Corrections for Electric Field and Pitch

D. Rubin for the Efield/Pitch team

and *thanks to Josh, Tyler, Renee, James, David T., Antoine, and Paul for slides and explanations*

July 15, 2020

Electric field and vertical pitching motion systematically shift ω_a

How do we determine that shift ?

- *Origin of Efield and pitch contributions*
- *Approximate Formulation – How do we quantify the effects?*
- *g-2 Ring model dependent uncertainties*
- *Measurement*
    - *Methods*
    - *Results*
    - *Uncertainties*

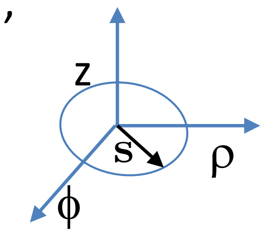
What is the contribution of electric field and pitch to the measured precession frequency?

We measure the rate of change of the projection of the polarization on the velocity

$$\frac{d}{dt}(\hat{\boldsymbol{\beta}}\cdot\mathbf{s}) = -\frac{e}{mc}\mathbf{s}_\perp \cdot \left[a_\mu \hat{\boldsymbol{\beta}} \times \mathbf{B} + \left(a_\mu - \frac{m^2c^2}{p^2}\right)\beta\mathbf{E}\right]$$

Jackson (11.171)

In a uniform field where $\mathbf{B} = B\hat{\mathbf{z}}$ $\hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\phi}}$ and $\mathbf{E} = 0$,

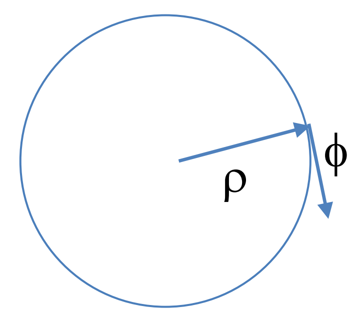


$$\mathbf{s} = |\mathbf{s}|(\cos\omega_a t\hat{\boldsymbol{\rho}} - \sin\omega_a t\hat{\boldsymbol{\phi}})$$

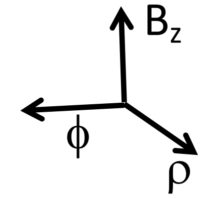
$$\mathbf{s}_\perp = \mathbf{s} - (\hat{\boldsymbol{\beta}}\cdot\mathbf{s})\cdot\hat{\boldsymbol{\beta}} = |\mathbf{s}|\cos\omega_a t\hat{\boldsymbol{\rho}}$$

$$\hat{\boldsymbol{\beta}}\cdot\mathbf{s} = -|\mathbf{s}|\sin\omega_a t$$

$$\omega_a = \frac{e}{mc}Ba_\mu$$



Pitch is that component of velocity parallel to $\mathbf{B}$

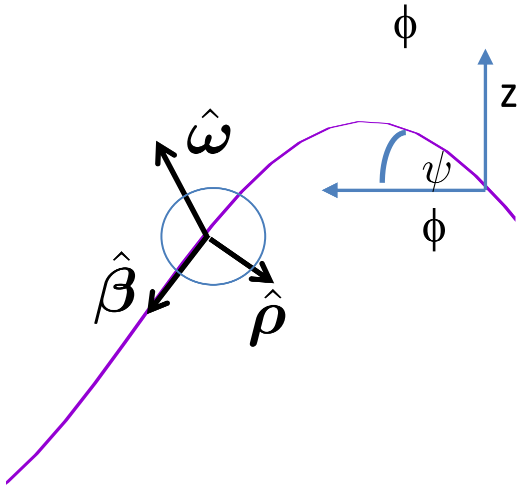


$$\hat{\boldsymbol{\beta}} = \cos\psi\hat{\boldsymbol{\phi}} + \sin\psi\mathbf{z}$$

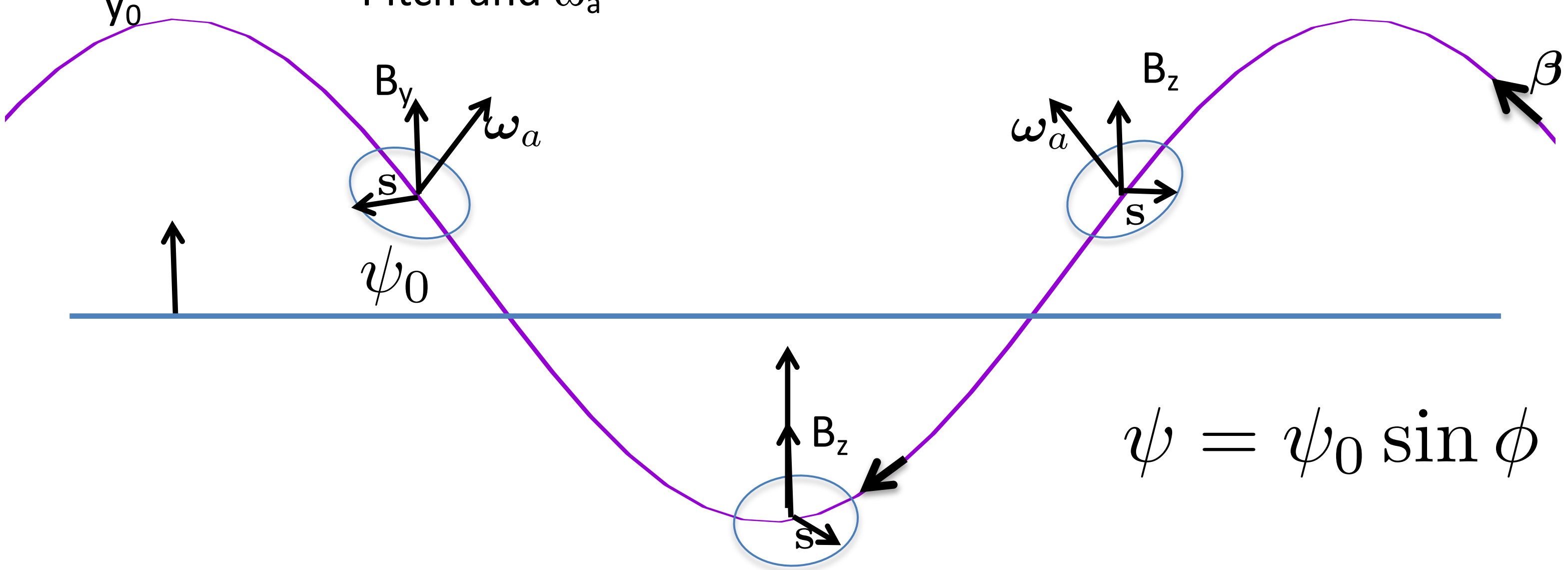
How much is the frequency shifted due to the pitch?

$$\mathbf{s}_{\perp} \to |\mathbf{s}|\cos(\omega_a + \Delta\omega)t\hat{\boldsymbol{\rho}}$$

$$\hat{\boldsymbol{\beta}} \cdot \mathbf{s} \to -|\mathbf{s}|\sin(\omega_a + \Delta\omega)t$$



Pitch and ω_a



The vertical motion is modulated by the quad *electric* field

If there were no magnetic field, projection of polarization on velocity is constant

At the magic momentum $\frac{d(\boldsymbol{\beta} \cdot \mathbf{s})}{dt} = 0$

The out of horizontal plane component of $\mathbf{s}$ follows the out of plane component of $\boldsymbol{\beta}$

$\boldsymbol{\omega}_a$ Is always in the plane defined by $\mathbf{B}$ and $\boldsymbol{\beta}$

Substitute the 'unperturbed' solution into our differential equation

$$\frac{d}{dt}(-|\mathbf{s}|\sin(\omega_a+\Delta\omega)t) \quad = \quad -\frac{e}{mc}|\mathbf{s}|\cos(\omega_a+\Delta\omega)t\hat{\boldsymbol{\rho}}\cdot\left[a_\mu(|\mathbf{B}|(\cos\psi\hat{\boldsymbol{\rho}}+\sin\psi\hat{\boldsymbol{\phi}})\right]$$

$$\frac{d}{dt}|\mathbf{s}|\sin(\omega_a+\Delta\omega)t) \quad = \quad \frac{e}{mc}|\mathbf{s}|\cos(\omega_a+\Delta\omega)t\left[a_\mu(|\mathbf{B}|\cos\psi)\right]$$

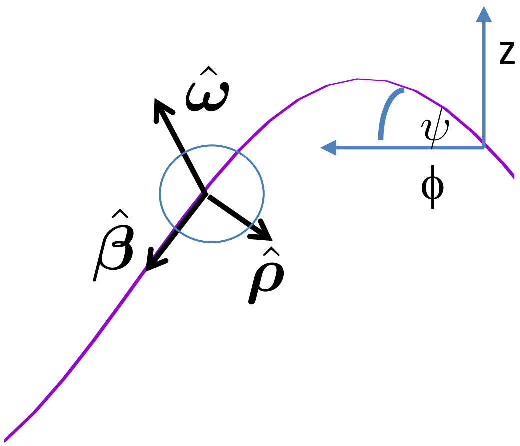
$$\frac{d}{dt}(|\mathbf{s}|\sin(\omega_a+\Delta\omega)t) = \frac{e}{mc}|\mathbf{s}|\cos(\omega_a+\Delta\omega)t\left[a_\mu(|\mathbf{B}|-|\mathbf{B}|(1-\cos\psi)\right]$$

$$\omega_a+\Delta\omega \quad = \quad \frac{e}{mc}\left[a_\mu(|\mathbf{B}|-|\mathbf{B}|(1-\cos\psi)\right]$$

$$\langle\Delta\omega\rangle = \frac{e}{mc}\left[a_\mu\langle-|\mathbf{B}|(1-\cos\psi)\rangle\right]$$

$$\frac{\Delta\omega_p}{\omega_a} = -\langle 1-|\hat{\boldsymbol{\beta}}\times\hat{\mathbf{B}}|\rangle \sim -\langle\frac{1}{2}\psi^2\rangle$$

How do we measure ψ ?



If motion (quad field) is linear

$$\begin{aligned} y &= \sqrt{a\beta}\cos\phi \\ \psi &= \psi_0 \sin\phi = \sqrt{\frac{a}{\beta}}\sin\phi \end{aligned}$$

Average over all ϕ for a given amplitude *a*

$$\langle \psi^2(a) \rangle_\phi = \frac{1}{2}\psi_0^2(a) = \frac{1}{2}\frac{\langle a \rangle}{\beta} = \frac{\langle y^2(a) \rangle}{\beta^2}$$

Pitch

$$\langle\psi^2(a)\rangle_\phi = \frac{1}{2}\psi_0^2(a) = \frac{\langle y^2(a)\rangle}{\beta^2}$$

$$C_p = -\langle 1 - |\hat{\boldsymbol{\beta}} \times \hat{\mathbf{B}}|\rangle \sim -\frac{1}{2}\langle\psi^2\rangle = -\frac{1}{2}\frac{\langle y^2\rangle}{\beta^2}$$

True in general for each muon

Assuming linear and continuous quads

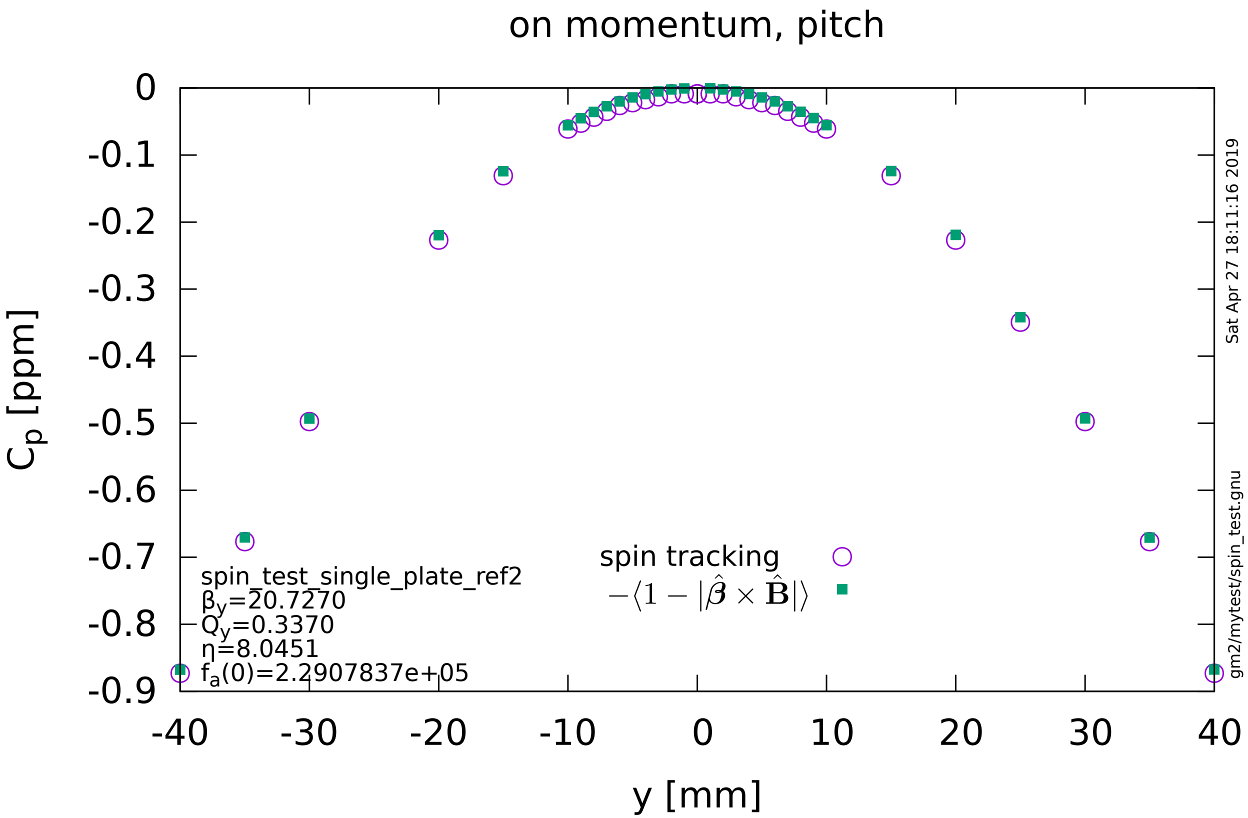
$$\beta = \frac{\sqrt{n}}{R_0}$$

$$C_p = -\frac{n\langle y^2\rangle}{2R_0^2}$$

$$\langle\psi^2(a)\rangle_\phi = \frac{1}{2}\psi_0^2(a) = \frac{1}{2}\frac{\langle a\rangle}{\beta}$$

$$\begin{aligned} y &= \sqrt{a\beta}\cos\phi \\ \psi &= \psi_0 \sin\phi = \sqrt{\frac{a}{\beta}}\sin\phi \end{aligned}$$

True for any muon ?



Excellent agreement with 'spin tracking' (integrating BMT equation)

For any muon in a distribution we can compute $-\langle 1-|\hat{\boldsymbol\beta}\times\hat{\mathbf{B}}|\rangle$ to determine the contribution of pitching motion to ω_a

What about our assumption of linear and continuous quads, in perfect alignment, with uniform voltage?

