Fast Rotation Fourier Method: Run-1 E-Field Correction Uncertainty

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Muon $g-2$ Collaboration Meeting
March 13, 2020

Outline

1. Introduction & Method Overview

- purpose
- implementation

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2. **Main Systematics & Outstanding Questions**
   - Fourier transform start time
   - positron energy threshold
   - momentum-time correlation

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   - Fourier transform start time
   - positron energy threshold
   - momentum-time correlation
3. **Current Run-1 Results**
   - systematic uncertainties
   - statistical uncertainty
   - CERN/χ^2 comparison

Purpose

E-Field Correction

fractional correction to ω_a from ESQs:

$$C_E \equiv \left\langle \frac{\Delta\omega_a}{\omega_a} \right\rangle = -2n(1-n)\beta^2 \frac{\langle x_e^2 \rangle}{R_0^2}$$

- $n =$ field index
- $\beta =$ muon velocity
- $R_0 =$ storage ring radius
- $x_e =$ equilibrium radius (relative to R_0)

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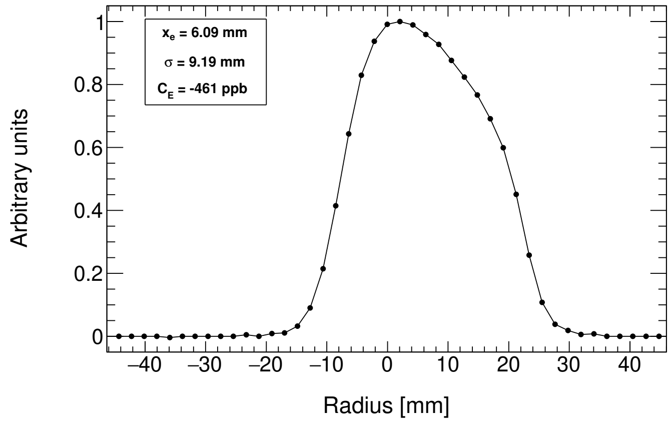
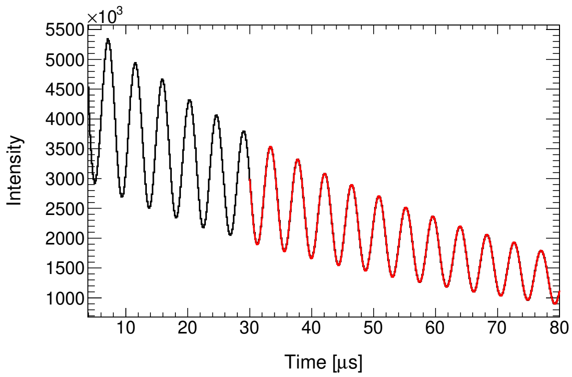


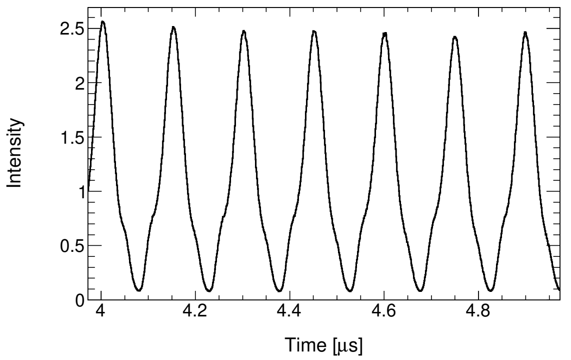
Radial Distribution

cyclotron radii in the storage ring:

$$\langle x_e^2 \rangle = \underbrace{\langle x_e \rangle}_{\text{mean}}{}^2 + \underbrace{\sigma_x}_{\text{std.}}{}^2$$

Fast Rotation Analysis



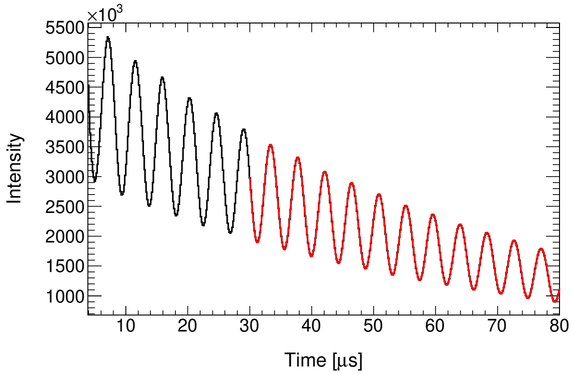


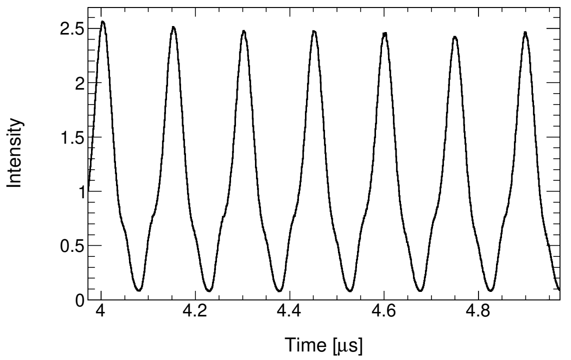
Fast Rotation Signal

cyclotron motion of the muon beam

- use coarse binning (149 ns) to fit ω_a wiggle
- use fine binning (1 ns) with wiggle divided out

Fast Rotation Analysis





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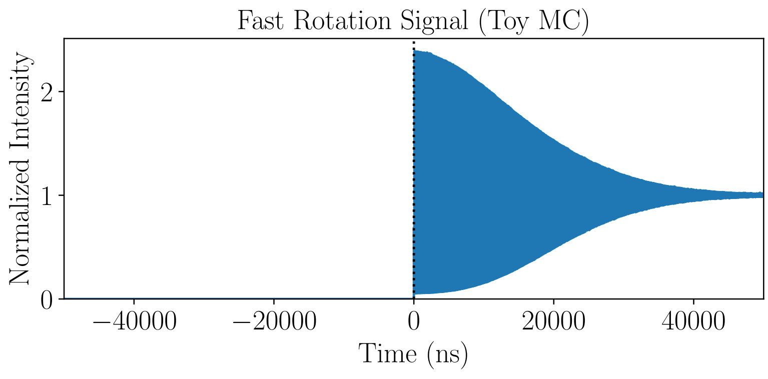
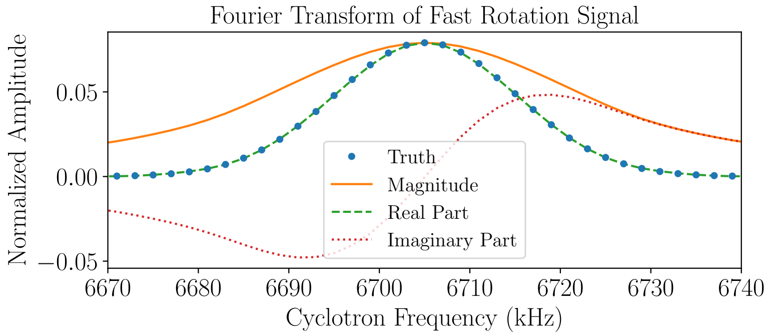
Two Analysis Methods

- CERN/χ^2 method: fit the signal, treating each bin in the radial distribution as a parameter
- **Fourier method**: use a Fourier transform to obtain the distribution of cyclotron frequencies, which can be converted to cyclotron radii

Cosine Transform





Fourier Transform requires fast rotation signal for **all time**, but we only observe $t > 0$.

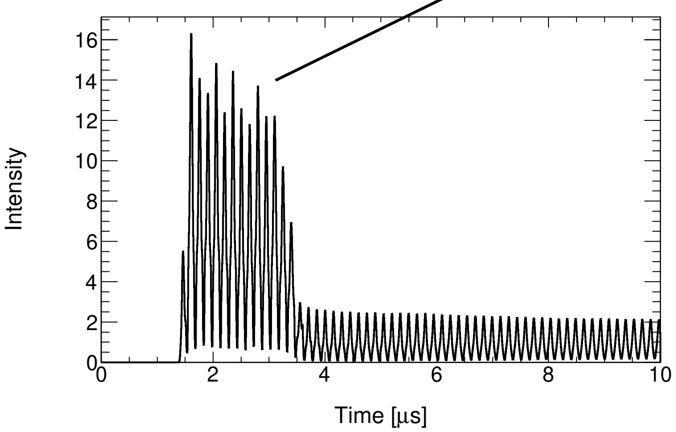
- cosine transform sees an even extension
- MC and analytical model support even symmetry about t_0 (center of first turn)

$$\hat{\mathcal{S}}(f) = \int_{t_0}^{\infty} \mathcal{S}(t) \cos[2\pi f(t - t_0)]\, dt$$

Start Time

Must start transform at $t_s \gtrsim 4\ \mu$s to discard early contamination:

$$\underbrace{\hat{\mathcal{S}}(f)}_{\text{full transform}} = \underbrace{\int_{t_0}^{t_s} \mathcal{S}(t)\cos[2\pi f(t-t_0)]\,dt}_{\text{discarded part}} + \underbrace{\int_{t_s}^{\infty} \mathcal{S}(t)\cos[2\pi f(t-t_0)]\,dt}_{\text{usable part}}$$



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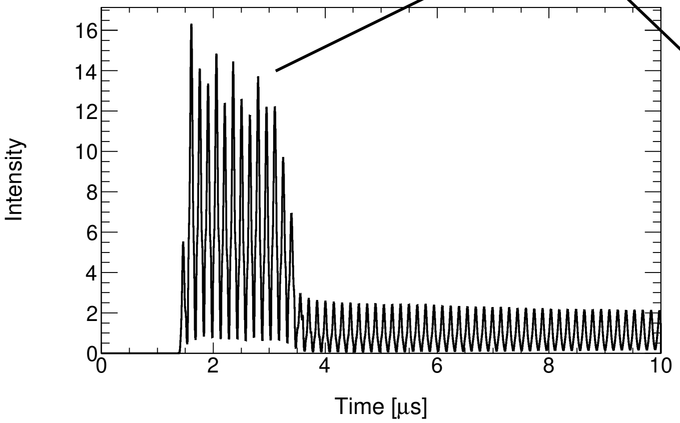

