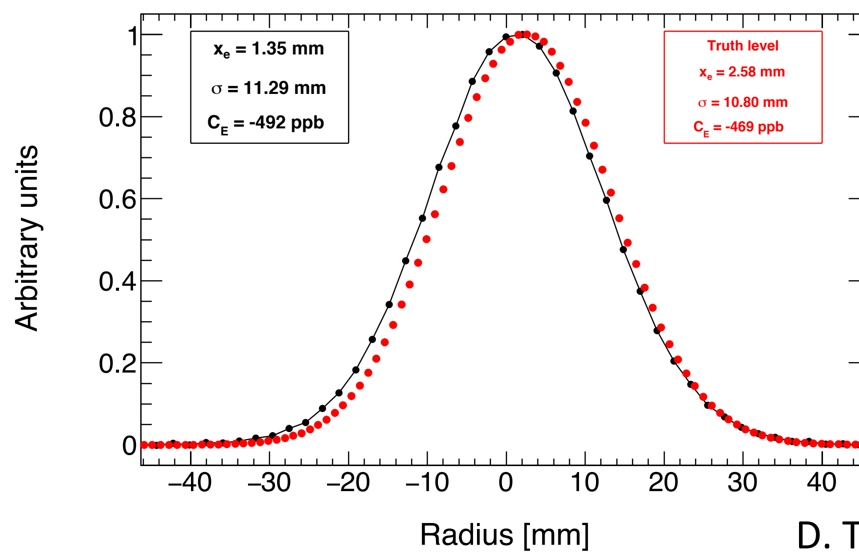
x_e = 5.20 mm
σ = 10.84 mm
C_E = -550 ppb
Truth level
x_e = 5.79 mm
σ = 10.71 mm
C_E = -564 ppb
Arbitrary units
Radius [mm]