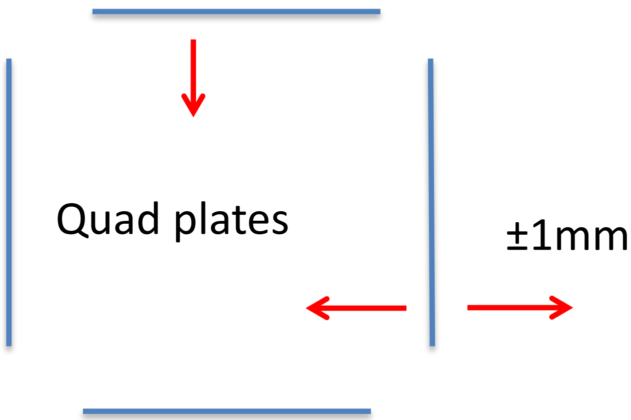
Test in simulation

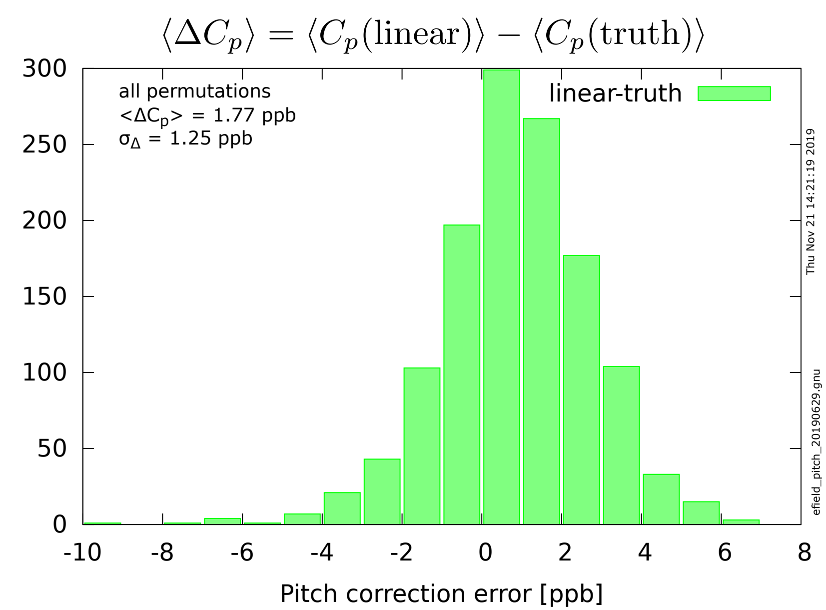
For each of 1280 permutations of plate offsets and voltage errors, inject and track a distribution of muons

$$C_p(\text{truth}) = -\langle 1 - |\hat{\boldsymbol{\beta}} \times \hat{\mathbf{B}}| \rangle$$

$$C_p(\text{linear}) = -\frac{n\langle y^2 \rangle}{2R_0^2}$$

$\Delta V_p = \pm 5\%\ V_0$

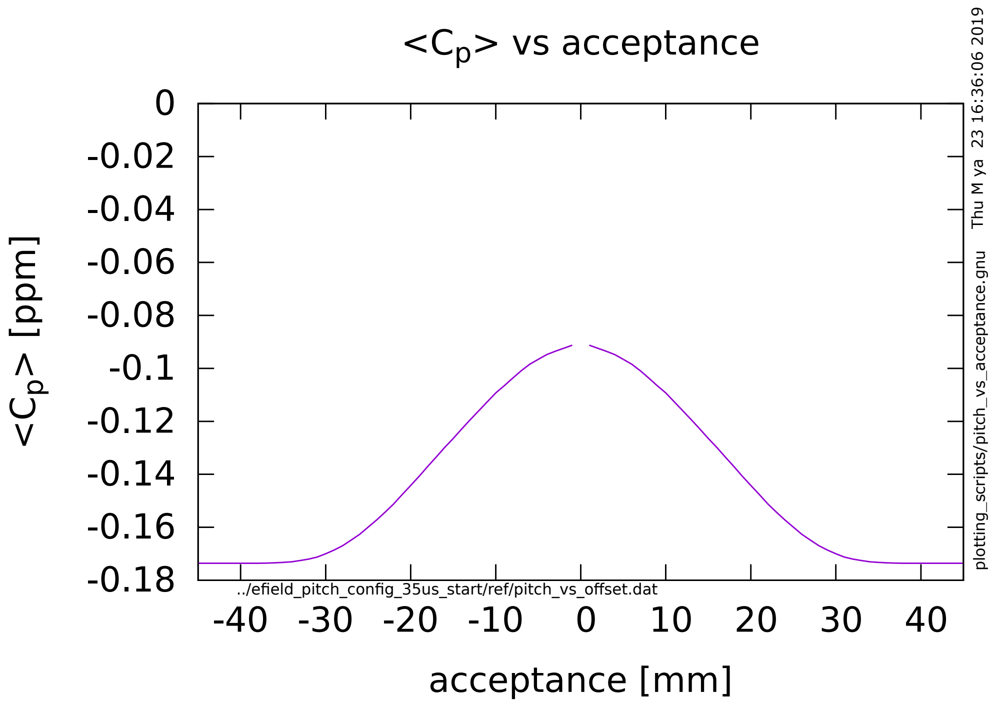




Pitch and acceptance

The maximum pitch angle is determined by the physical aperture (collimators)
The average pitch correction for the distribution depends on the vertical acceptance of the detector

If aperture is 45mm then

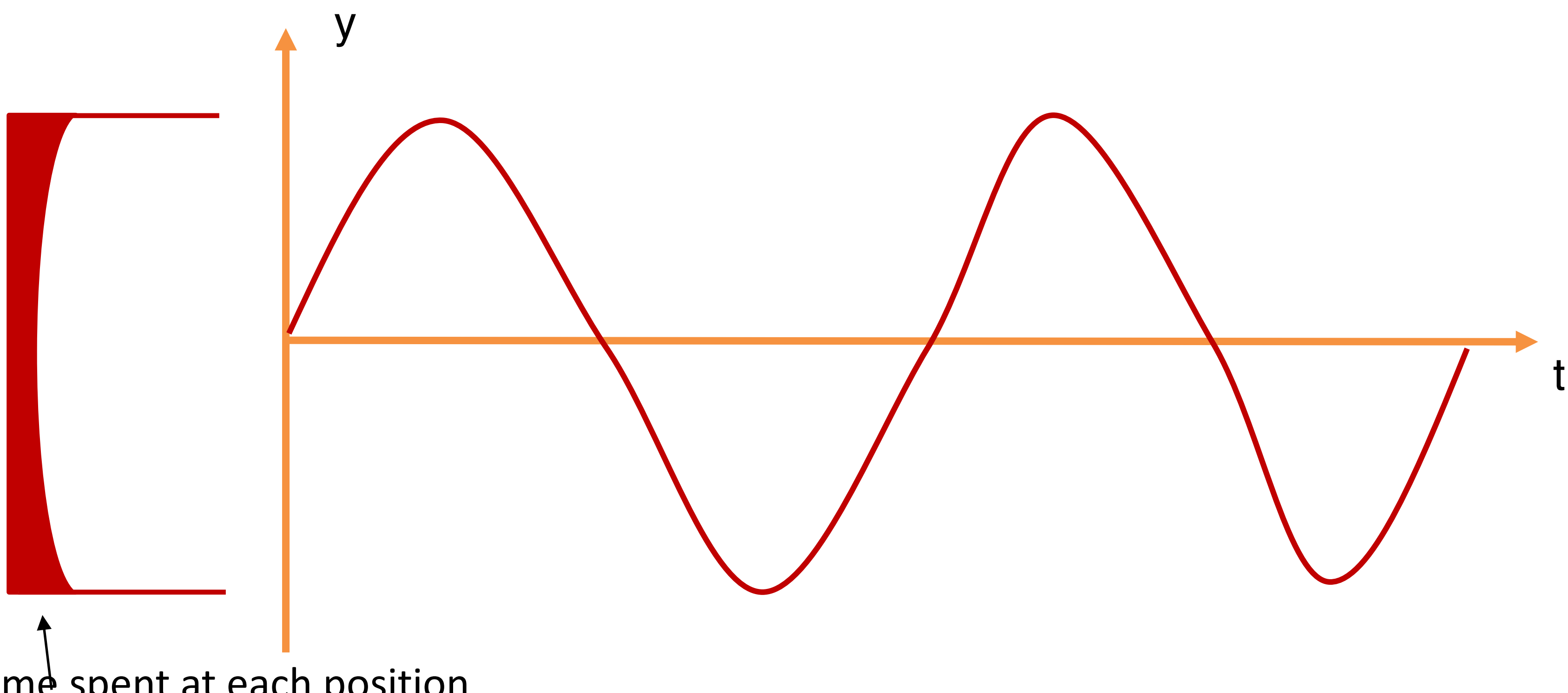


Acceptance	C_p [ppm]
± 1 mm	-0.09
± 10 mm	-0.11
± 40 mm	-0.175

$$y = \sqrt{a\beta}\cos\phi$$

We measure vertical position. We prefer vertical amplitude

$$\langle \psi^2(a) \rangle_\phi = \frac{1}{2}\psi_0^2(a) = \frac{1}{2}\frac{\langle a \rangle}{\beta}$$

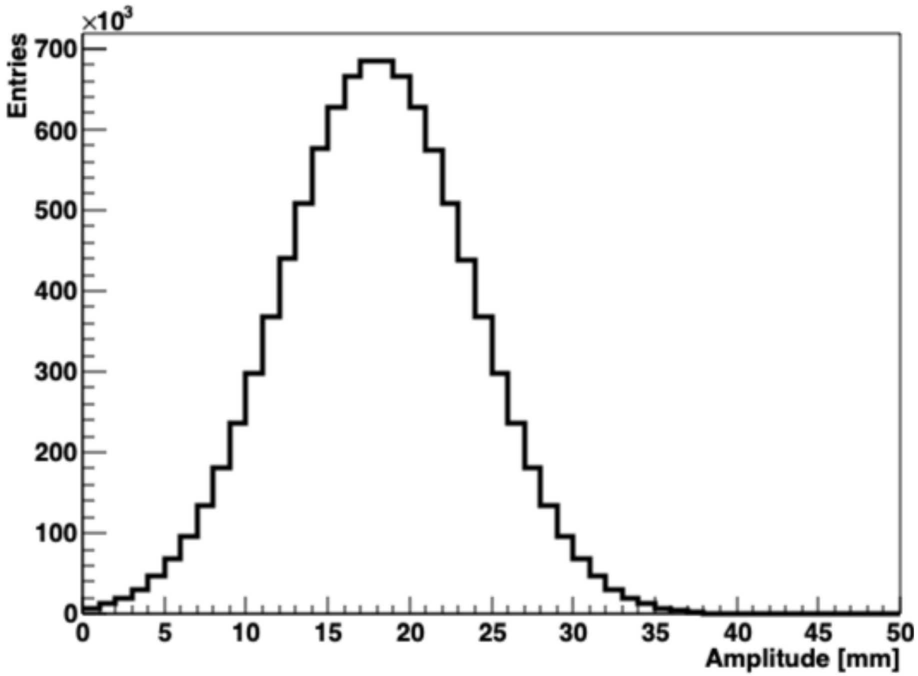


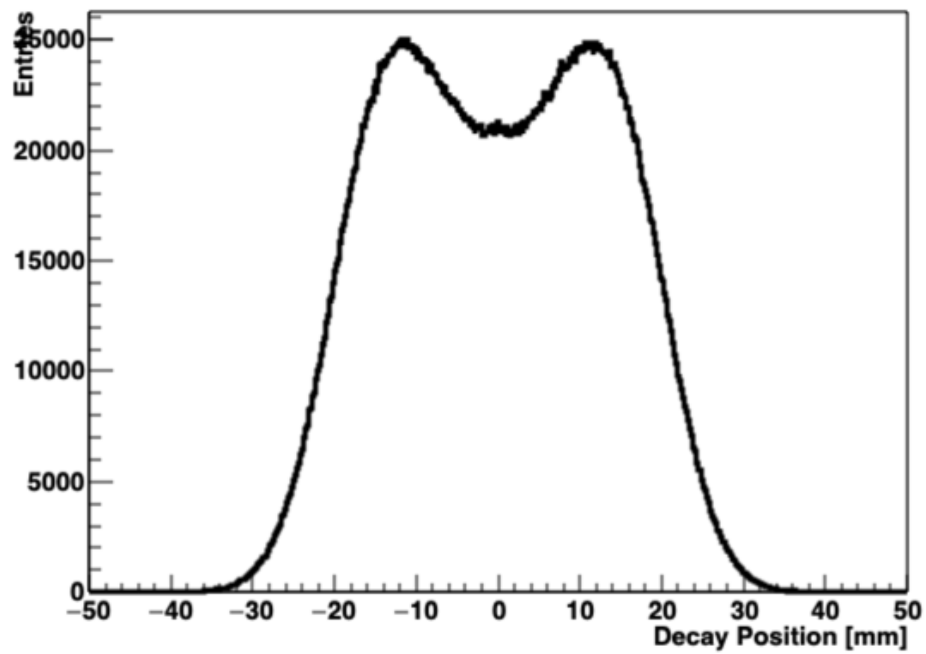
Time spent at each position
(same as decay distribution)

Next few slides courtesy J. Mott

Toy MC: Amplitudes to Positions

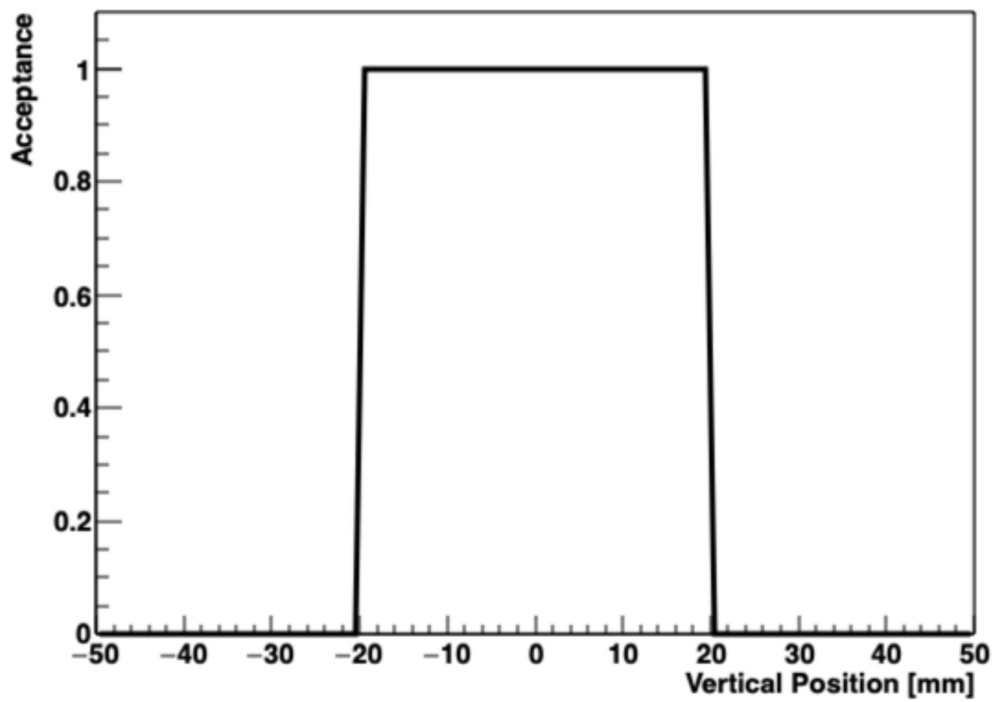
Distribution of amplitudes → Distribution of measured positions