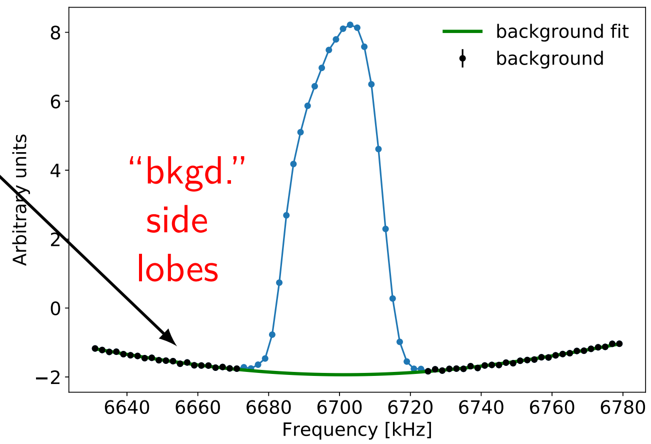

Background Fit

"Background" side lobes are the missing cosine transform from t_0 to t_s:

$$\Delta(f) = \underbrace{\int_{t_0}^{t_s} \mathcal{S}(t)\cos[2\pi f(t - t_0)]\, dt}_{\text{discarded part}}$$

Application

- choose trial frequency distribution $\hat{\mathcal{S}}(f)$
- plug in inverse cosine transform for $\mathcal{S}(t)$
- fit $\Delta(f)$ to side lobes and remove
- repeat over range of t_0 for best fit

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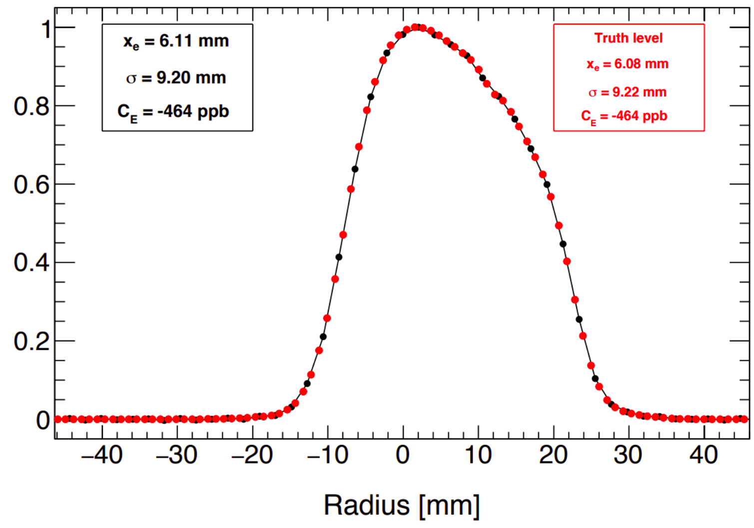
When all is said and done...



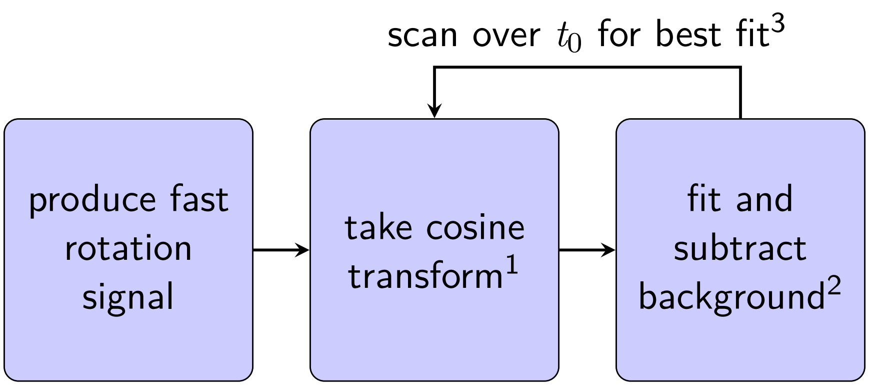
Toy MC using realistic distributions is correct within ∼1 ppb.

Fourier Method

produce fast rotation signal

[1]implicitly mirrors the signal over t_0, filling in for $t < 0$
[2]because of the discarded signal between t_0 and start time $t_s \gtrsim 4~\mu$s
[3]sets the point of symmetry for the cosine transform's even extension

Fourier Method



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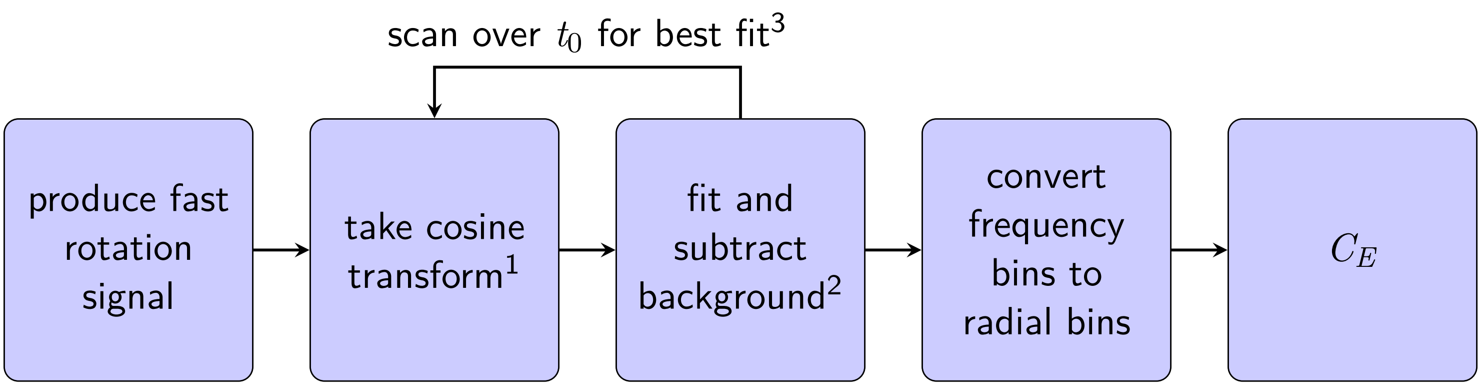

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Systematic Uncertainties

Fourier Method Parameters

- background fit model
- start time
- end time
- frequency bin width
- background definition threshold
- background removal threshold

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Fast Rotation Signal Production

- wiggle fit model
- energy threshold
- momentum-time correlation

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Fast Rotation Signal Production

- wiggle fit model
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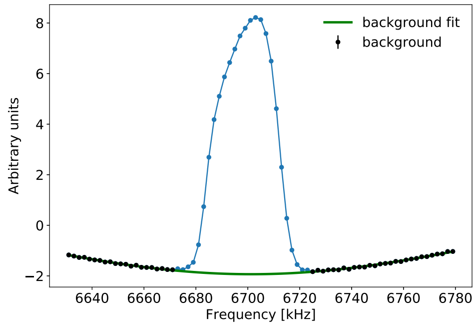
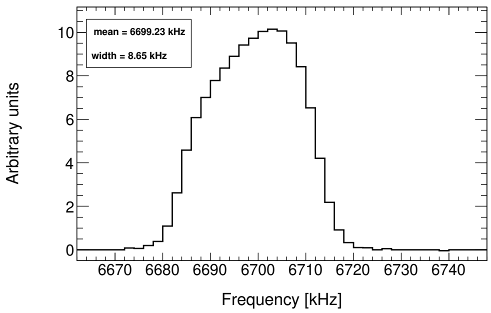
Others

- quad alignment & voltage errors

Cosine Transform Start Time

$t_s = 4\ \mu$s





Cosine Transform Start Time

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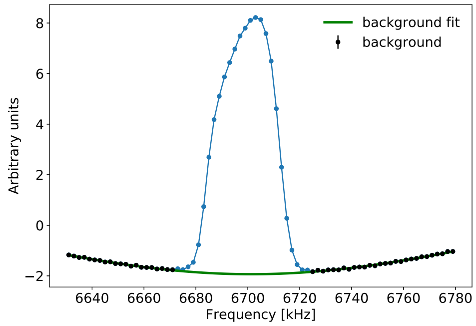
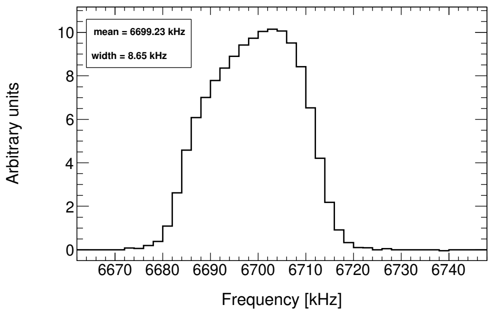
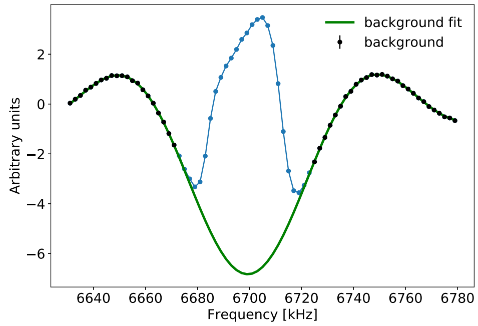
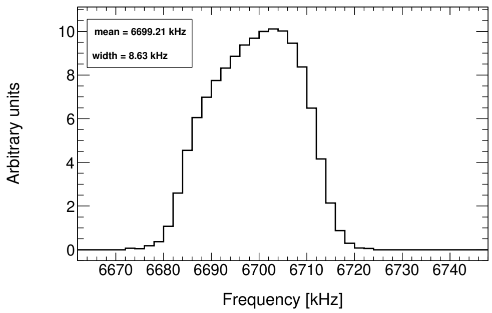