T. Barrett

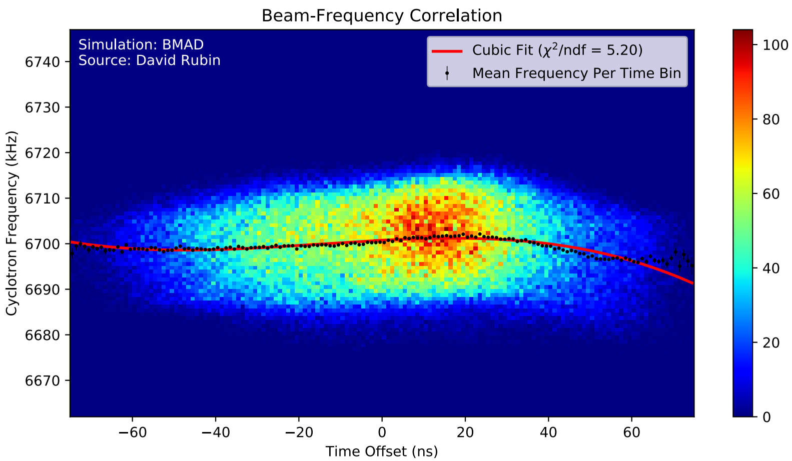


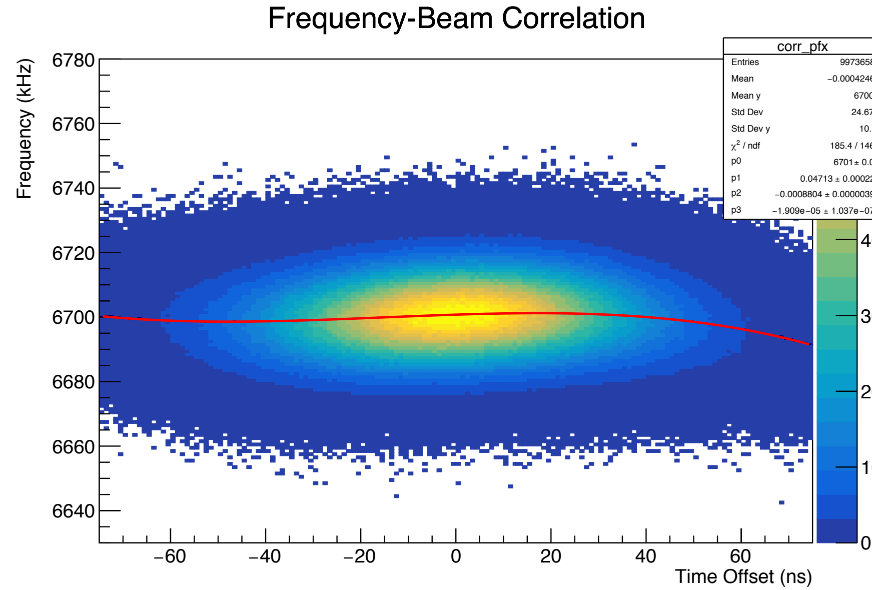


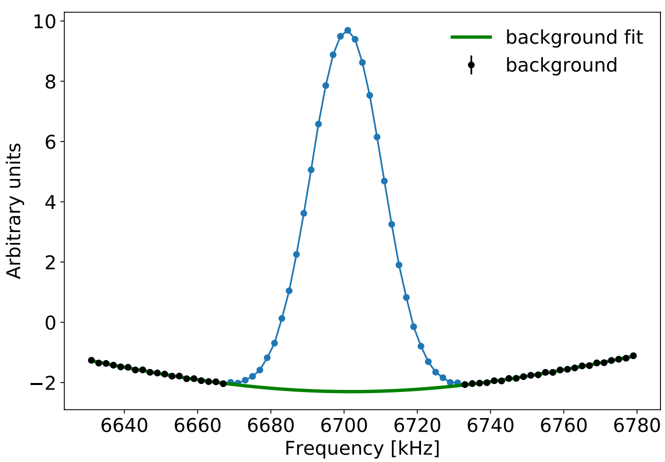


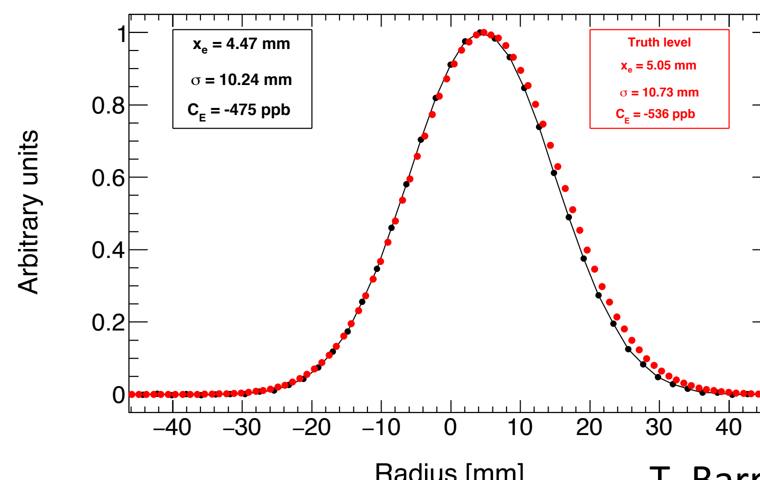


D. Tarazona
T. Barrett

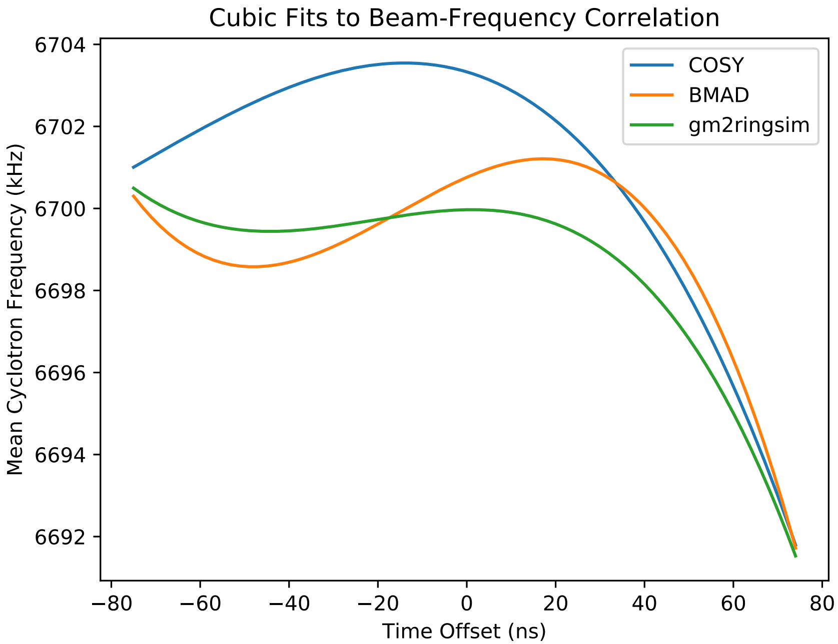

Beam-Frequency Correlation
Simulation: BMAD
Source: David Rubin
Cubic Fit (χ^2/ndf = 5.20)
Mean Frequency Per Time Bin
Cyclotron Frequency (kHz)
Time Offset (ns)


Frequency-Beam Correlation
corr_pfx
Entries 9973658
Mean −0.0004246
Mean y 6700
Std Dev 24.67
Std Dev y 10.1
χ^2 / ndf 185.4 / 146
p0 6701 ± 0.0
p1 0.04713 ± 0.00022
p2 −0.0008804 ± 0.0000039
p3 −1.909e−05 ± 1.037e−07
Frequency (kHz)
Time Offset (ns)


background fit
background
Arbitrary units
Frequency [kHz]


x_e = 4.47 mm
σ = 10.24 mm
C_E = -475 ppb
Truth level
x_e = 5.05 mm
σ = 10.73 mm
C_E = -536 ppb
Arbitrary units
Radius [mm]