Toy MC: Simple Calo Acceptance

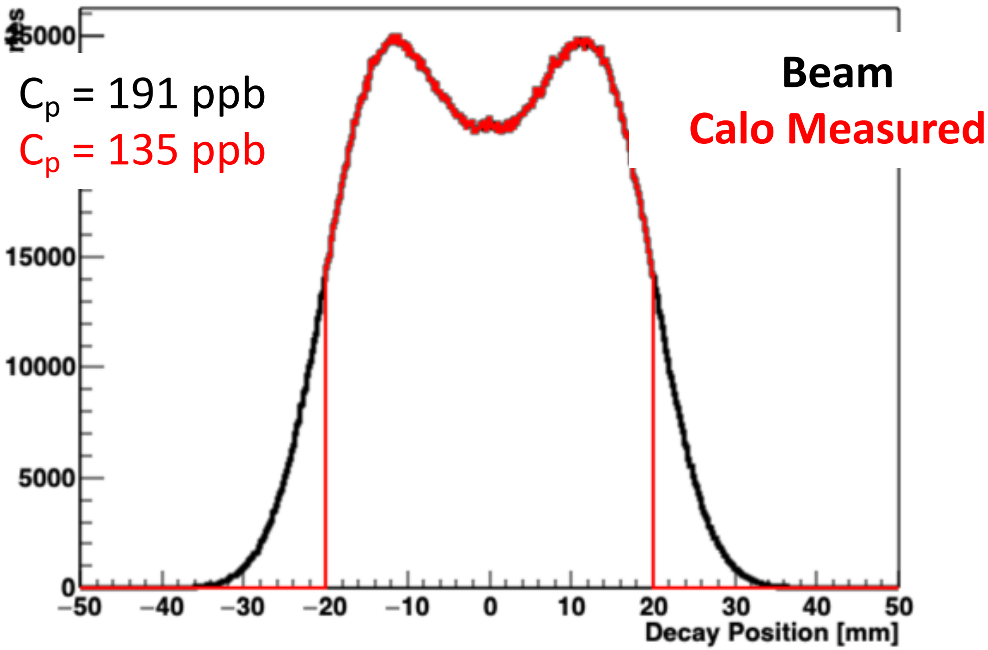
- Suppose this box function defines the calo acceptance

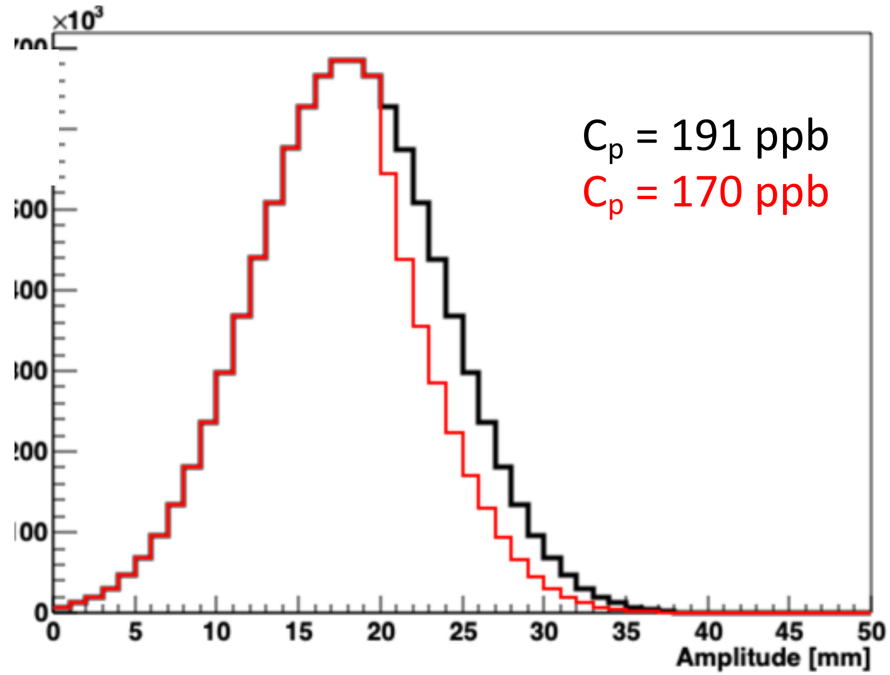


Toy MC: Simple Calo Acceptance

Imagine a calo acceptance of +/- 20 mm

Measured position distribution ⟷ Amplitude distribution



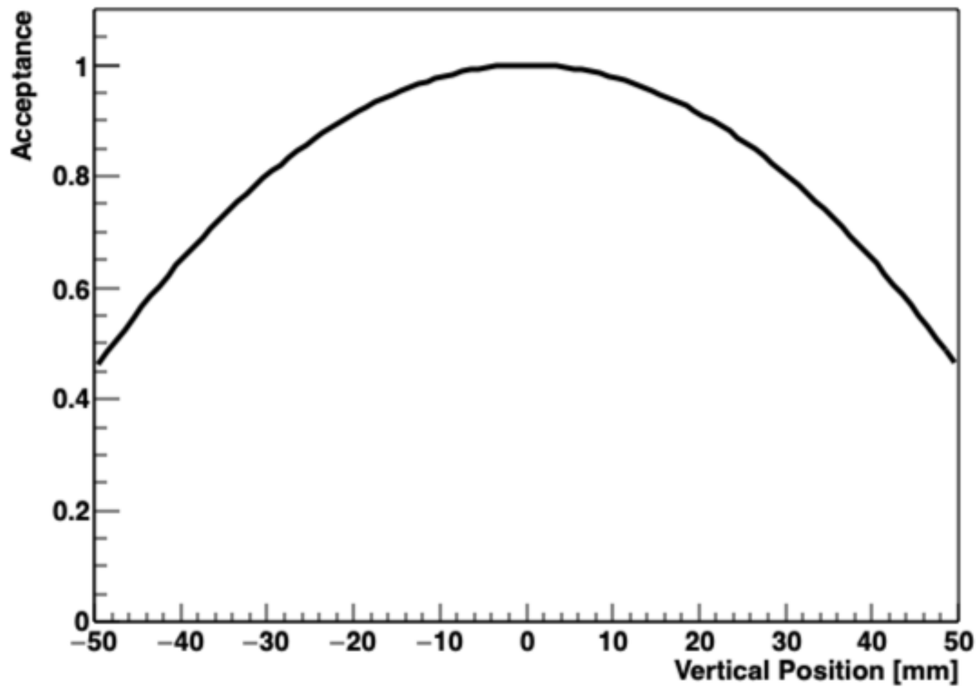


<y^2> underestimates angles and C_p

<a> accounts for all angles

Toy MC: Realistic Calo Acceptance

- More realistic calo acceptance:

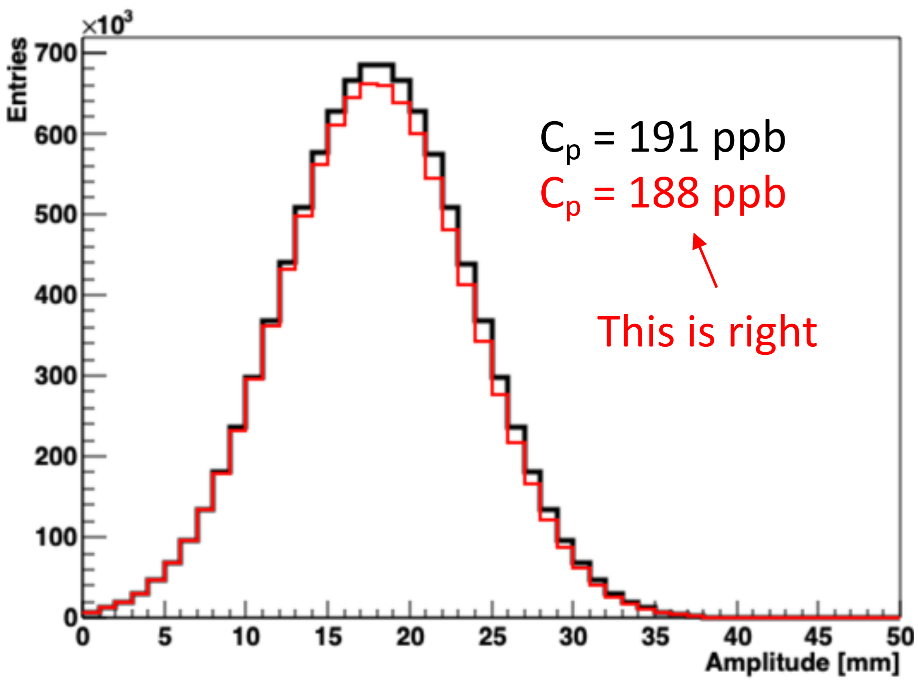


- Uses gm2ringsim (gas gun) and E > 1700 MeV. All the calos combined.

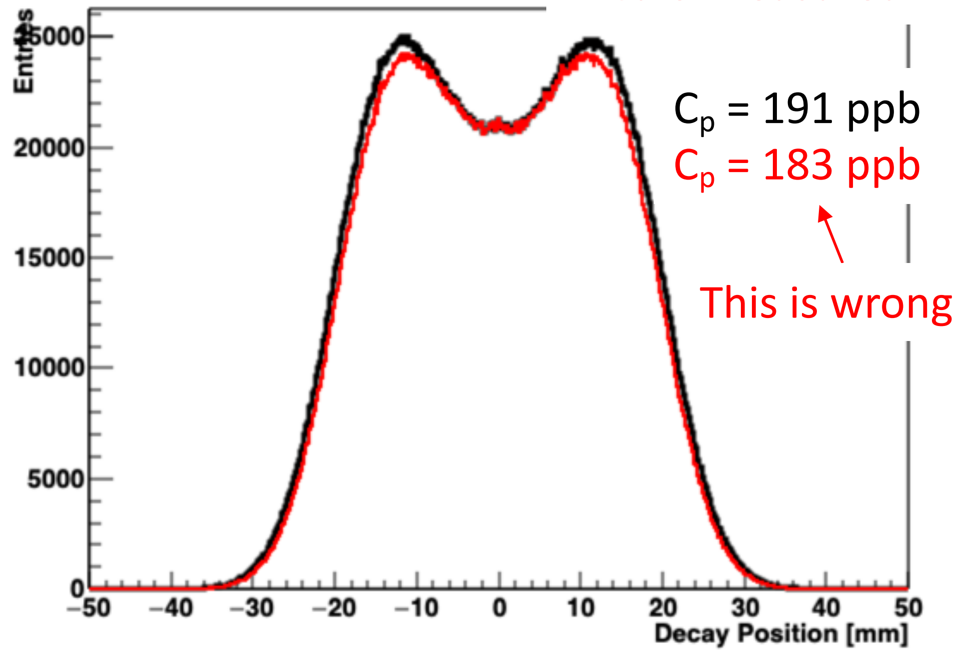
Toy MC: Realistic Calo Acceptance

- Effect is the same as before, but more subtle.