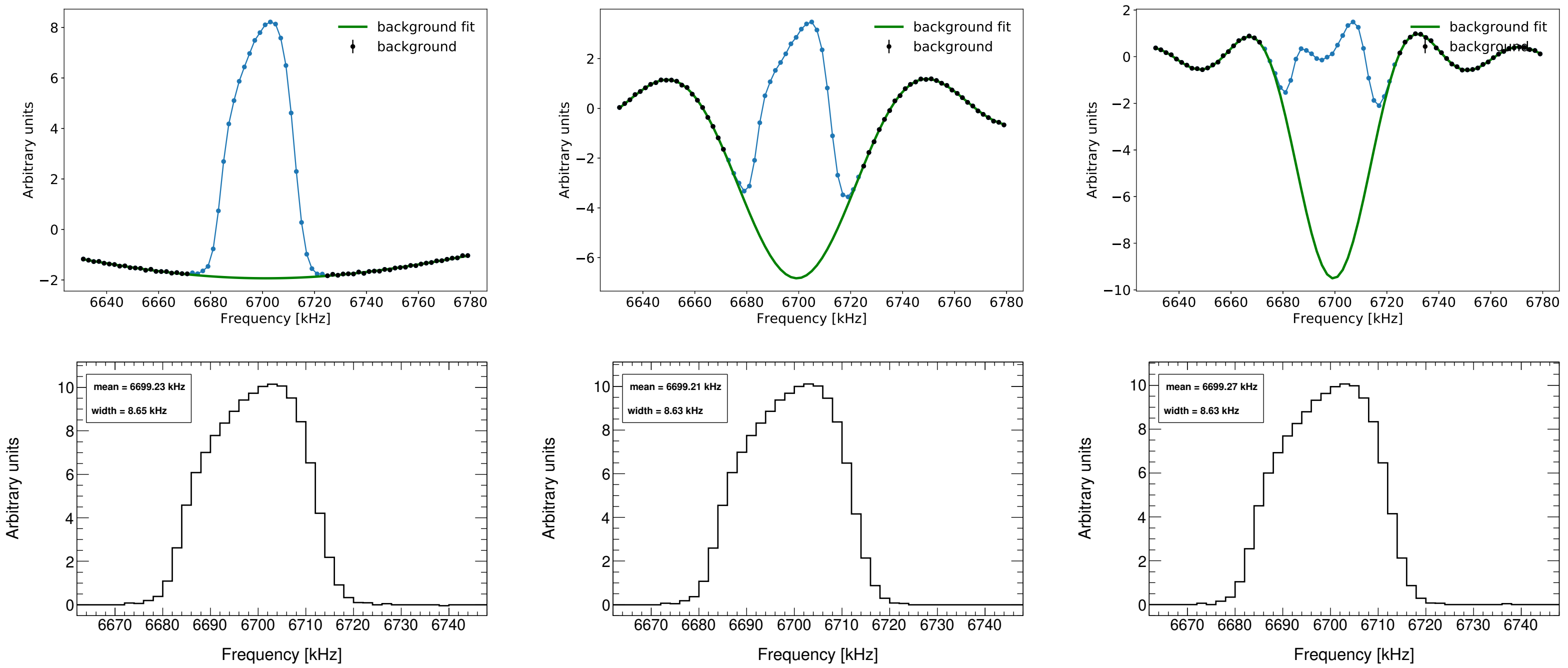
$t_s = 15\ \mu\text{s}$





Cosine Transform Start Time

$t_s = 4\ \mu s$ $\qquad$ $t_s = 15\ \mu s$ $\qquad$ $t_s = 25\ \mu s$



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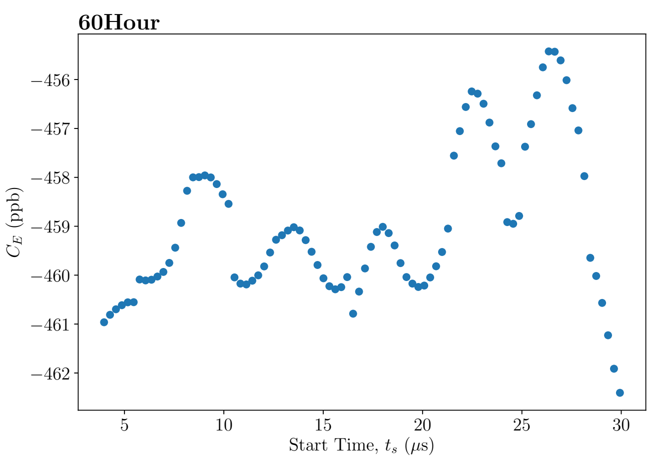

Competing Effects

- background fit only viable for $t_s \lesssim 25\ \mu$s
  - and increasingly difficult along the way
- beam may change due to scraping up to $\sim$30 μs
- hard to disentangle these contributions

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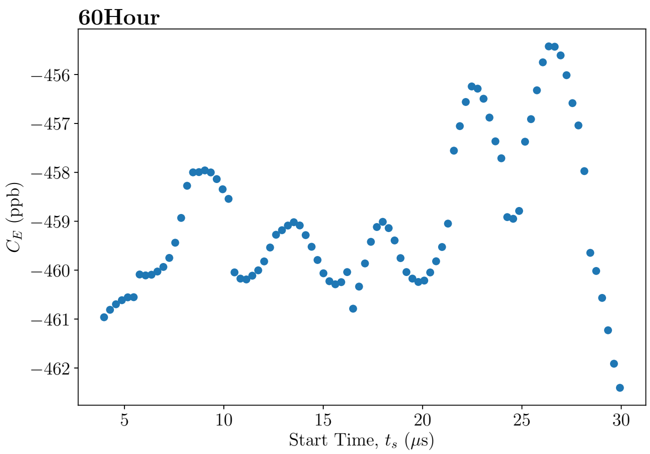

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MC Studies

- in progress (Josh): MC start time scans with changing momentum distribution

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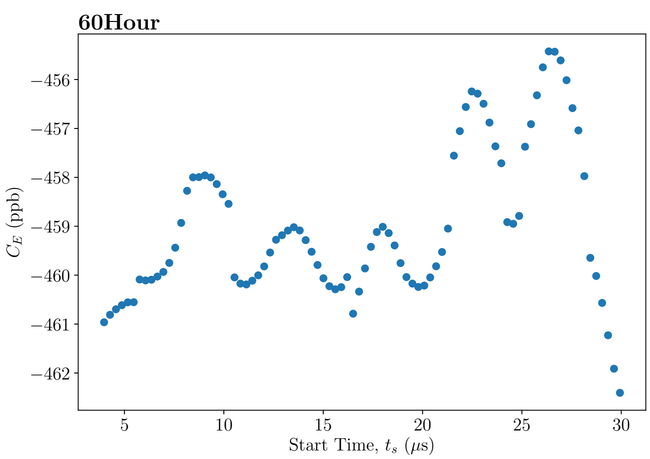

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- beam may change due to scraping up to ∼30 μs
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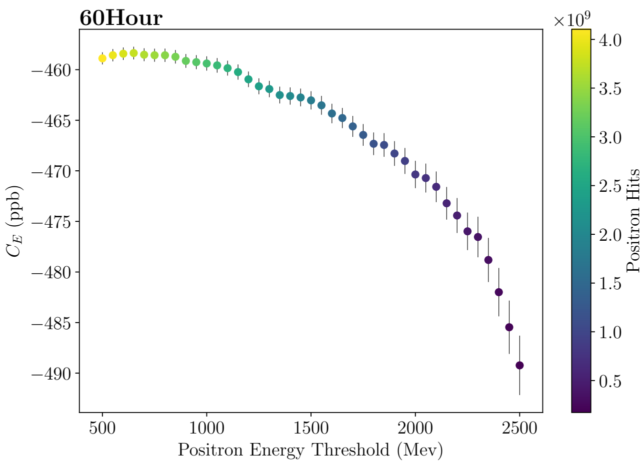
Current Estimate

half-width of spread in C_E between 4 – 25 μs

Positron Energy Threshold



- so far, included as systematic uncertainty

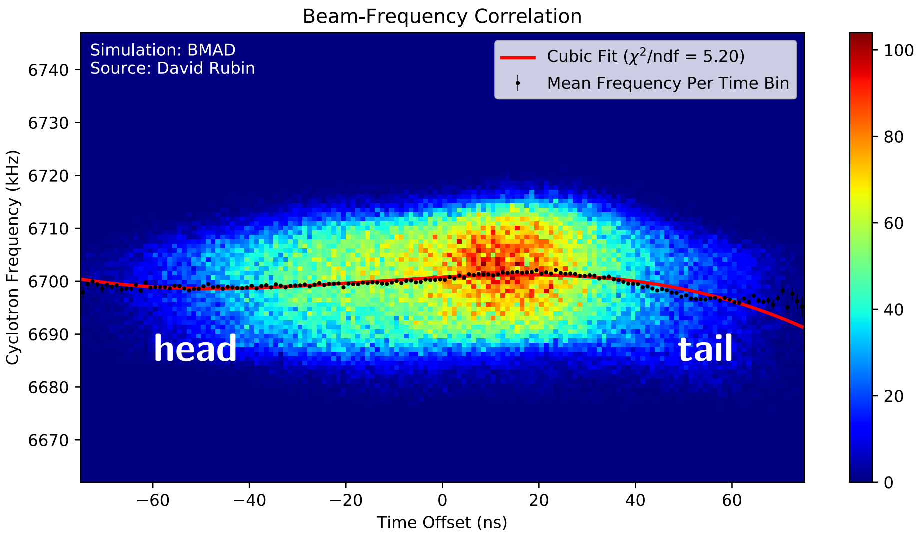
Possible Change?