T. Barrett



$$f(t) = p_0 + p_1 t + p_2 t^2 + p_3 t^3$$

Coefficient	COSY	BMAD	gm2ringsim
p_3 (kHz/ns^3)	$-5.6(9) \times 10^{-6}$	$-1.9(1) \times 10^{-5}$	$-1.13(6) \times 10^{-5}$
p_2 (kHz/ns^2)	$-1.26(3) \times 10^{-3}$	$-8.8(4) \times 10^{-4}$	$-7.3(2) \times 10^{-4}$
p_1 (kHz/ns)	$-3.2(3) \times 10^{-2}$	$+4.7(3) \times 10^{-2}$	$+1.8(1) \times 10^{-3}$
p_0 (kHz)	$+6703.33(5)$	$+6700.76(6)$	$+6699.97(3)$

- Time-momentum correlation introduces systematic error in Fourier method
- The correlation will depend on kicker parameters
- Modeling the correlation may be problematic

60-hour summary

Fourier Fast rotation analysis

$$x_e = 6.11 \pm 0.01(\mathrm{stat}) \pm 0.36(\mathrm{syst})\ \mathrm{mm}$$

$$\sigma = 9.21 \pm 0.01(\mathrm{stat}) \pm 0.33(\mathrm{syst})\ \mathrm{mm}$$

$$C_E = -463 \pm 1(\mathrm{stat}) \pm 27(\mathrm{syst})\ \mathrm{ppb}$$

Alignment/voltage systematic

$$\pm 8.7\ (\mathrm{misalignment\ syst})\ \mathrm{ppb}$$

Correlation

$$\pm 80\ (\mathrm{correlation\ syst})\ \mathrm{ppb}$$

Summary

E-field

- Correlation systematic dominates uncertainty of E-field contribution
- We are exploring refinements to Fourier method to mitigate effects of correlatio
- Redefine t_0 to peak of fast rotation signal (maximal bunching point) ?

- χ^2 method? Parameterize initial distribution and fit to S(t).
  Correlation can be included in that parameterization

Or perhaps a hybrid of Fourier and χ^2 method?

Pitch

- Effects of misalignments are small
- Remaining uncertainty dominated by relative acceptance
  In progress

Average over all ϕ for a given amplitude *a*

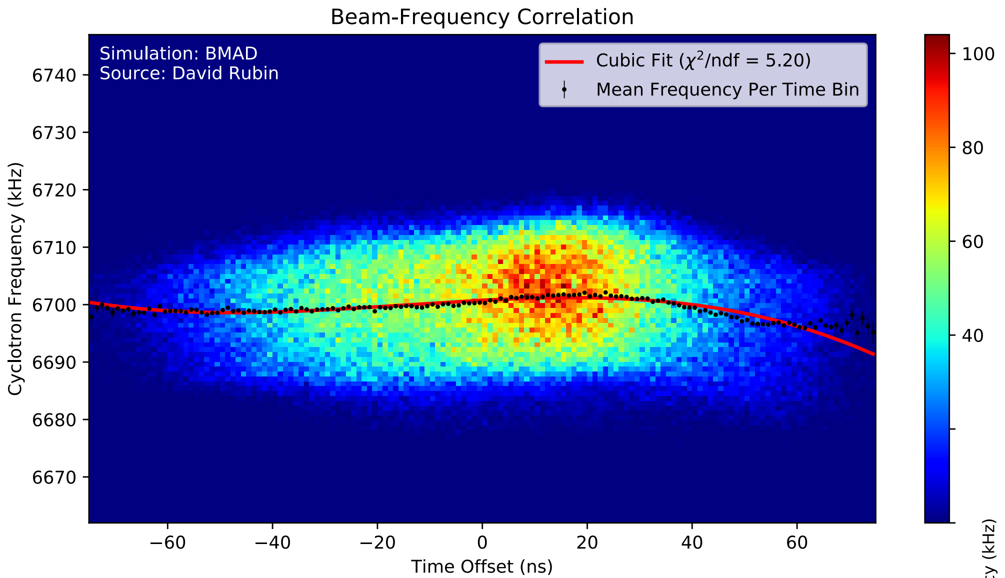
$$
\begin{aligned}
y &= \sqrt{a\beta}\cos\phi \\
\psi &= \psi_0 \sin\phi = \sqrt{\frac{a}{\beta}}\sin\phi \\
\langle\psi^2(a)\rangle_\phi &= \frac{1}{2}\psi_0^2(a) = \frac{\langle y^2(a)\rangle_\phi}{\beta^2}
\end{aligned}
$$