Beam
Calo Measured



<a> accounts for all angles



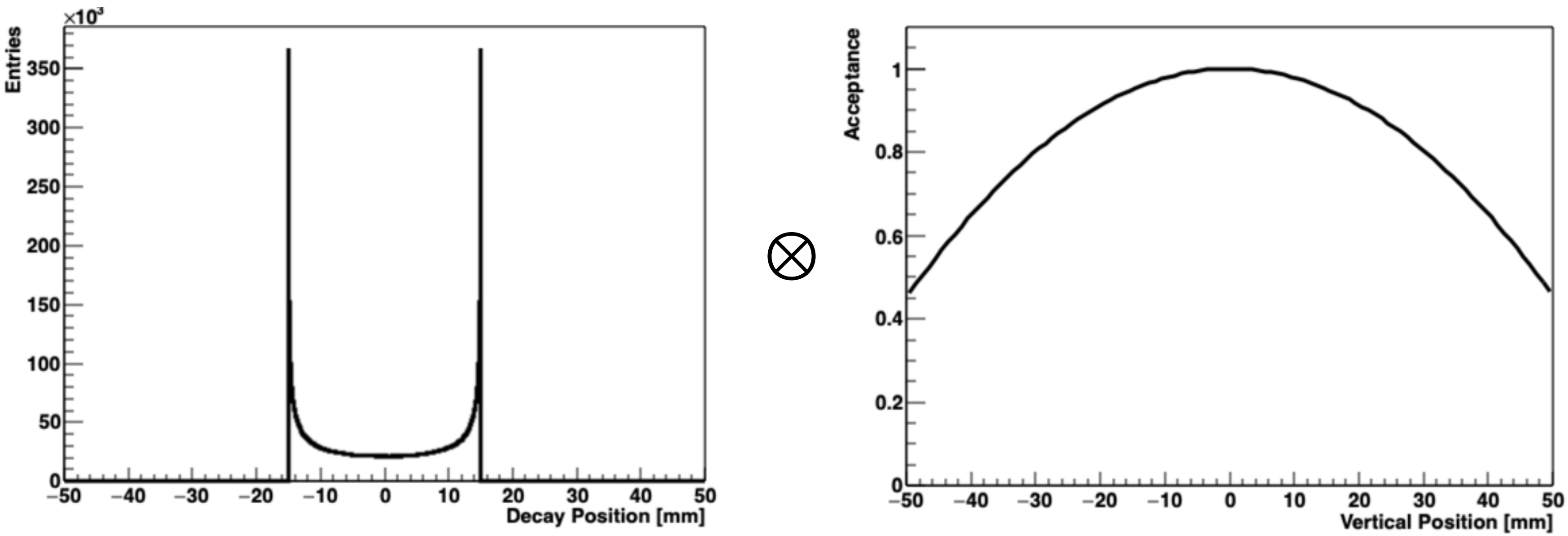
$<y^2>$ underestimates angles and C_p

- In this made up toy MC case, we're only off by 5 ppb

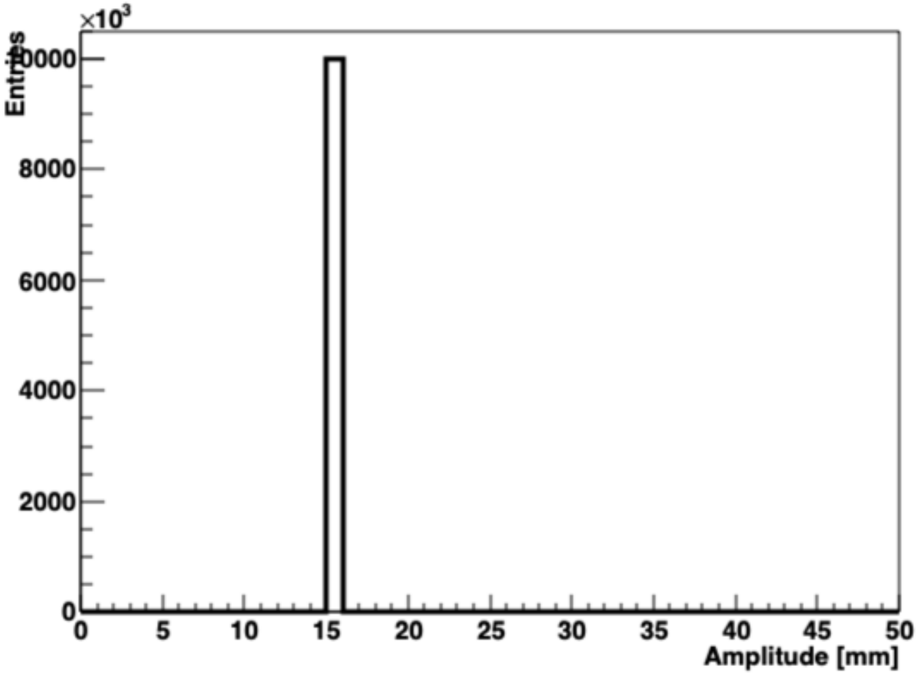
Calo Acceptance & Weighting

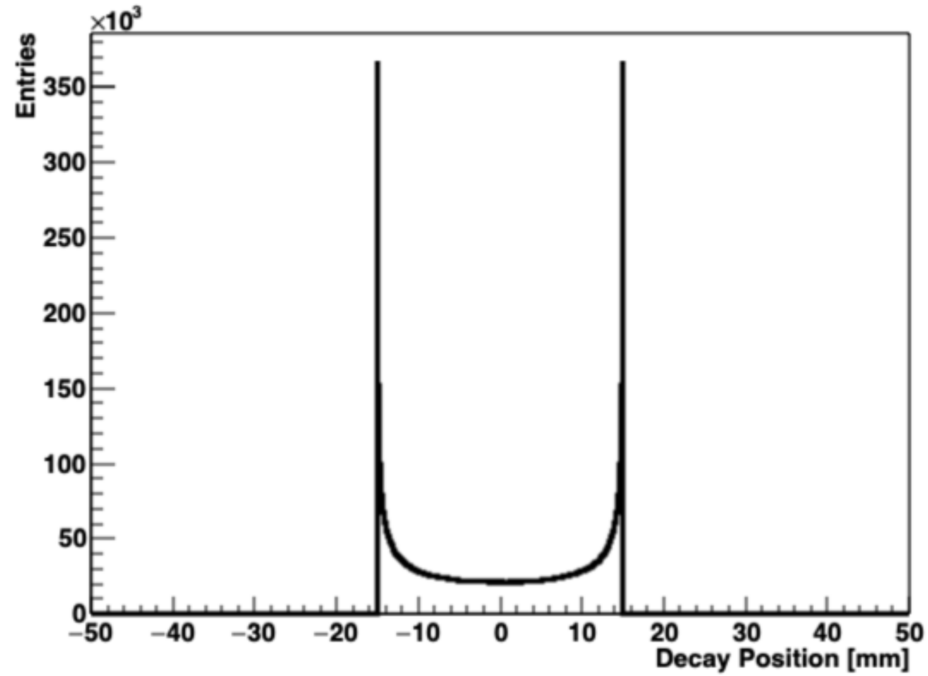
- If we have a betatron amplitude, we can calculate its appropriate weighting based on the calo acceptance maps

- We just need to multiply the two distributions together to get how often the muon decays as seen by calos:



For any given betatron amplitude, A, we can compute probability of measuring a particular position y





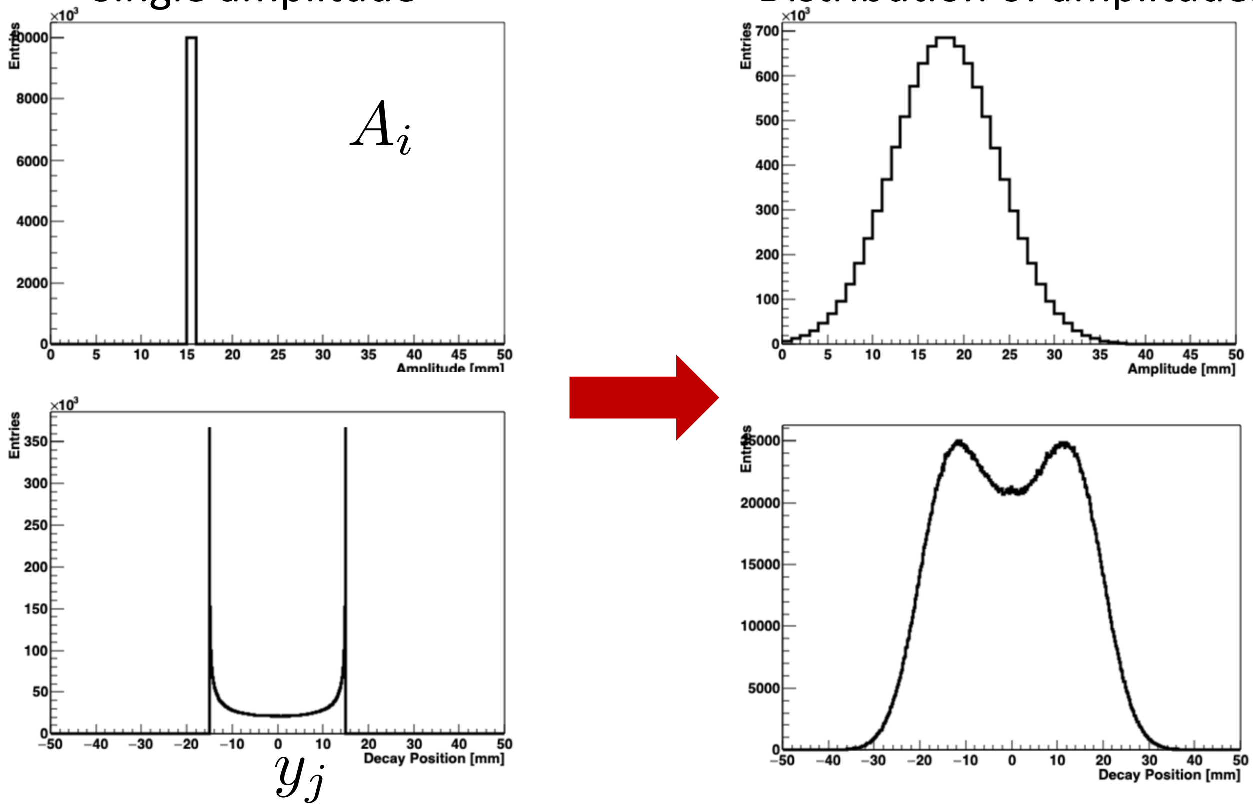
$$N(y, A) = \begin{cases} \frac{1}{\sqrt{1-(y/A)^2}} & |y| < A \\ 0 & \text{otherwise} \end{cases}$$

Single amplitude

Distribution of amplitudes

Amplitude distribution

Position distribution



Probability that amplitude A_i lands at position y_j is β_{ij}

If there are f_i entries in amplitude bin i then there will be $N_{ij} = \beta_{ij} f_i$ entries in position bin j

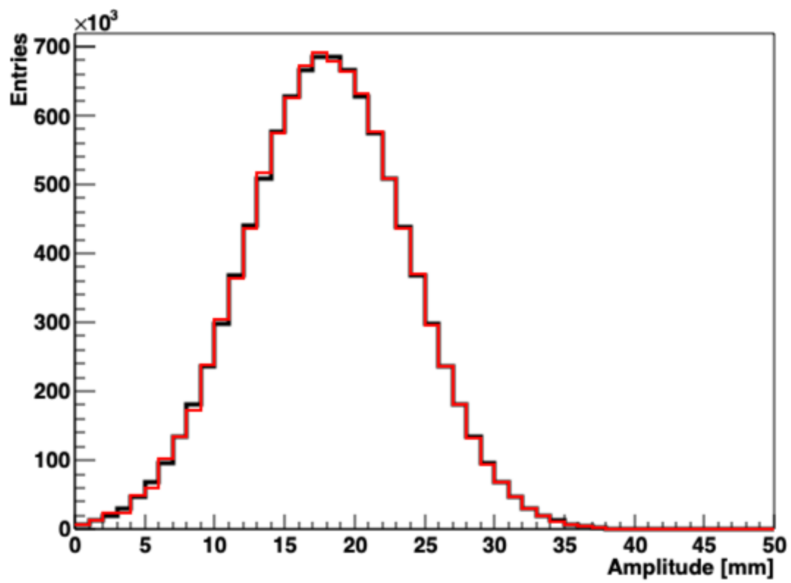
Summing over amplitude bins $$\hat{N}_j = \sum_i \beta_{ij} f_i$$

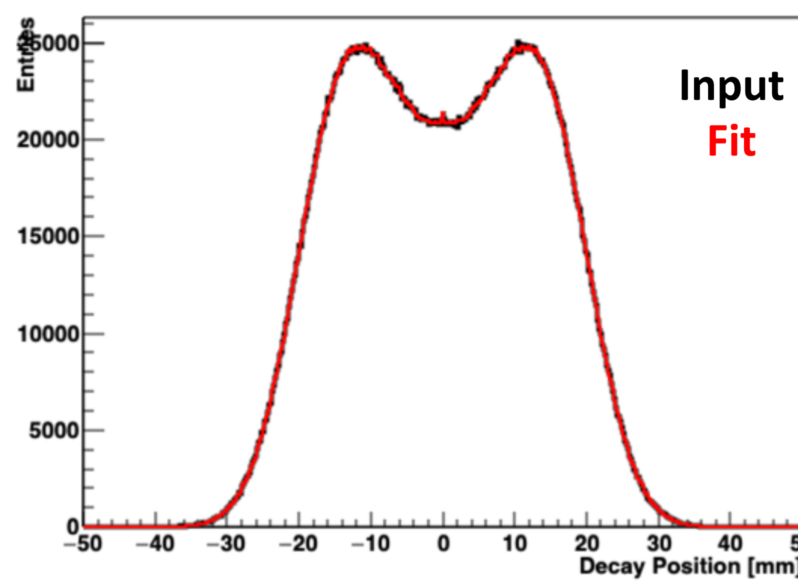
We want to determine the number of entries f_i in each amplitude bin A_i based on the number of entries in position bins N_j

Measure N_j. Compute β_{ij}. And minimize χ^2 to determine f_i

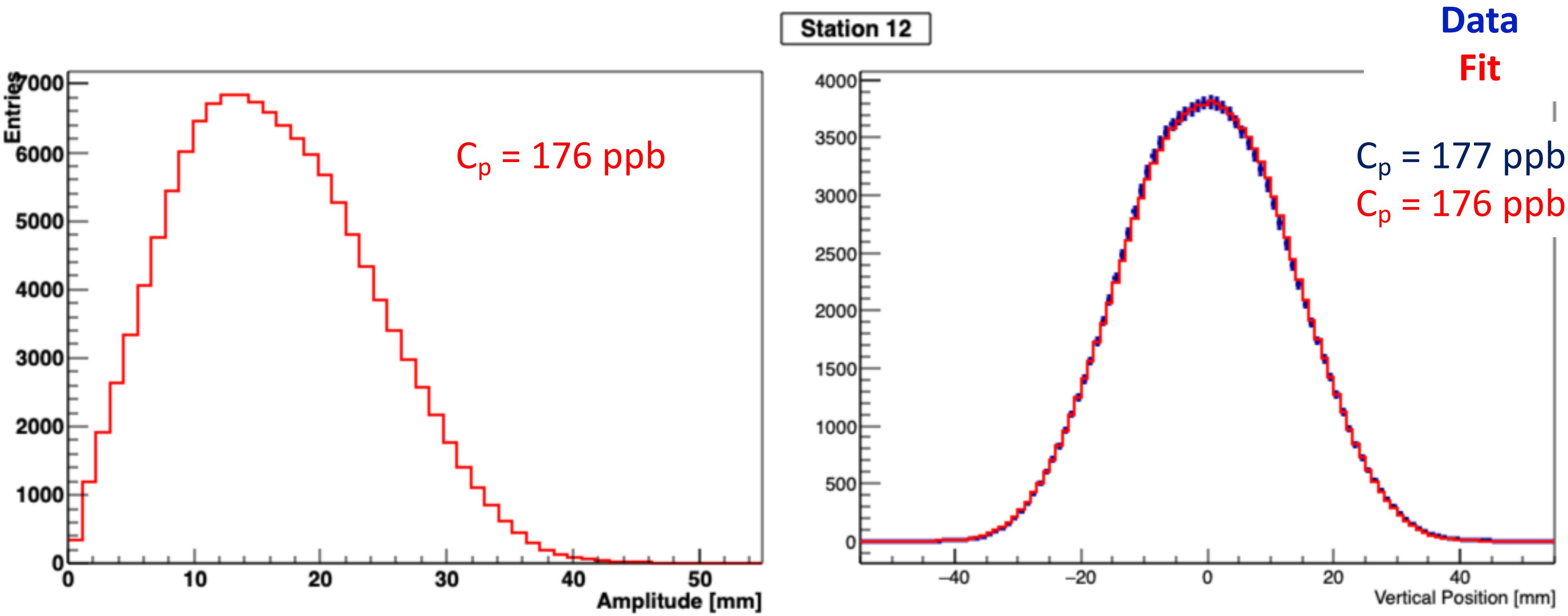
$$\chi^2 = \sum_j \frac{\sum_i \left(\beta_{ij} f_i - N_j\right)^2}{\sigma_{N_j}^2}$$