- provide C_E over a range of finite energy bins
- let each ω_a analyzer average/weight the appropriate energy bins to more closely match their own ω_a methodology

Current Estimate

half-width of spread in C_E between 500 – 2500 MeV

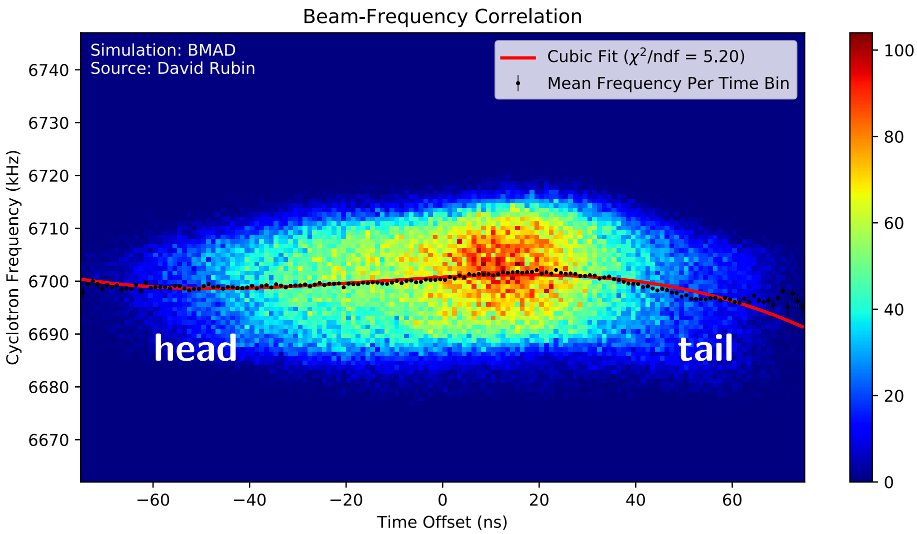
Momentum-Time Correlation



Cause for Concern

- method relies on even extension of fast rotation signal
- kickers introduce changing momentum across beam profile
- beam debunches differently forward and backward in time
- even symmetry spoiled!

Momentum-Time Correlation



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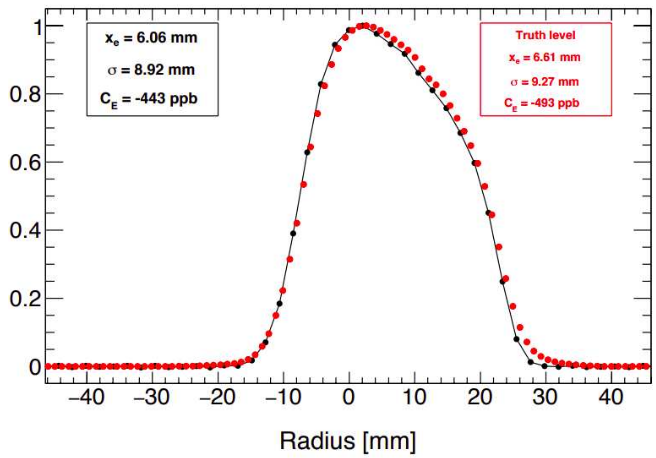
Takeaway

momentum-time correlation $\Longrightarrow$ cosine transform “sees” wrong fast rotation signal

Momentum-Time Systematic



Uncertainty Estimation

Using simulations from BMAD (D. Rubin), COSY (D. Tarazona), gm2ringsim (R. Fatemi)...

- fit the mean frequency along beam profile
- apply frequency bias to Toy MC
  - use realistic distributions from each dataset
- take maximum discrepancy from truth

Large effect: from gm2ringsim, Josh's MC estimates ∼50–60 ppb discrepancy.

Combined Systematic Uncertainty (e.g. 60Hour)

Consider possible correlations among **method parameters**:

Systematic	δC_E (ppb)
Bkgd. Fit Model	0.4
Start Time	2.4
End Time	4.1
Freq. Bin Width	3.5
Bkgd. Definition	3.7
Bkgd. Removal	2.4
Quadrature Sum	7.4
Linear Sum	16.5
Average	11.9

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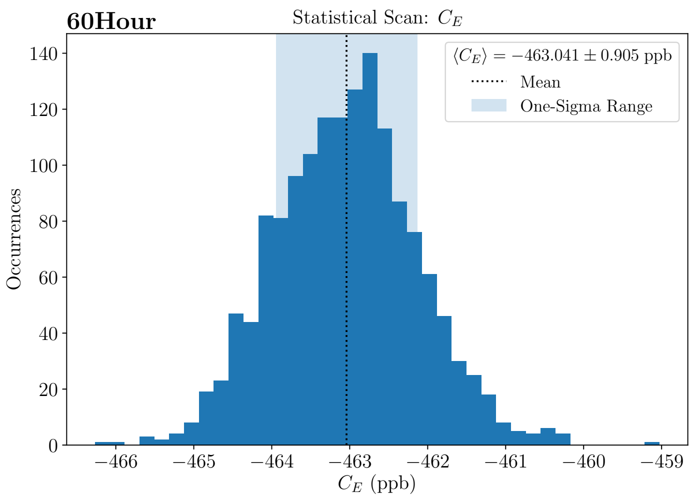
Assume the rest are independent:

Systematic	δC_E (ppb)
Method Parameters	11.9
Wiggle Fit Model	0.9
Energy Threshold	15.4
Momentum-Time Correlation	50
Quad Alignment/Voltage	8.7
Quadrature Sum	54.4

Statistical Uncertainty



- apply Poisson fluctuations to fast rotation bins
- repeat analysis 1500 times
- take std. dev. as statistical uncertainty

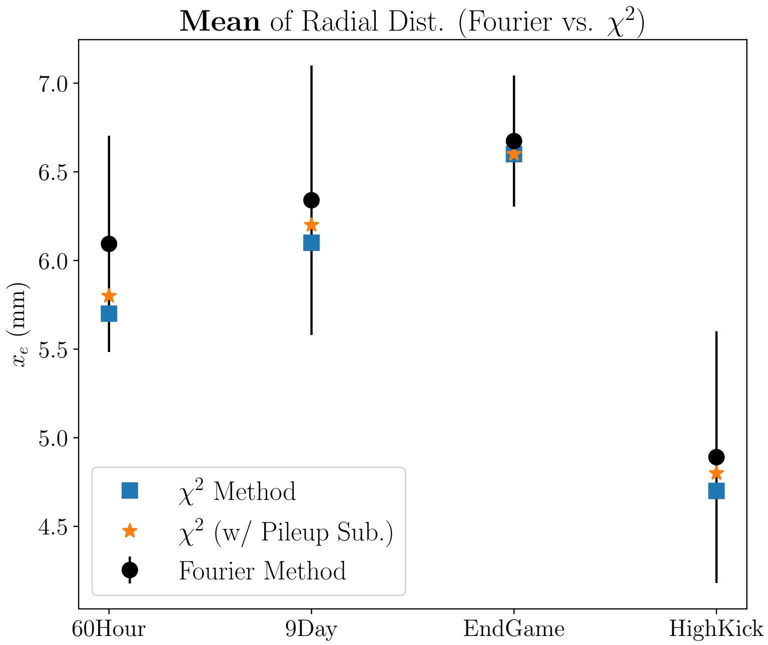
Current Run-1 Results

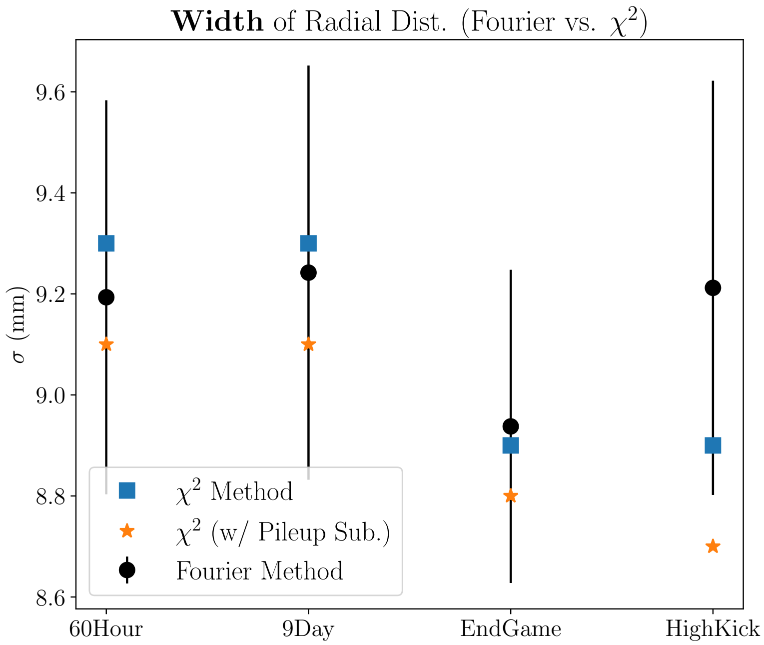
Preliminary Results (as of March 12, 2020)

Dataset	n^1	Kick^2 (kV)	C_E (ppb)	σ_{syst} (ppb)	σ_{stat} (ppb)
60Hour	0.1075	130.0	−461.0	54.4	0.91
9Day	0.1197	130.0	−522.7	73.1	0.82
EndGame	0.1066	124.5	−468.0	35.9	0.34
HighKick	0.1198	137.0	−453.0	59.3	0.96

[1] Using 9-parameter fits from our fast rotation production.
[2] Sum of three settings, on average (GM2-doc-15781).

Comparison with CERN/χ^2 Method





Latest CERN/χ^2 results from Rob Carey (GM2-doc-21672).