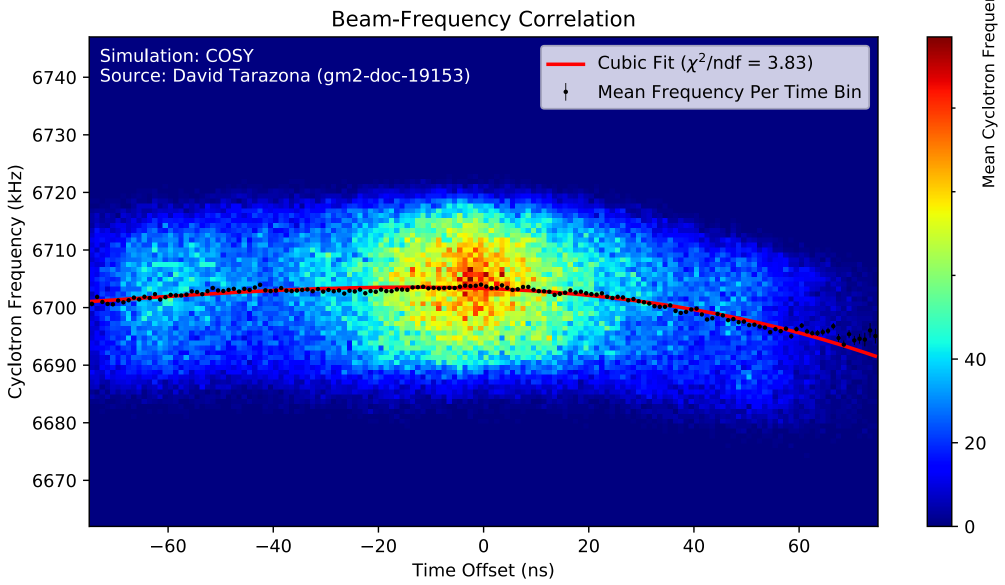
Assumes linearity

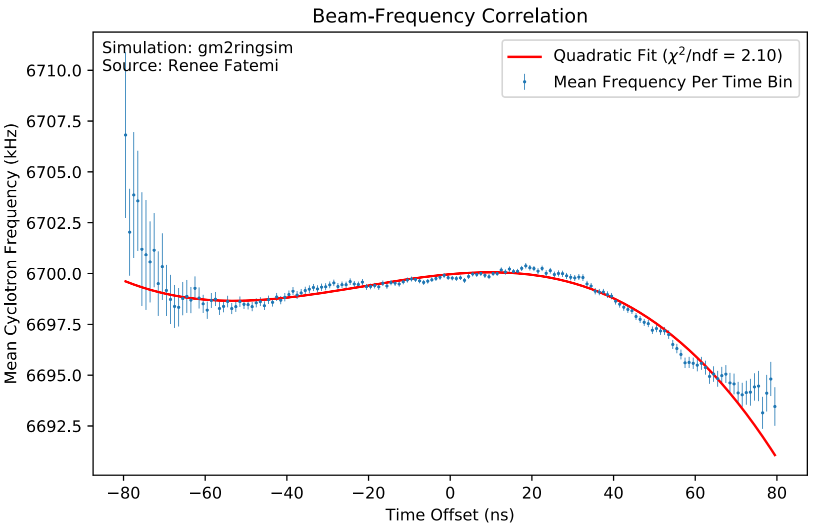
$$\beta(y) = \frac{R_0}{\sqrt{n(y)}}$$

We know that the effective quad index decreases with amplitude y

$$|C_p| < \frac{n\langle y^2\rangle}{2R_0^2}$$


Beam-Frequency Correlation
Simulation: BMAD
Source: David Rubin
Cubic Fit (χ^2/ndf = 5.20)
Mean Frequency Per Time Bin
Cyclotron Frequency (kHz)
Time Offset (ns)


Beam-Frequency Correlation
Simulation: COSY
Source: David Tarazona (gm2-doc-19153)
Cubic Fit (χ^2/ndf = 3.83)
Mean Frequency Per Time Bin
Cyclotron Frequency (kHz)
Time Offset (ns)


Beam-Frequency Correlation
Simulation: gm2ringsim
Source: Renee Fatemi
Quadratic Fit (χ^2/ndf = 2.10)
Mean Frequency Per Time Bin
Mean Cyclotron Frequency (kHz)
Time Offset (ns)

Many thanks to the

Efield/pitch working group

T. Barrett, R. Carey, A. Chapelain, J. Crnkovic, P. Debevec,J. Fagin, R. Fatemi, M. Kargiantoulakis, A. Lorente, J. Mott, J. Price, D. Tarazona, T. Walton