Works just great with Toy MC



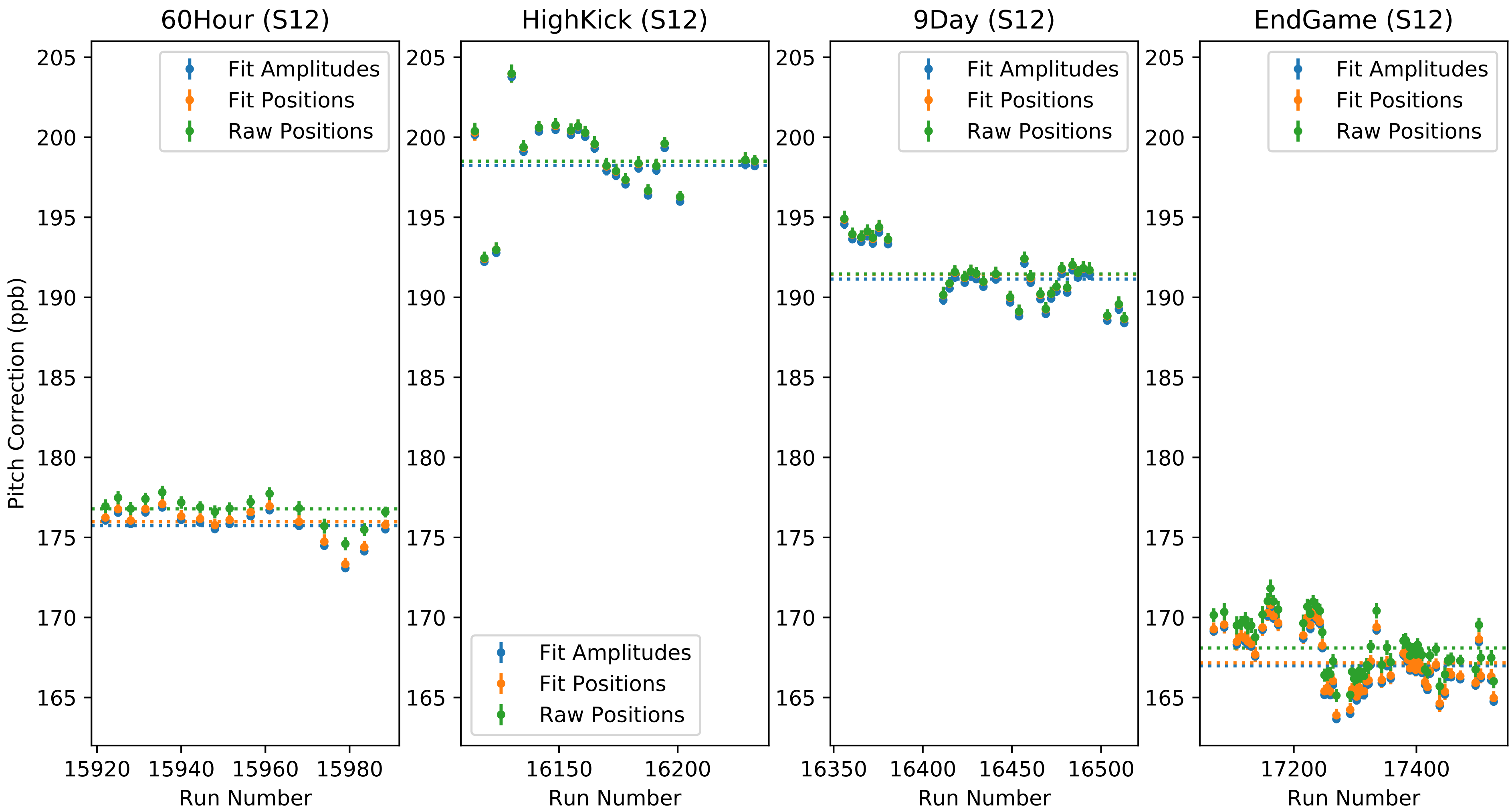


Data: Amplitude Extraction in Action



- Looks reasonable, and also a small effect

Run 1 Tracker S12

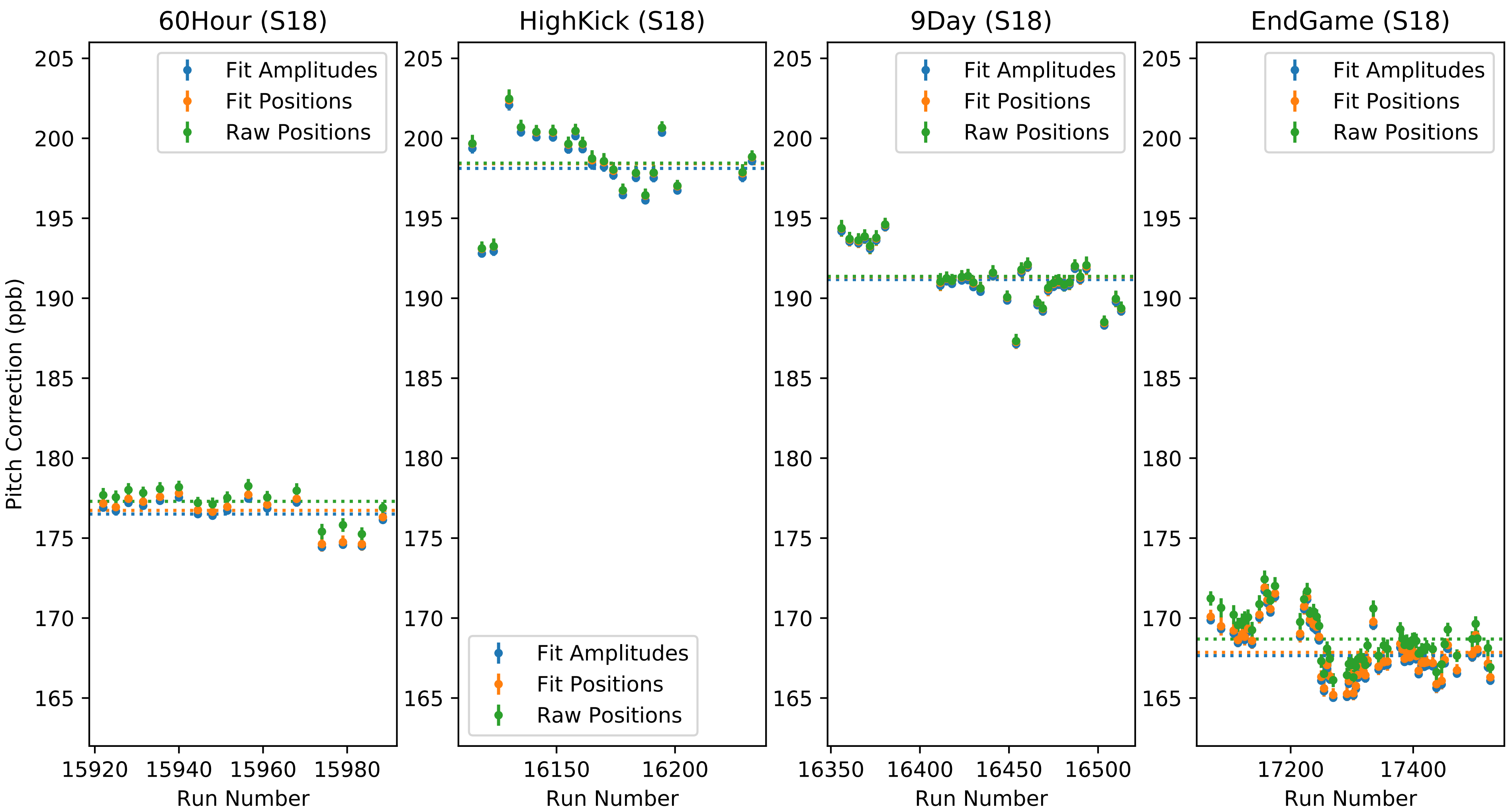


Raw positions – corrected for tracker resolution and acceptance

Fit amplitudes – f_i

Fit positions – based on fitted amplitudes

Run 1 tracker S18



Raw positions – corrected for tracker resolution and acceptance
Fit amplitudes – f_i
Fit positions – based on fitted amplitudes

Convert from position to amplitude space to properly account for angular acceptance

Next steps

- Station by station acceptance correction
- Modulation of β –function around the ring
- Apply calo acceptance to ‘measured’ amplitude distribution
- Determine systematic model dependent uncertainty
- Fit vertical frequency to measure vertical tune to get n

Pitch reconstruction method systematics σ_{C_p} (ppb)

source	Run 1a	Run 1b	Run 1c	Run 1d
Tracking	8.6	8.6	8.6	8.6
Vertex Res.	3	3	3	3
Acceptance	10	10	10	10
Model	10	10	10	10
quad. sum	16.8	16.8	16.8	16.8

Estimates (Acceptance, Model)

Run independent systematics σ_{C_p} (ppb)

	Run 1a	Run 1b	Run 1c	Run 1d
Ring Model	1.3	1.3	1.3	1.3

Preliminary

C_p **(ppb)**

	Run 1a	Run 1b	Run 1c	Run 1d
Tracker 12	$-176.79\pm(0.10)_{\rm stat}$	$-198.51\pm(0.10)_{\rm stat}$	$-191.46\pm(0.08)_{\rm stat}$	$-168.09\pm(0.06)_{\rm stat}$
Tracker 18	$-177.30\pm(0.10)_{\rm stat}$	$-198.45\pm(0.10)_{\rm stat}$	$-191.37\pm(0.08)_{\rm stat}$	$-168.69\pm(0.06)_{\rm stat}$

Efield correction

$$\frac{d}{dt}(\hat{\boldsymbol{\beta}}\cdot\mathbf{s}) = -\frac{e}{mc}\mathbf{s}_\perp\cdot\left[a_\mu\hat{\boldsymbol{\beta}}\times\mathbf{B}+\left(a_\mu-\frac{m^2c^2}{p^2}\right)\beta\mathbf{E}\right]$$

$$\sim \quad -\frac{e}{mc}\mathbf{s}_\perp\cdot\left[a_\mu\hat{\boldsymbol{\beta}}\times\mathbf{B}-2a_\mu\frac{\Delta p}{p_m}\beta\mathbf{E}\right] \qquad (11.171)$$

Substitution of our 'unperturbed' solution gives us

$$\rightarrow \frac{\Delta\omega_E}{\omega_a} = C_e \quad = \quad \frac{-2\langle\frac{\Delta p}{p}\beta E_\rho\rangle}{B}$$

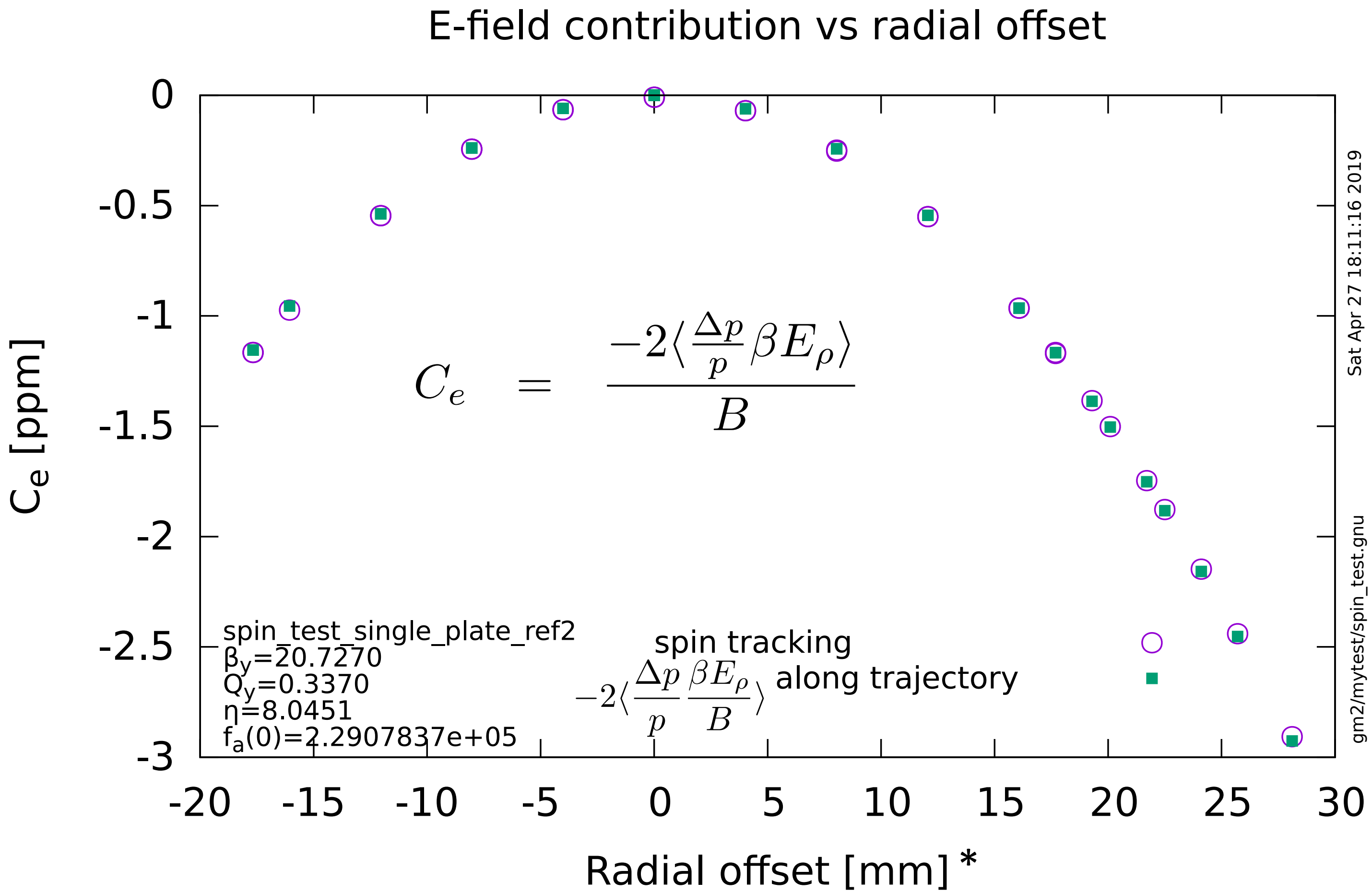
True for each muon

Assuming continuous linear quad fields

$$\left\{\begin{array}{lcl} \langle E_\rho\rangle & = & n\left(\frac{\beta B}{R_0}\right)x_e \\ \frac{\Delta p}{p} & = & \frac{x_e}{\eta} \\ C_e & \sim & -2\beta^2 n(1-n)\frac{x_e^2}{R_0^2} \end{array}\right.$$

Assuming linear and continuous quads

True for any muon ?



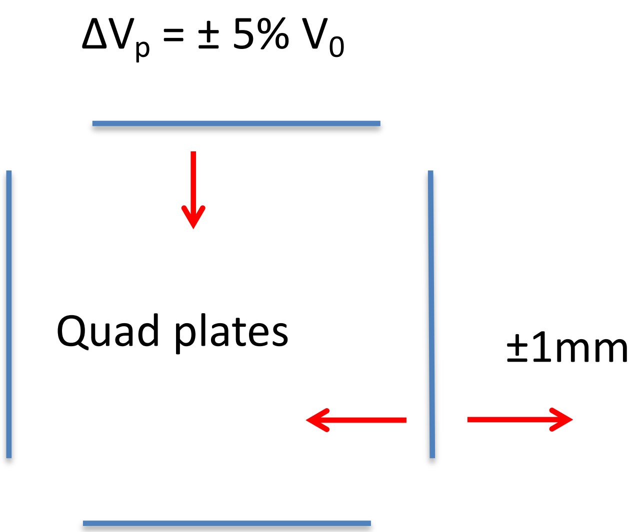
*(No vertical motion)

Excellent agreement with 'spin tracking' (integrating BMT equation)

What about our assumption of linear and continuous quads, in perfect alignment, with uniform voltage?