Summary

Fourier Method

- use cosine transform to recover cyclotron frequency distribution
  - mirrors the measured fast rotation signal about t_0
- fit and subtract "background" due to late start time t_s

Ongoing Work

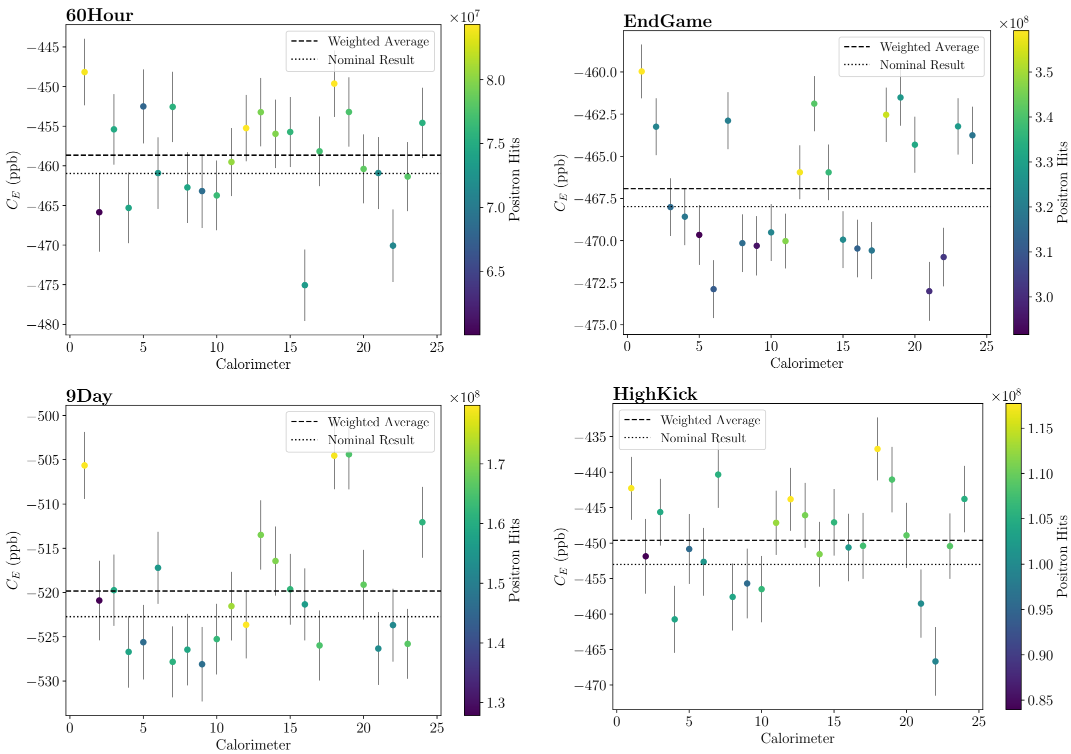
- energy-binned C_E analysis
- investigating contributions to start time scan
- including field index uncertainty
- mitigating impact of momentum-time correlation

Acknowledgment

This material is based upon work supported by the National Science Foundation Graduate Research Fellowship under Grant No. DGE-1650441.

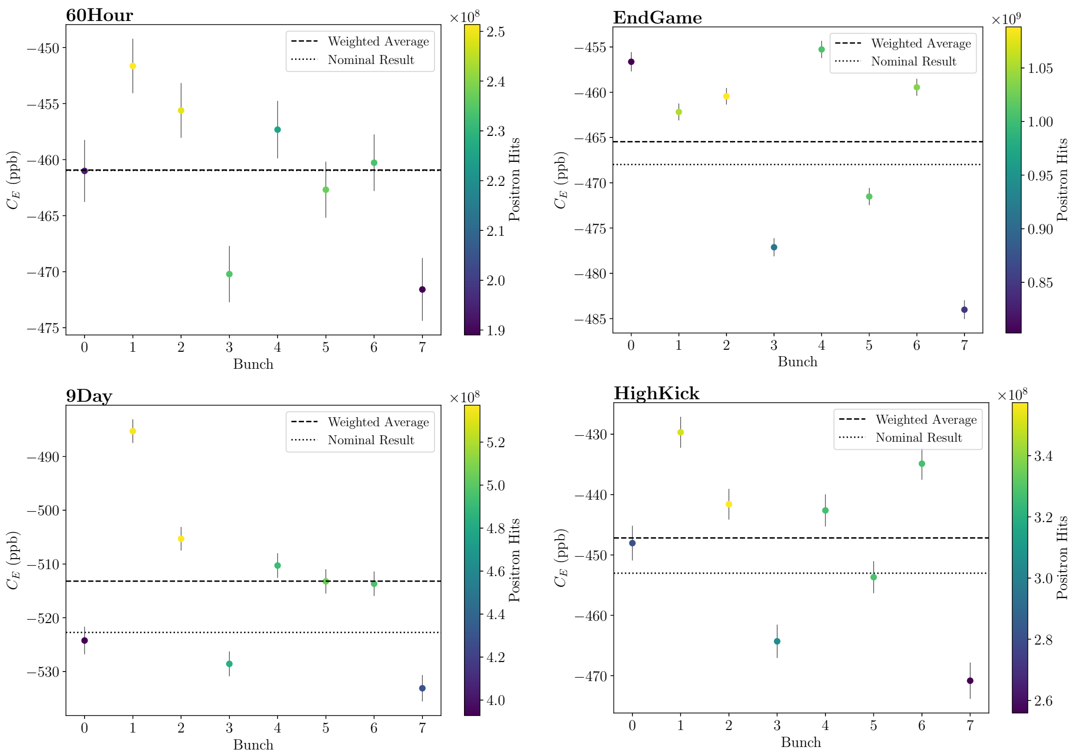
Backup Slides

Per-Calorimeter Analysis



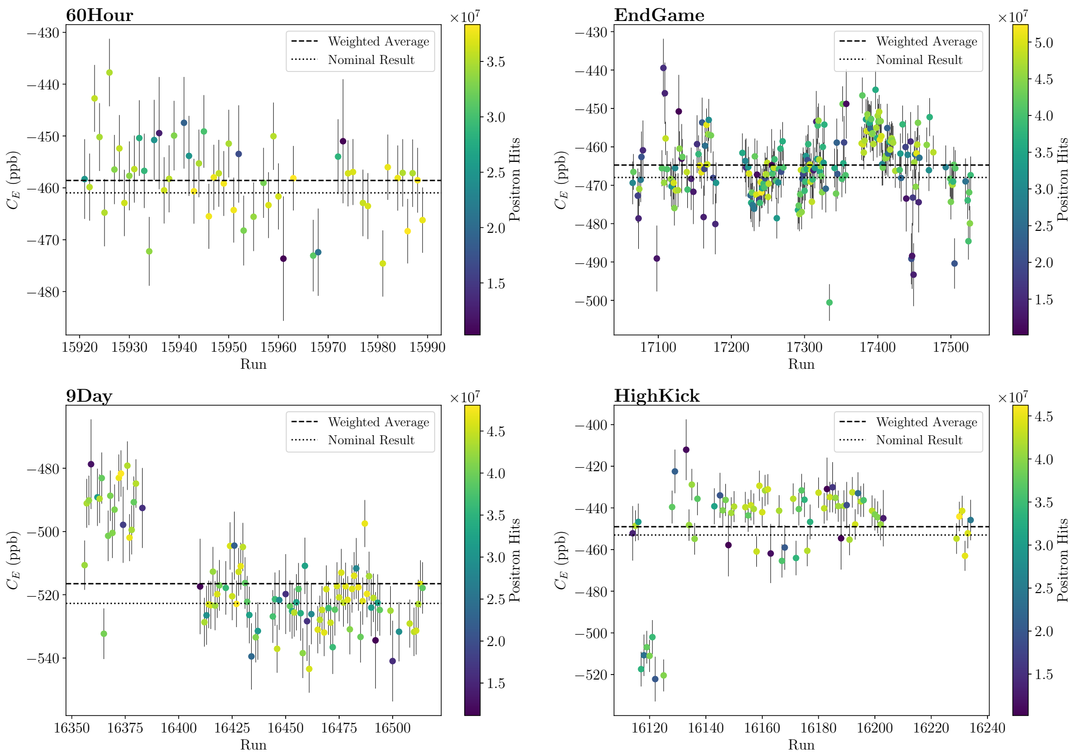
- statistical error bars
  - scaled from nominal statistical uncertainty
- nominal result and weighted average differ by only a few ppb

Per-Bunch Analysis



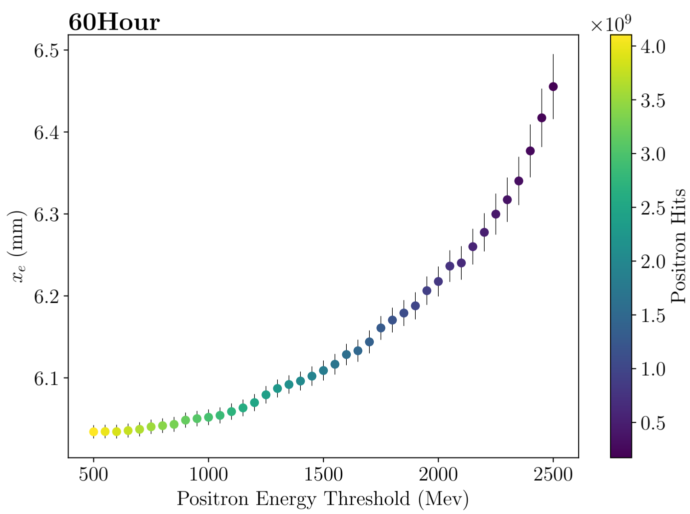
- statistical error bars
  - scaled from nominal statistical uncertainty
- nominal result and weighted average differ by $\lesssim 10$ ppb

Per-Run Analysis



- statistical error bars
  - scaled from nominal statistical uncertainty
- nominal result and weighted average differ by $\lesssim 5$ ppb
- some populations of outliers for 9Day and HighKick