Test in simulation

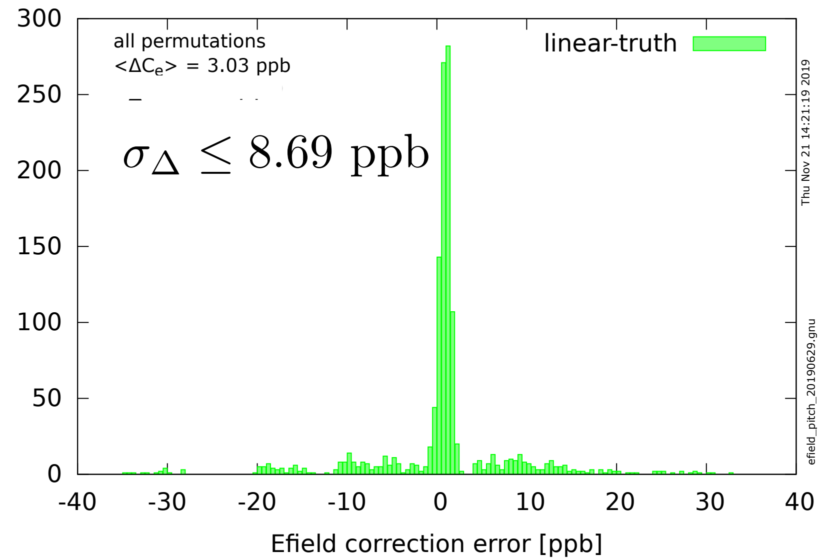
For each of 1280 permutations of plate offsets and voltage errors, inject and track a distribution of muons



$$C_e(\text{truth}) = -2\frac{\langle \frac{\Delta p}{p}\beta E_\rho \rangle}{B}$$

$$C_e(\text{linear}) = -2\beta^2 n(1-n)\frac{\langle x_e^2 \rangle}{R_0^2}$$

$$\langle \Delta C_e \rangle = \langle C_e(\text{linear}) \rangle - \langle C_e(\text{truth}) \rangle$$



Contribution to ω_a from E-field

$$C_e \sim -2\beta^2 n(1-n)\frac{x_e^2}{R_0^2}$$

Measurement of radial closed orbit, x_e => E-field correction

The revolution frequency is related to x_e as $\omega_R = \dfrac{c\beta}{R + x_e}$

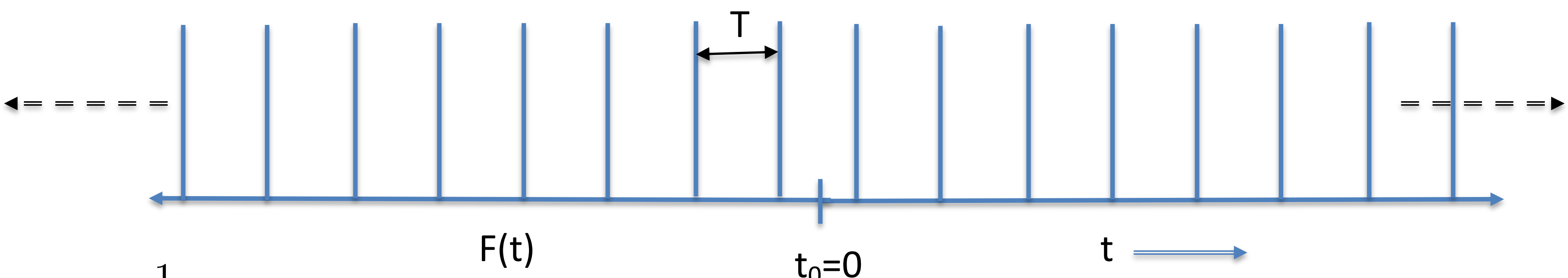
Fourier method: Measure frequency distribution of the 'Fast Rotation' signal $S(t)$

'Fast Rotation' signal is all calo hits vs time above threshold

t

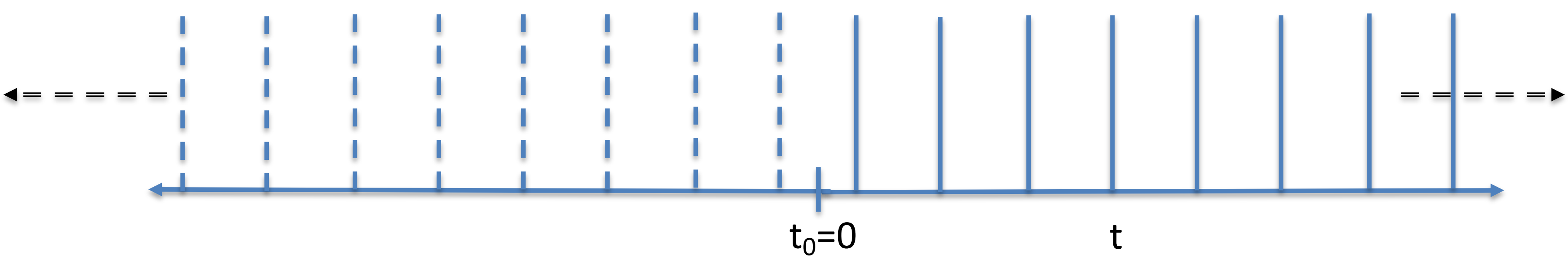
Suppose a single momentum is circulating in the ring, forward and backward in time from $t = 0$ to $t = \pm\infty$

Then $S(t)$ is a Dirac comb

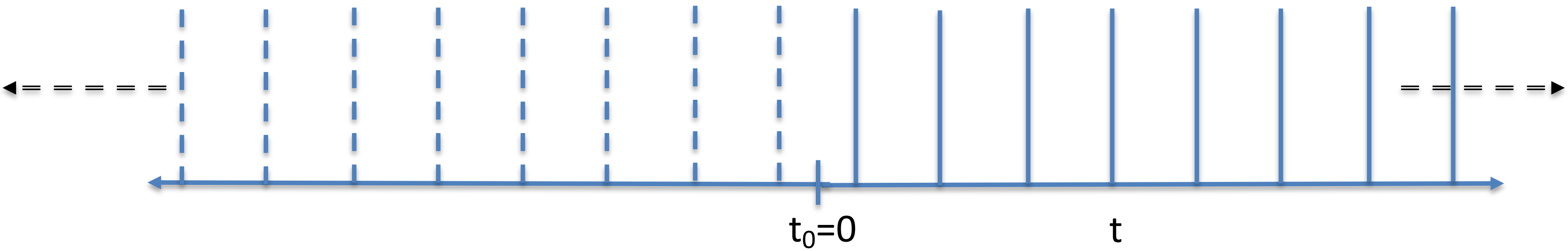


$$F(\omega) = \frac{1}{2} + \sum_{n=1} \cos(n\omega T)$$

Frequency spectrum is $\omega_0 = 2\pi/T$ and all harmonics

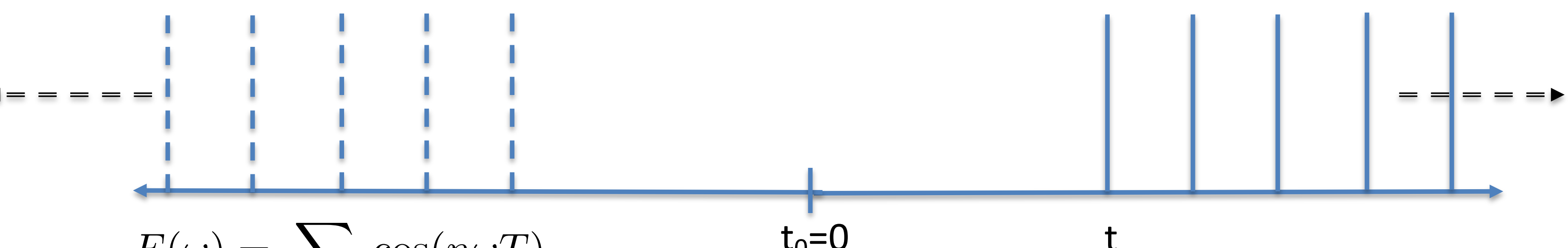


As long as we know t_0, and that the distribution that evolves forward in time is mirror symmetric with respect to the distribution that would evolve backward in time, then we anticipate even symmetry, and we enforce the symmetry with the cosine transform



Start time

But the first few microseconds after injection are contaminated, and are missing from our signal



$$F(\omega) = \sum_{n=n_s} \cos(n\omega T)$$

Now the cosine transform will necessarily include ‘unphysical’ frequencies

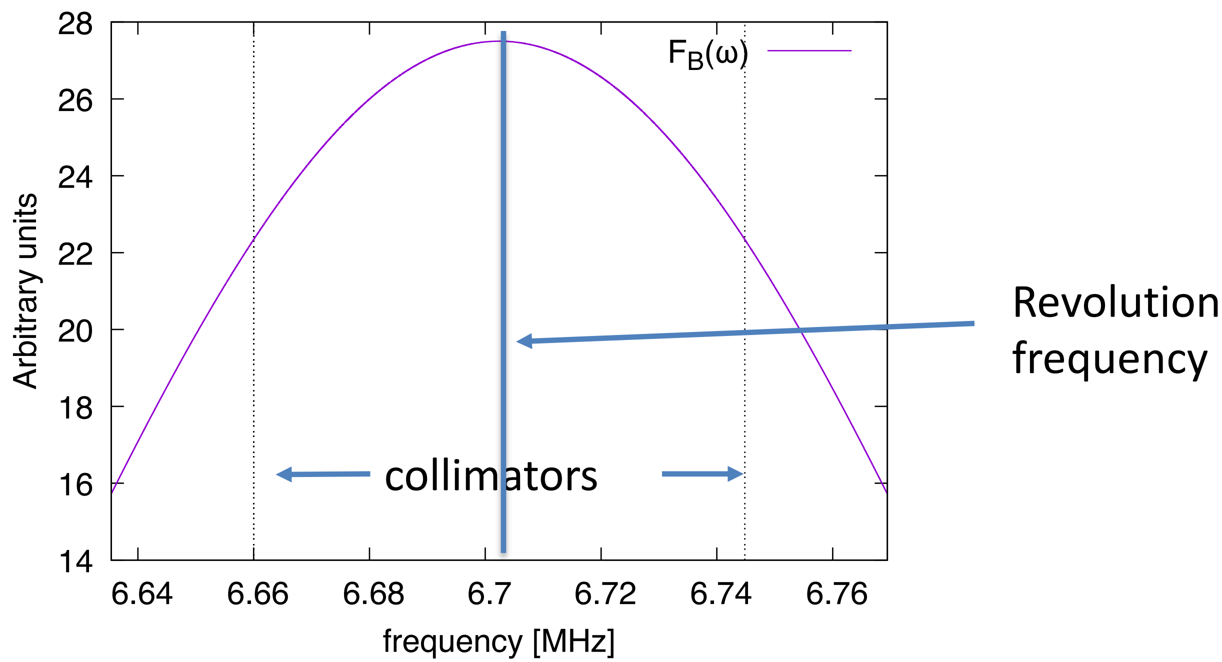
The piece missing from our frequency spectrum

$$F(\omega) = \left\{ \frac{1}{2} + \sum_{n=0} \cos(n\omega T) \right\} - \left\{ \sum_{n=n_s} \cos(n\omega T) \right\}$$

We call the missing piece, Background

$$= F_B(\omega) = \frac{1}{2} + \sum_{n=0}^{n_s} \cos(n\omega T)$$

$$n_s = 27 \ (\sim 4\mu\text{s})$$



Fourier transform is linear function of $S(t)$

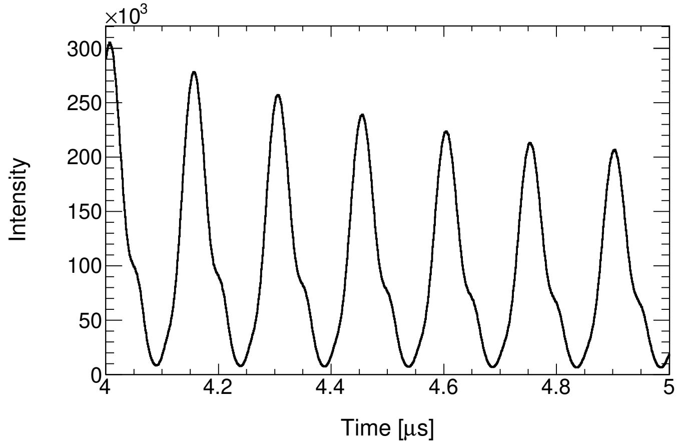
Generalize from a single muon with a single revolution frequency to a distribution with a range of frequencies.

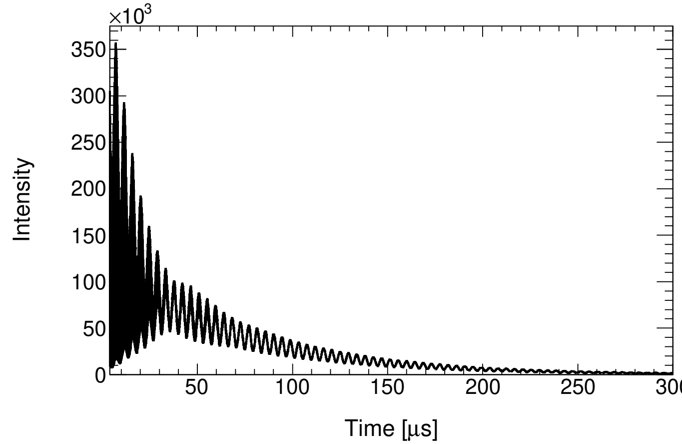
Then the cosine transform of $S(t)$ gives us the frequency distribution

We need to determine

- t_0 (symmetry point)
- Start time ('background')
- End time (Muons decay)
- Background fit (functional form)
- Frequency bin width

Raw calo hits vs time

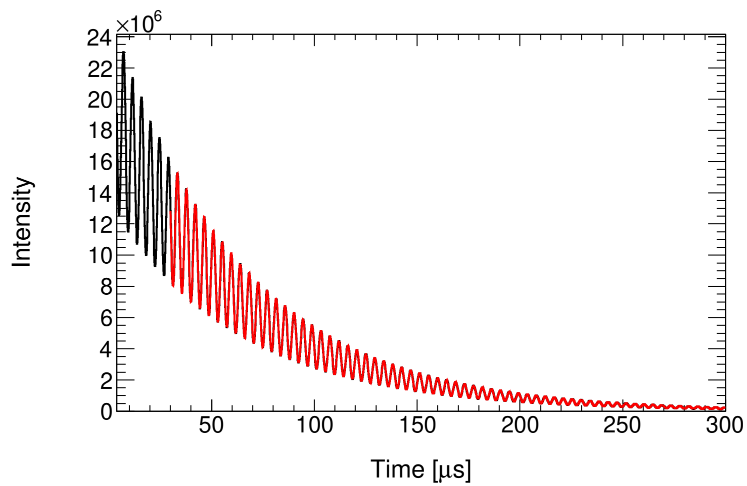




Remove all irrelevant frequencies

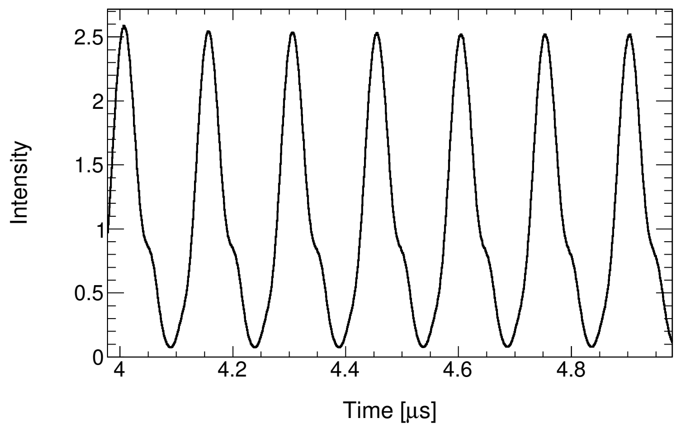
Nine parameter fit

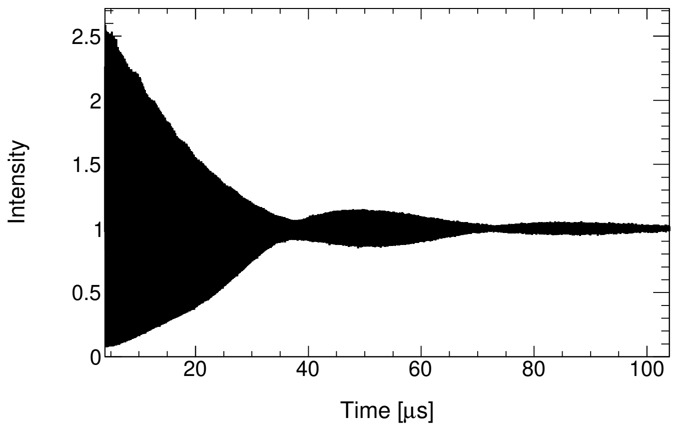
$$N(t) = N_0 \cdot e^{-t/\tau_\mu}[1 + A \cdot cos(\omega_a t + \phi)] \cdot e^{-t/\tau_{cbo}}[1 + A_{cbo} \cdot cos(\omega_{cbo} t + \phi_{cbo})],$$