Positron Energy Threshold



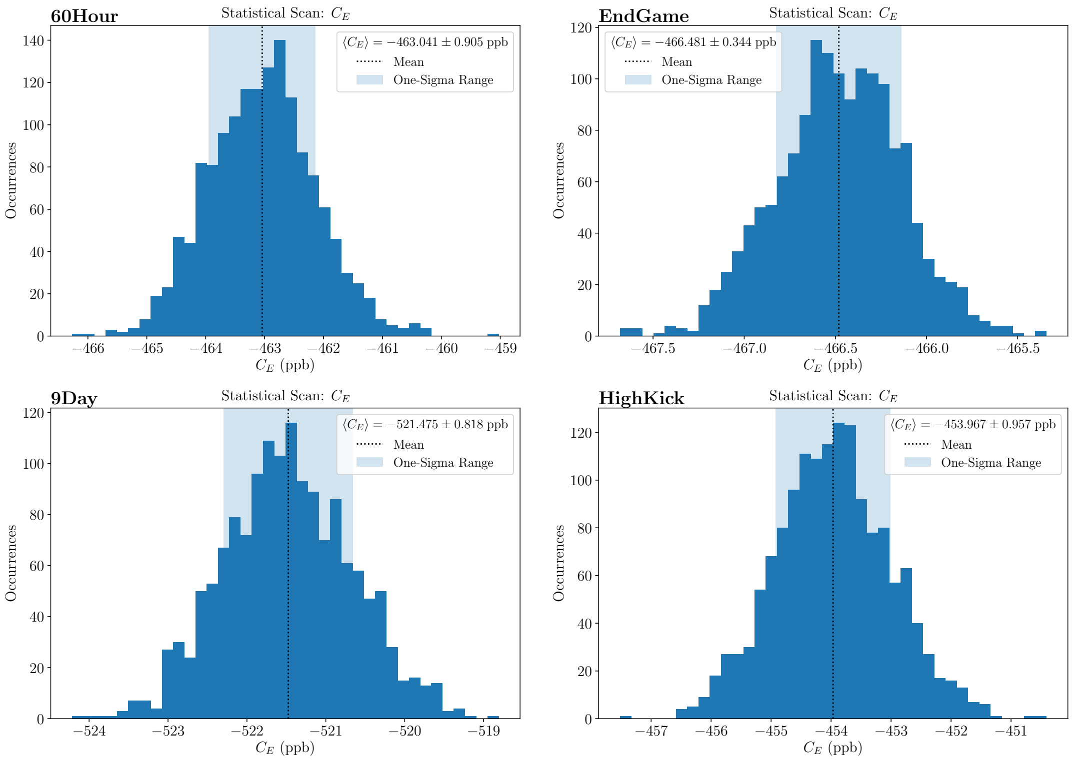
Possible Hypothesis

- calorimeter acceptance vs. positron energy vs. muon radius?
- at low muon radius, high-energy positron curves too little and misses calo?
- higher threshold $\rightarrow$ more missed positrons at low muon radius $\rightarrow$ bias toward higher muon radius?

Simulation

more verification needed, but preliminary comparison with gm2ringsim by Renee agrees well for 500, 1000, 1500 MeV thresholds

Statistical Uncertainty



- apply Poisson fluctuations to fast rotation bins
- repeat analysis 1500 times
- standard deviation taken as statistical uncertainty

Momentum-Time Correlation

Possible Solutions

- look for better approximations of even symmetry (i.e. move t_0)
  - successful only for linear correlation
- measure the real correlation within the ring (post-kick)
- ditch the cosine transform and extrapolate $t < 0$ some other way (machine learning?)

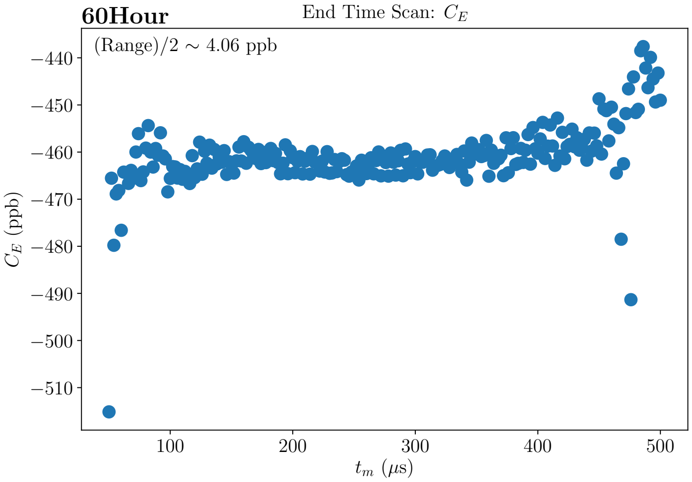
End Time, t_m

Concept

end of cosine transform

Estimate

half-width of spread over 150 – 300 μs



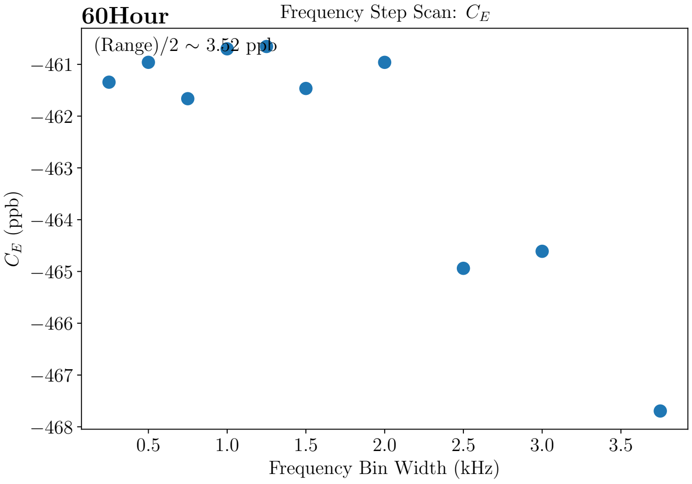
Frequency Bin Width

Concept

step size in frequency domain

Estimate

half-width of spread over 0.25 – 3.75 kHz



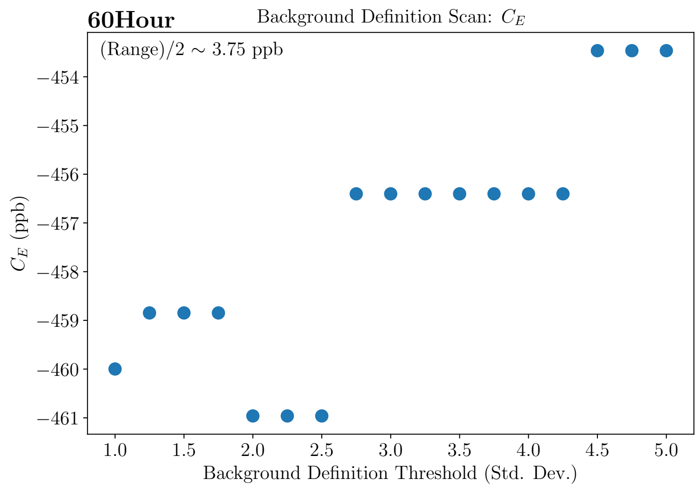
Background Definition Threshold

Concept

- start with unphysical outer bins
- fit bkgd. and calculate residuals
- include inner bins within a few std. devs.
- iterate 3 – 4 times for best fit and t_0

Estimate

half-width of spread over 1 – 5 std. devs.



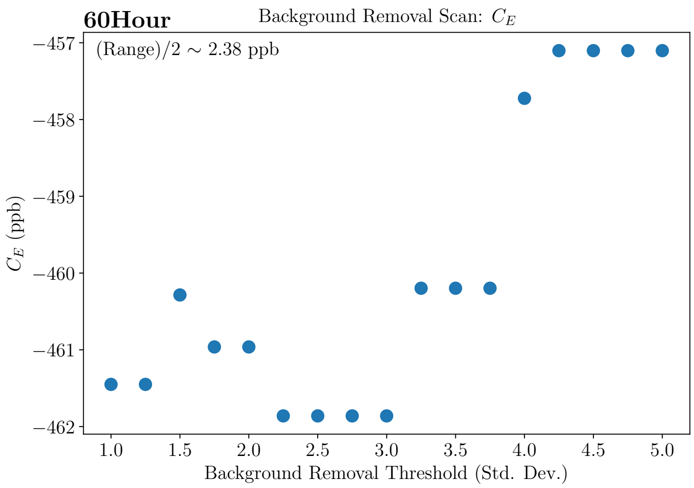
Background Removal Threshold

Concept

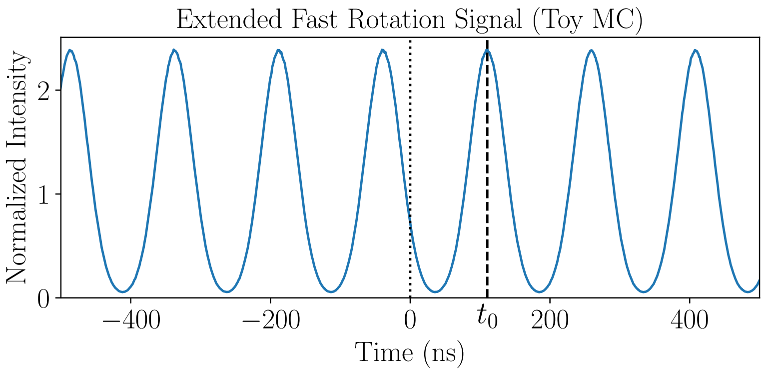
- fit and subtract background
- residual noise remains
- inside fit region, nullify any bins within a few std. devs. of zero

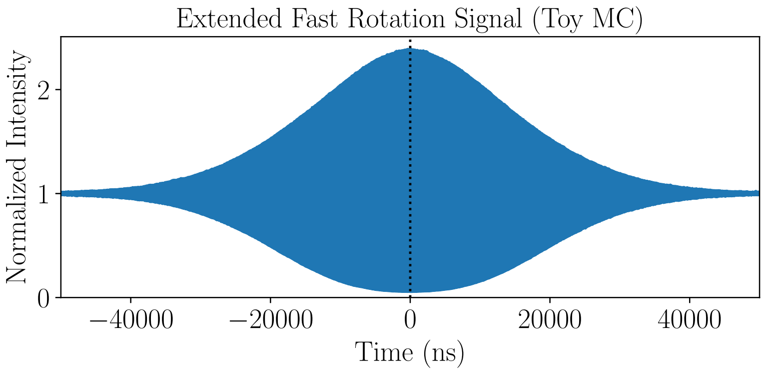
Estimate

half-width of spread over 1 – 5 std. devs.



Empirical Motivation



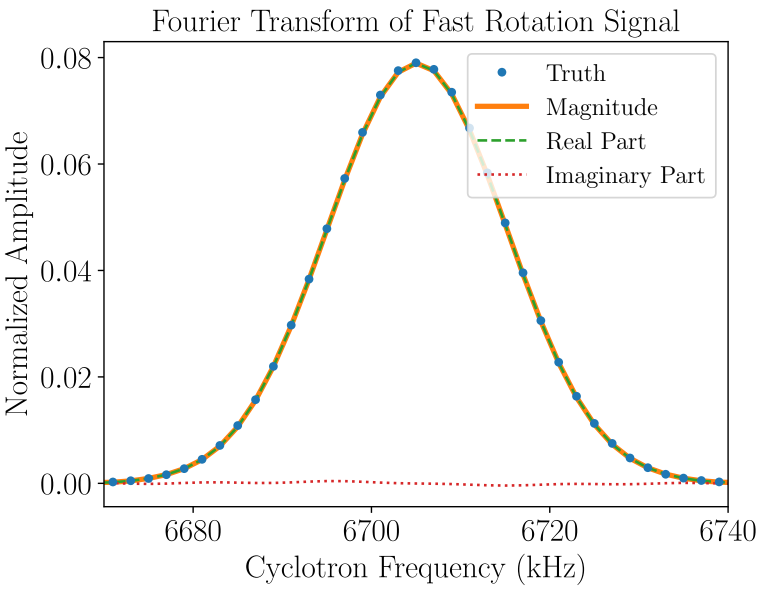


To motivate this, let's start with the purest Toy MC, and then add constraints one-by-one.