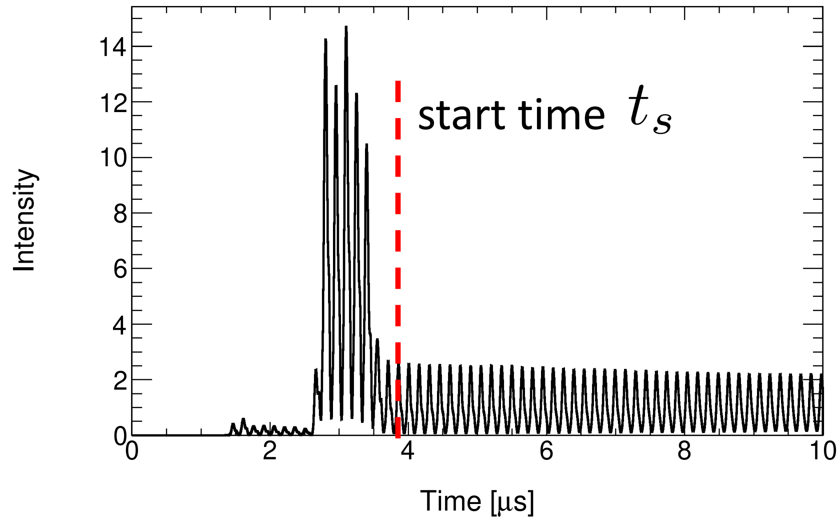


Divide raw hits vs time by $N(t)$ of 9 parameter fit

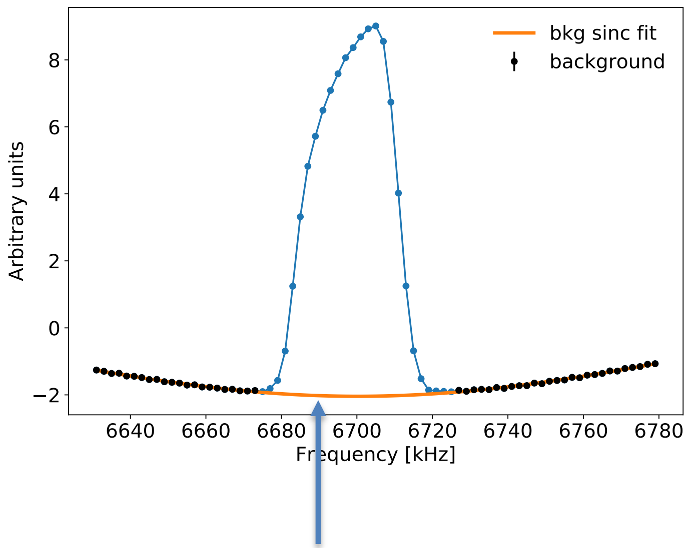
→ *Fast Rotation signal*



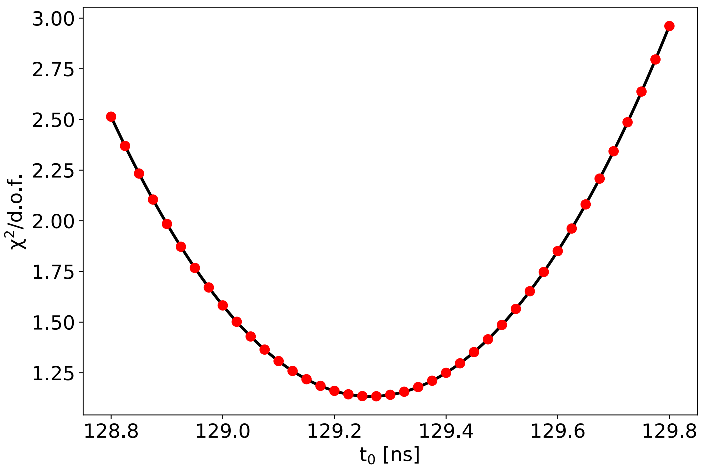




$$F(\omega) = \int_{t_s}^{t_m} S(t) \cos \omega (t - t_0) dt$$



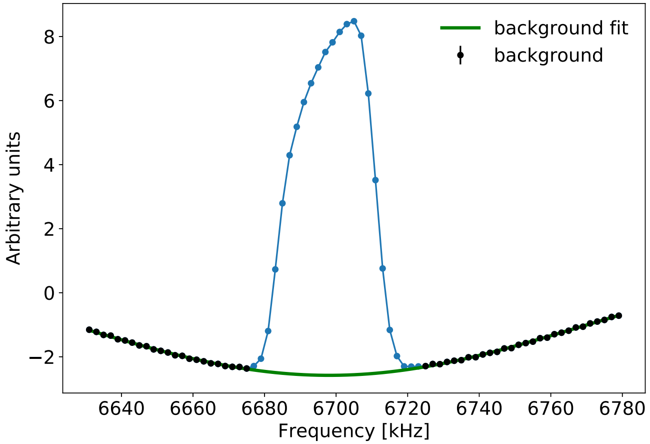
Background



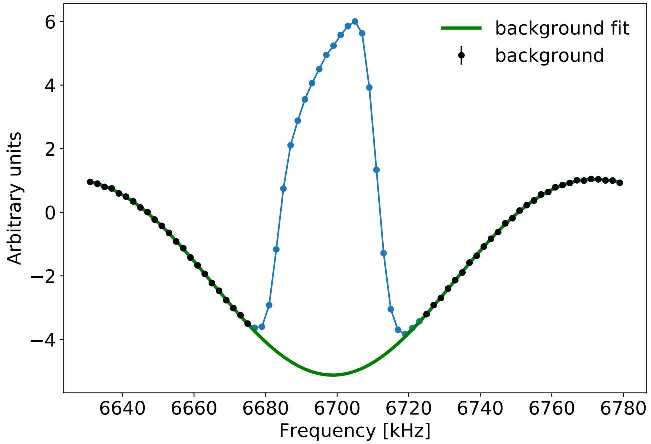
Start time scan

Dependence on start time

$$F(\omega) = \int_{t_s}^{t_m} S(t)\cos\omega(t - t_0)dt$$



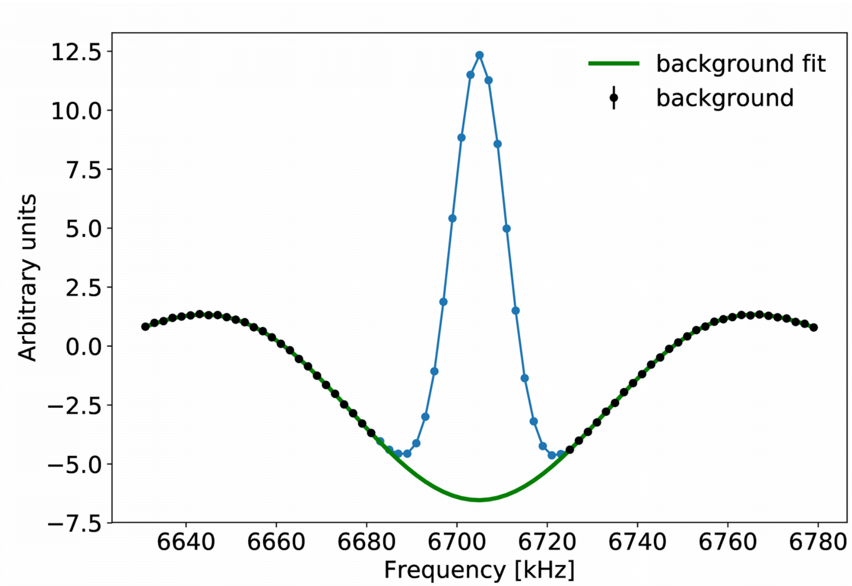
5us

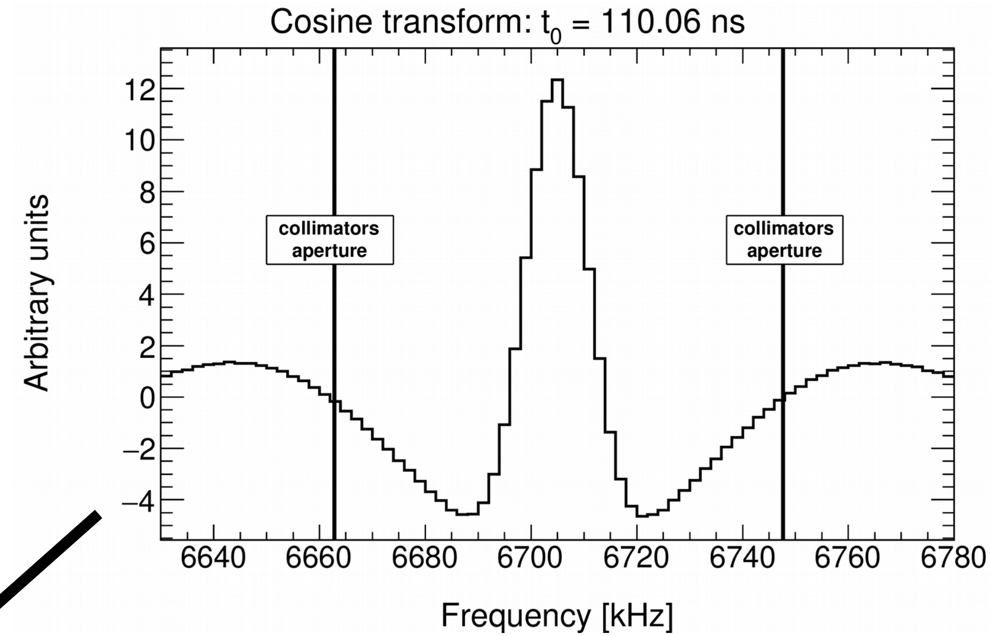


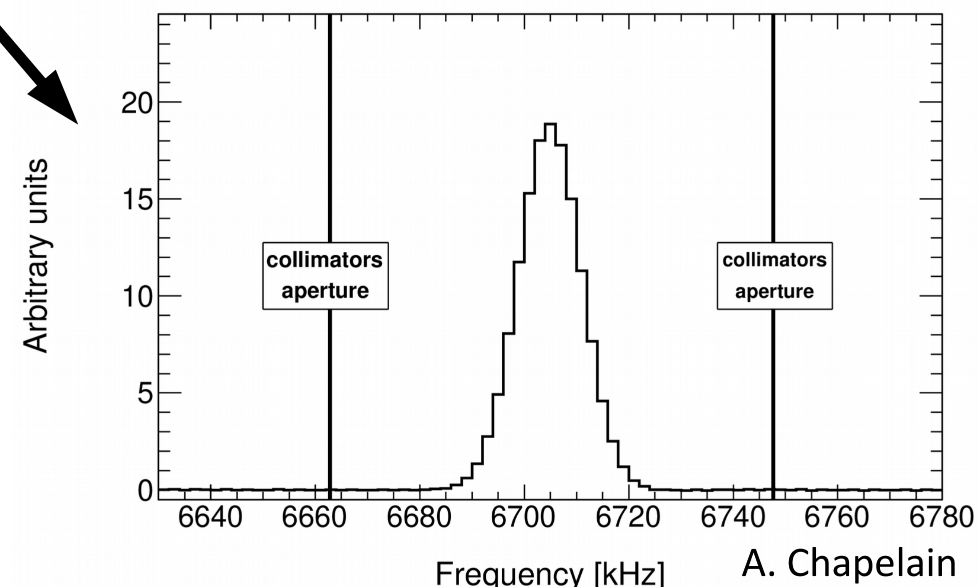
10us

Data missing from the fast rotation signal distorts frequency spectrum

Cornell Fourier implementation: correct for the so-called “background” (caused by missing early time) via a direct background fit.



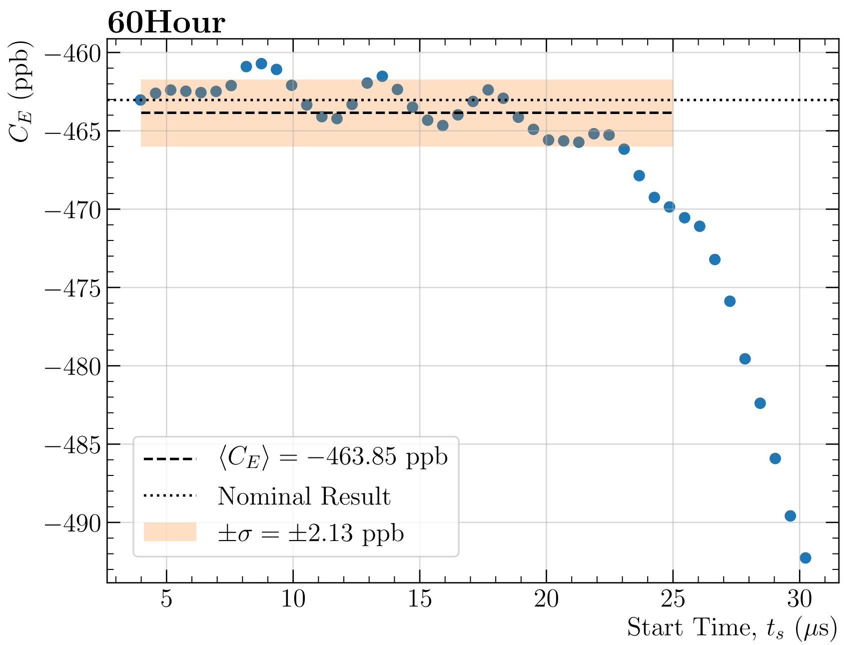


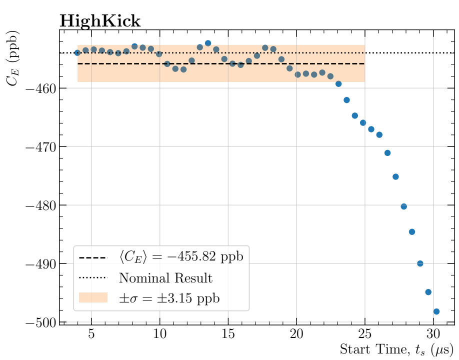


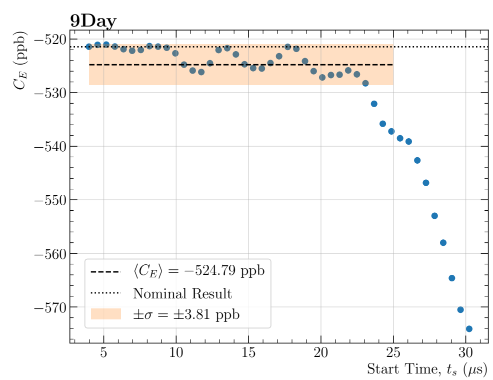
“background” = size bands region outside of the main peak