Toy Monte Carlo

- place a detector plane in the ring
  - t_0 = average time on "first" turn
- for each muon...
  - choose cyclo. frequency f (Gaussian)
  - choose time offset τ (Gaussian)
  - calculate all detector crossing times
  - bin into the fast rotation histogram

Empirical Motivation



- **magnitude** of Fourier transform recovers the true frequency distribution
- so does the **real part**
  - the signal was symmetric in time

Analytical Motivation

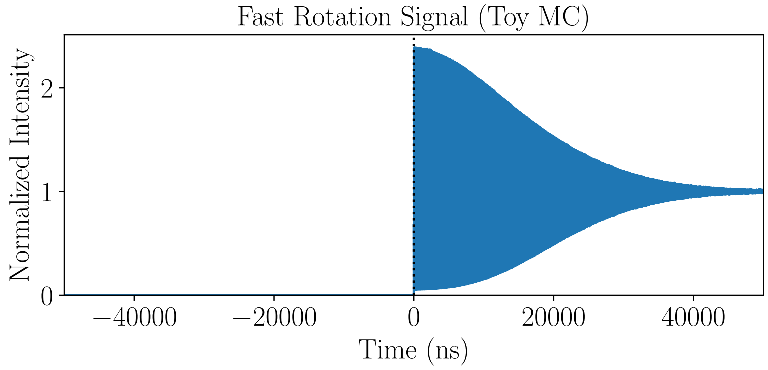
We can also turn our Toy MC algorithm into a formula:

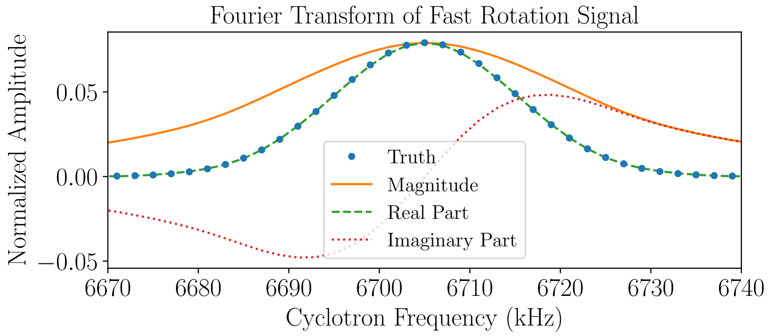
$$\underbrace{\mathcal{S}(t)}_{\substack{\text{fast rotation}\\ \text{signal}}} \propto \underbrace{\sum_{n=-\infty}^{\infty}}_{\substack{\text{sum}\\ \text{all turns}}} \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} df\, d\tau\, \rho(f)\, \xi(\tau)}_{\substack{\text{sum all muons, using}\\ \text{frequency distribution } \rho(f)\\ \text{and beam profile } \xi(\tau)}} \underbrace{\delta\left[(t-t_0) - \left(\frac{n}{f} + \tau\right)\right]}_{\text{time of detection on } n\text{th turn}}$$

The Fourier transform also recovers the input frequency distribution:

$$\underbrace{\left|\int_{-\infty}^{\infty} \mathcal{S}(t)\, e^{i(2\pi f)(t-t_0)}\, dt\right|}_{\substack{\text{magnitude of}\\ \text{Fourier transform}}} \propto \underbrace{\rho(f)}_{\substack{\text{original}\\ \text{frequency}\\ \text{distribution}}}$$

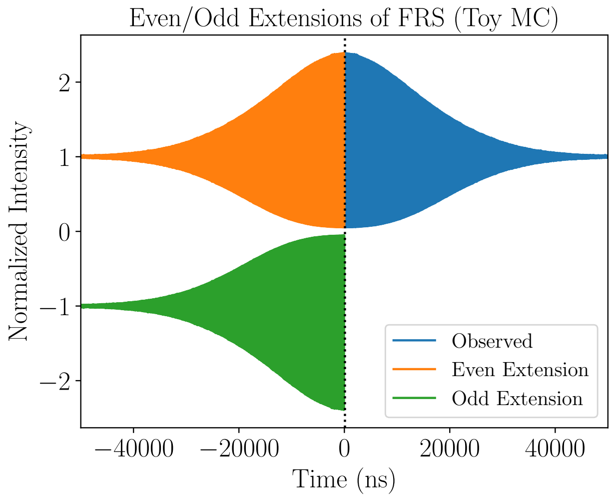
Injection Constraint





- this assumes we have $\mathcal{S}(t)$ on $-\infty < t < +\infty$
- realistically, only measure $t > 0$
- what if we remove the signal for $t < 0$?
- magnitude $\neq$ truth
  - broadened by unphysical frequencies
- **real part** still works! why?

Cosine Transform



- real part of Fourier transform is the **cosine transform**
- can always split signal into even and odd parts:

$$\mathcal{S}(t) \equiv \underbrace{\frac{1}{2}\Big[\mathcal{S}(t) + \mathcal{S}(-t)\Big]}_{\text{manifestly even}} + \underbrace{\frac{1}{2}\Big[\mathcal{S}(t) - \mathcal{S}(-t)\Big]}_{\text{manifestly odd}}$$

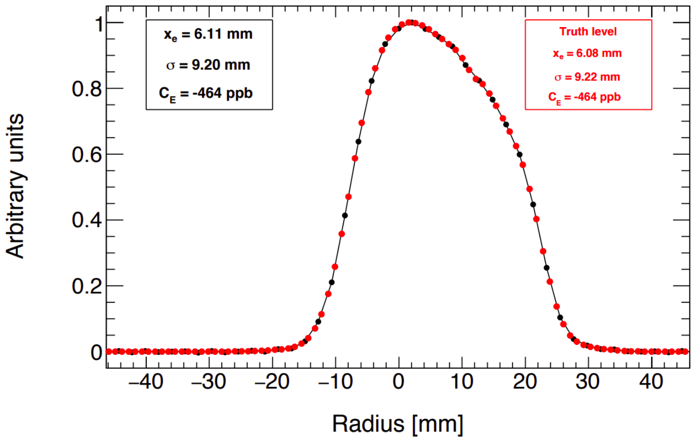
- cosine transform picks out the **even part**
- in our case, even part is an **even extension**
  - restores the missing signal for $t < 0$

Cosine transform yields correct frequency distribution by mirroring signal about t_0.

Asymmetric Beam Profile

Our analytical formula **agrees** with even symmetry (assuming symmetric beam profile).

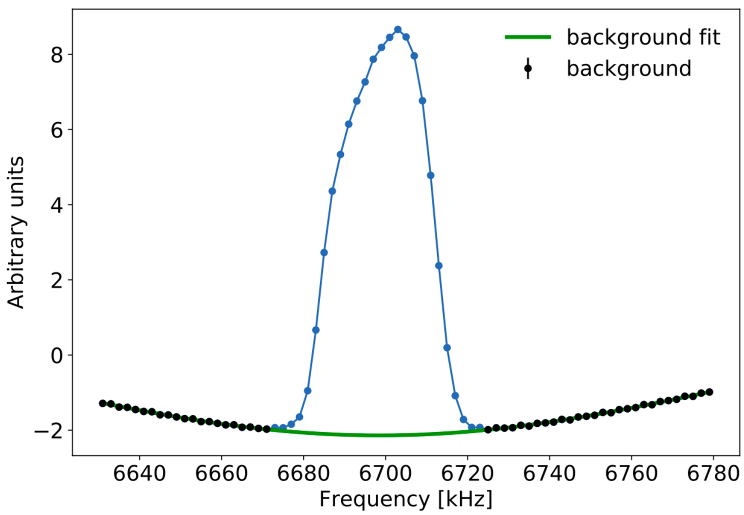
- What happens with an asymmetric (i.e. realistic) beam profile?



- MC from Josh
  - realistic frequency distribution (60Hour)
  - realistic beam profile (60Hour)
- symmetry approximation still holds

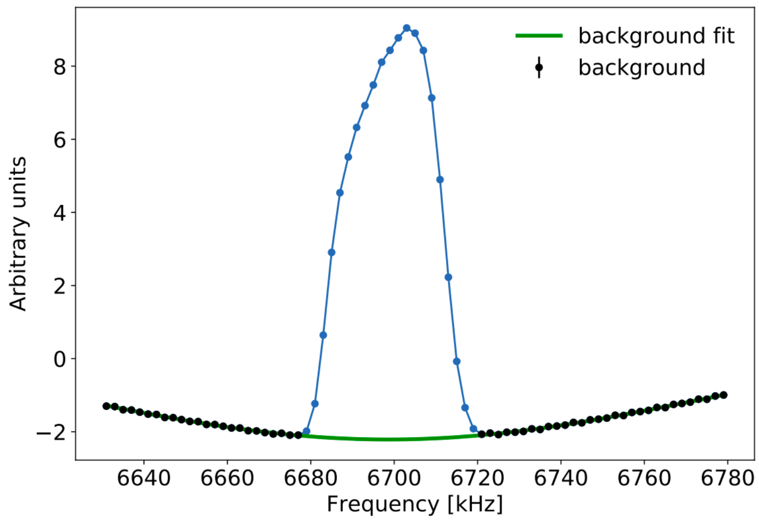
Momentum-Time Correlation

Two Toy MC, one with correlation and one without.



no correlation

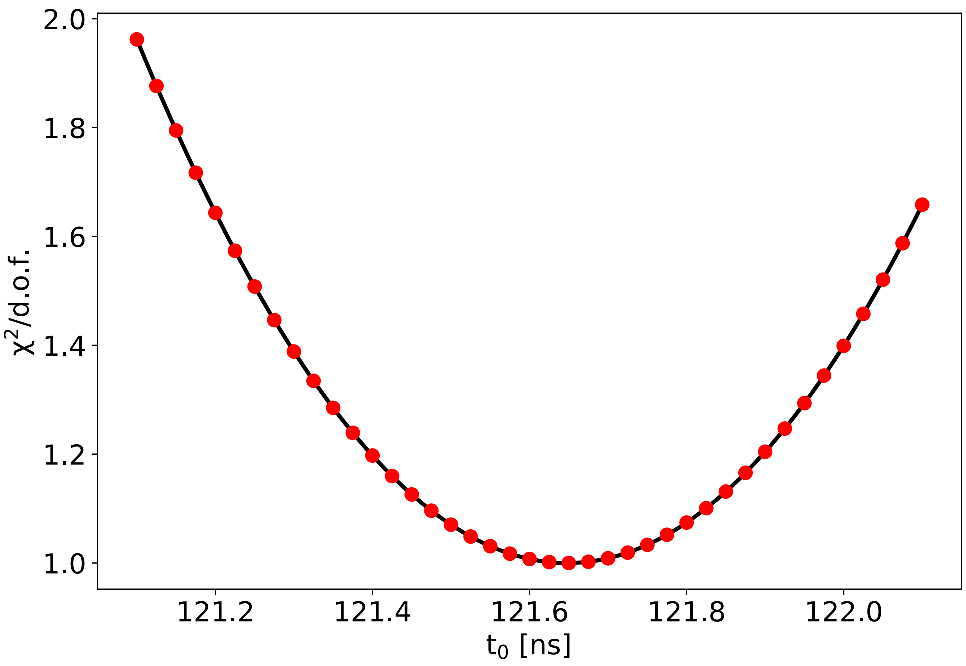
$\sim$1 ppb discrepancy from truth

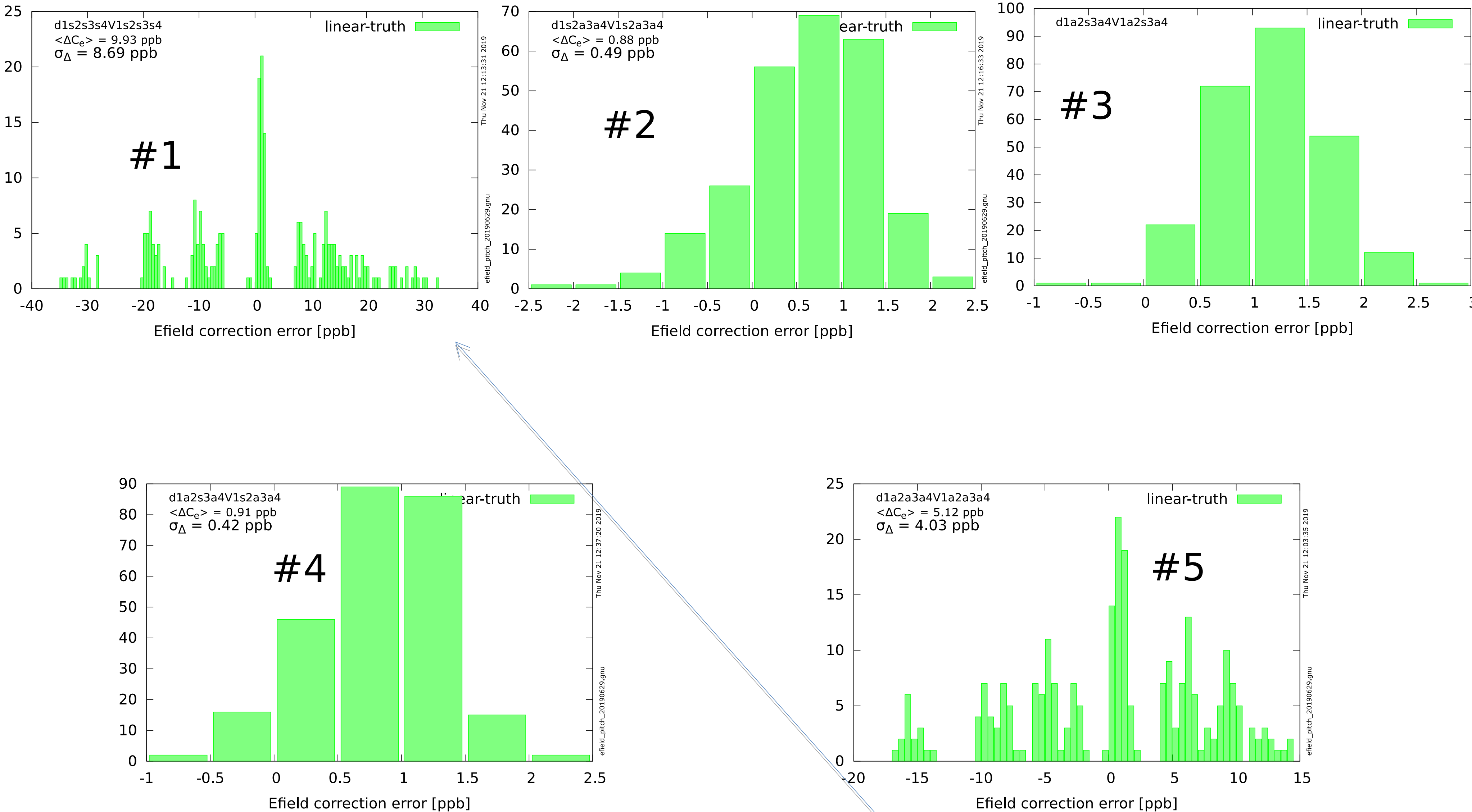


gm2ringsim-inspired correlation

$\sim$50 ppb discrepancy from truth

χ^2 for t_0 Determination (e.g. 60Hour)





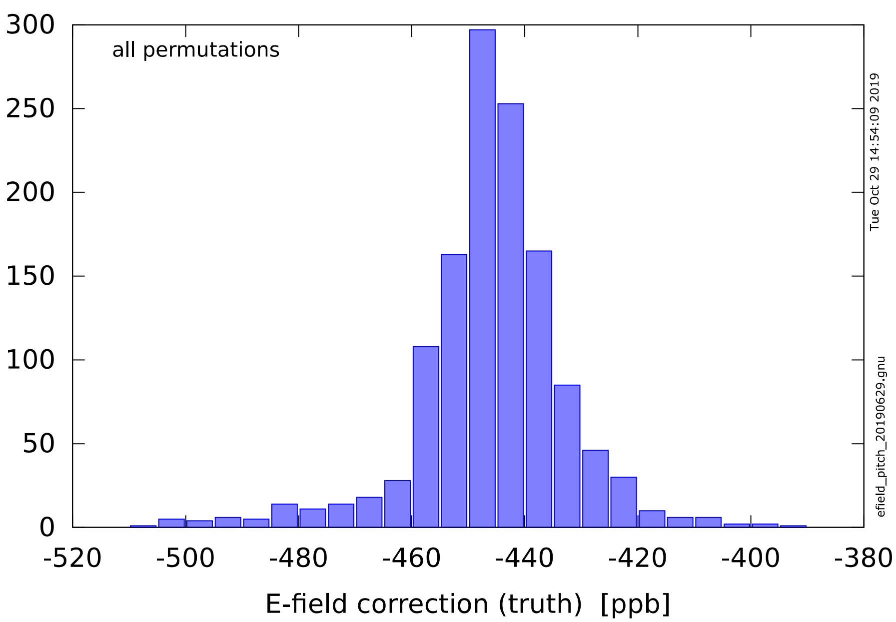
$$C_e(\text{meas}) = -2\beta^2 n_x(1-n_x)\frac{\langle x_e^2\rangle}{R_0^2}$$

$$|C_e(\text{meas}) - C_e(\text{truth})| < 35 \text{ ppb}$$

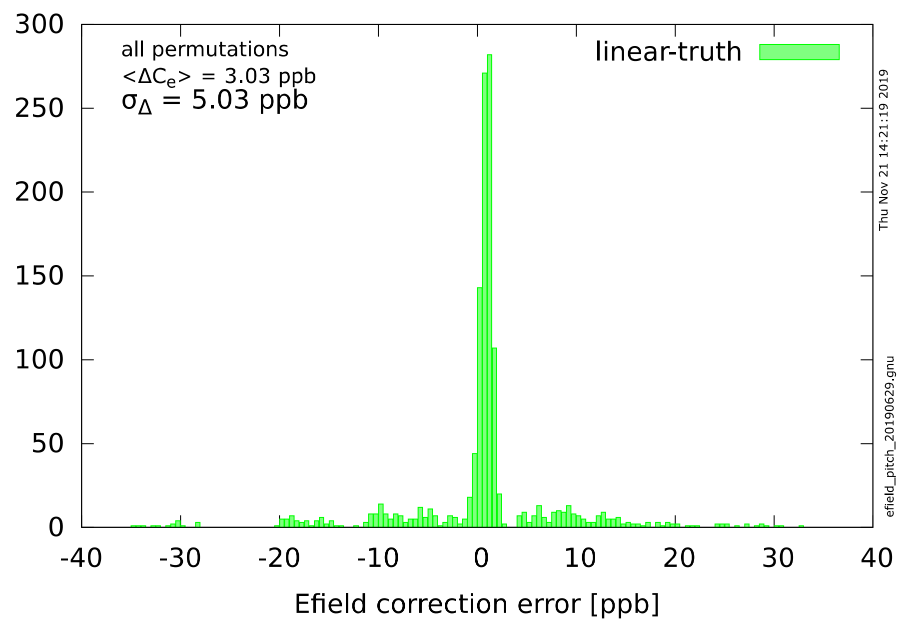
$$\sigma_\Delta \leq 8.69 \text{ ppb}$$

D. Rubin

All 1280 configurations



truth



measured - truth

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The betatron frequencies of the horizontal and vertical motion of the centroids => Qx, Qy

$$n_y = Q_y^2$$
$$n_x = 1 - Q_x^2$$