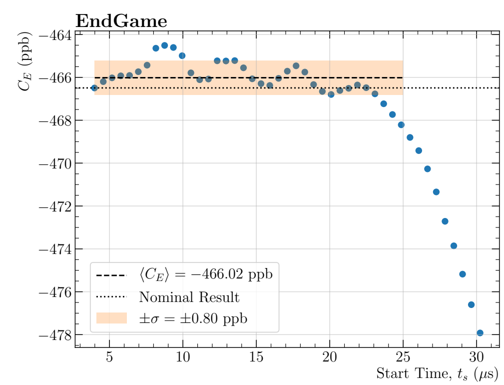
A. Chapelain

Dependence on start time

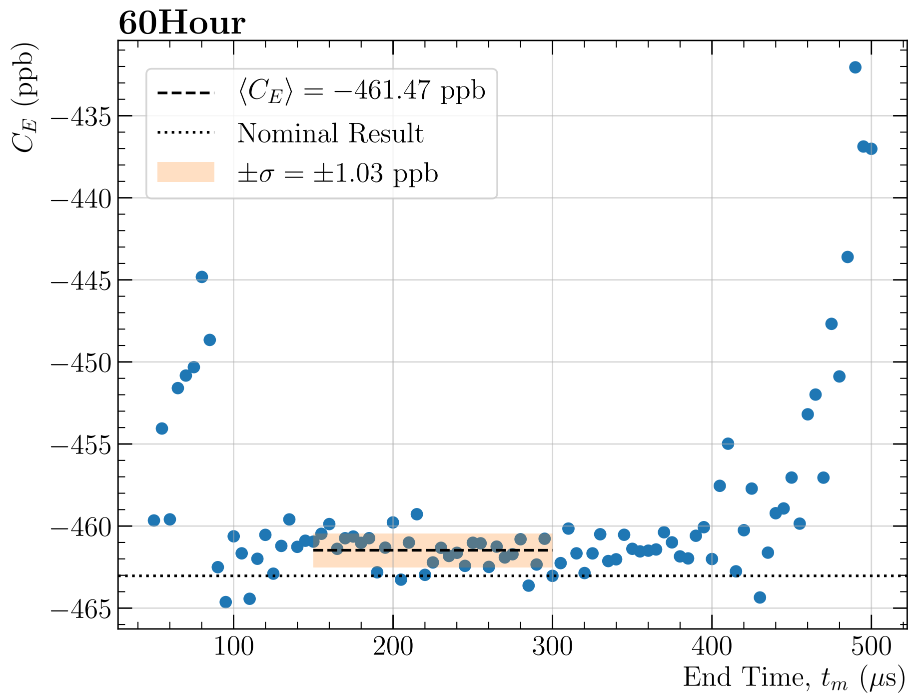


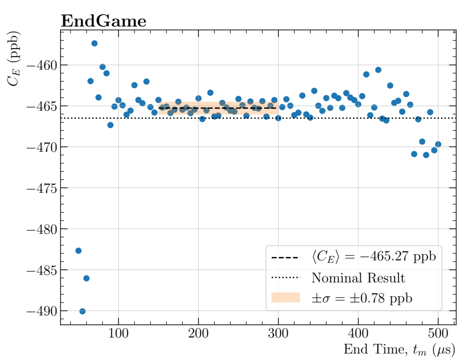


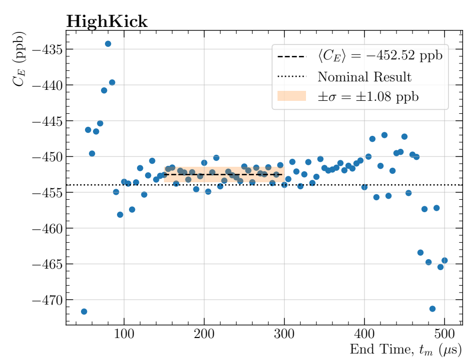


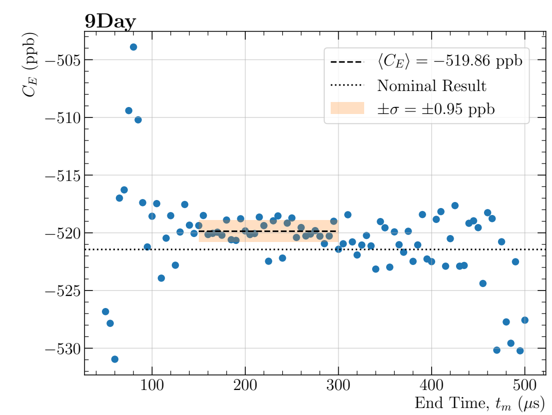


Dependence on End time

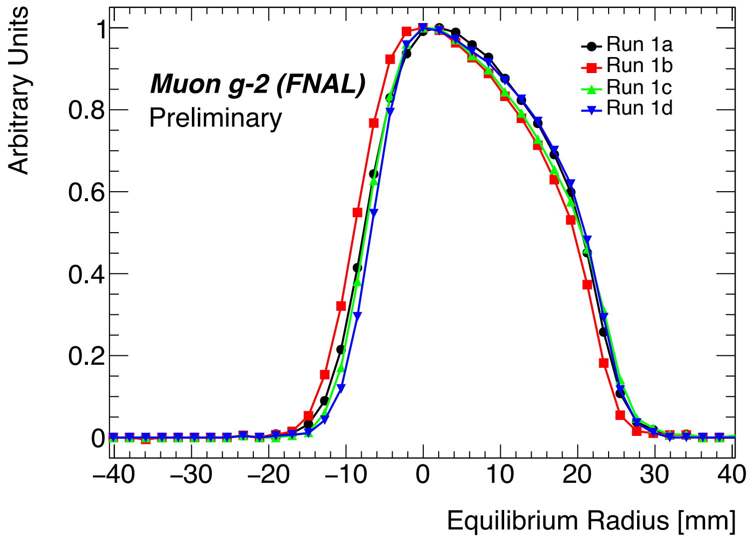






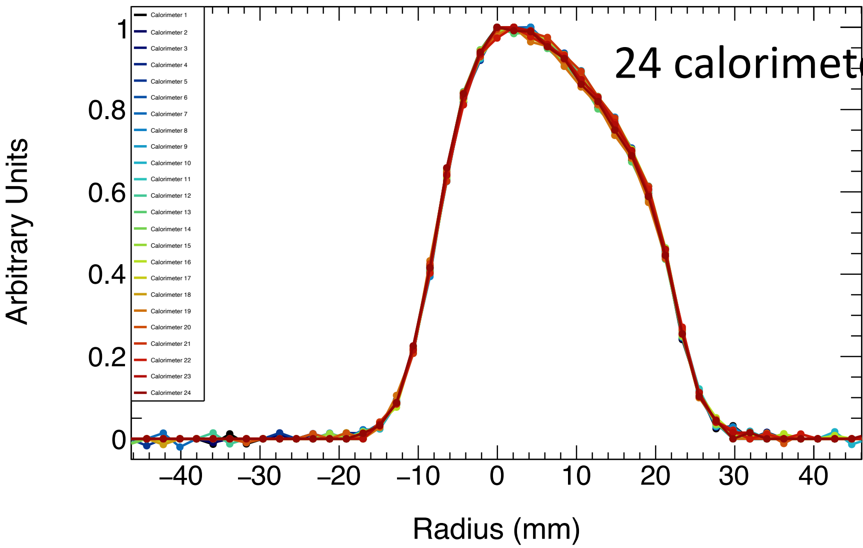


Run dependence

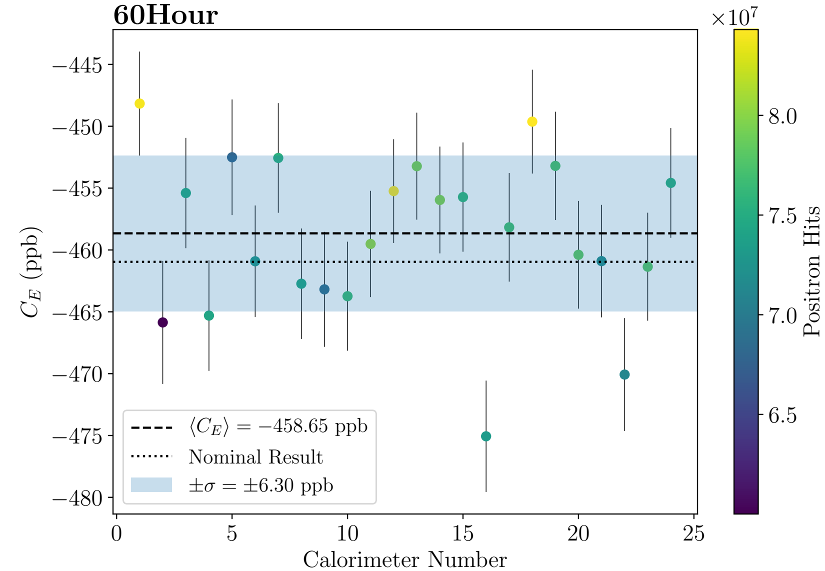


Dataset	f_{cbo} (kHz)	n	C_E (ppb)
60Hour	370.44	0.1075	−461
9Day	413.64	0.1197	−523
EndGame	367.05	0.1066	−468
HighKick	414.29	0.1198	−453

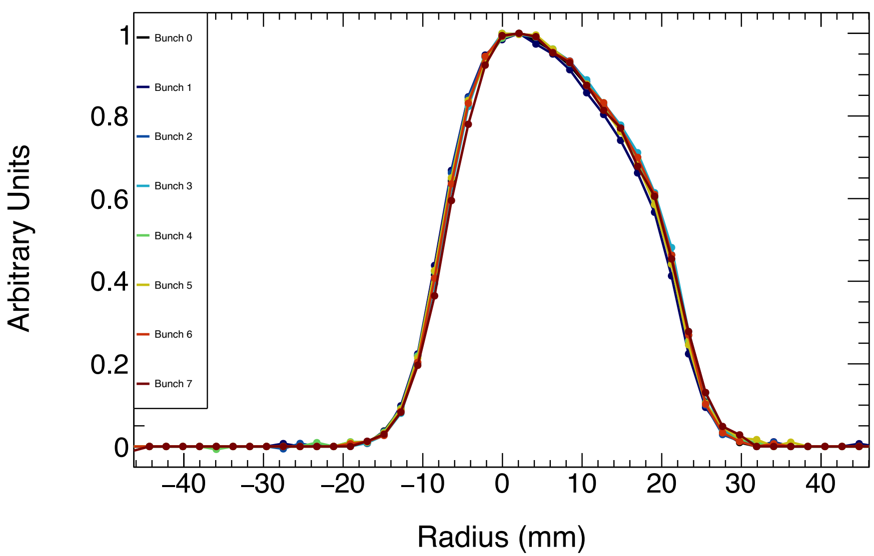
Calorimeter dependence

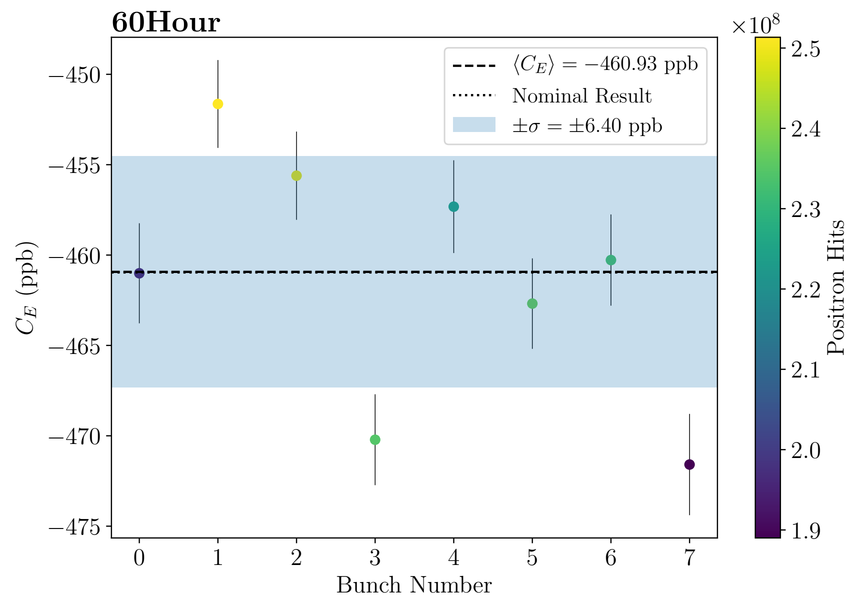


24 calorimeters



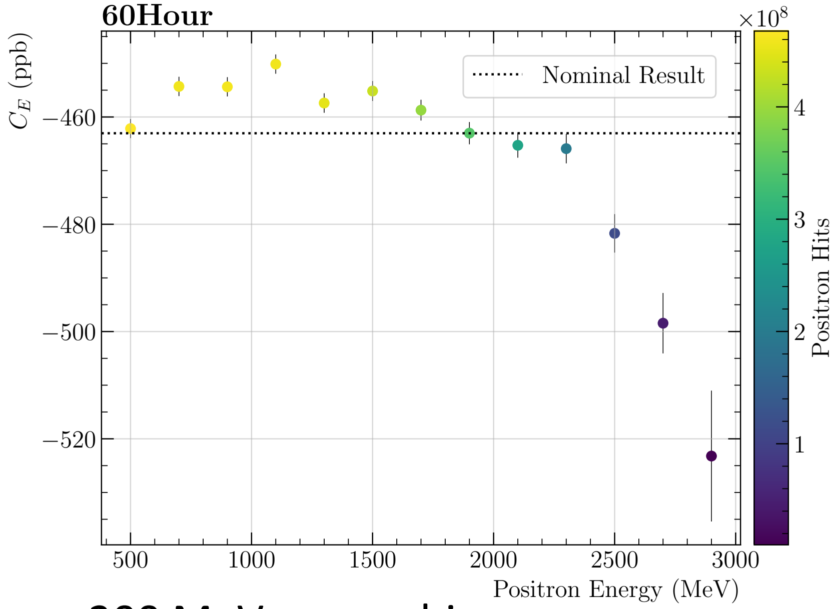
Bunch dependence



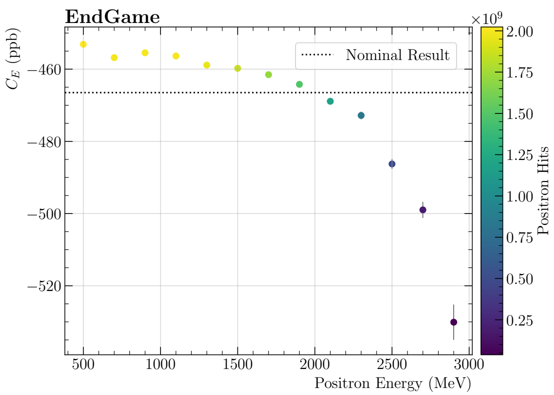


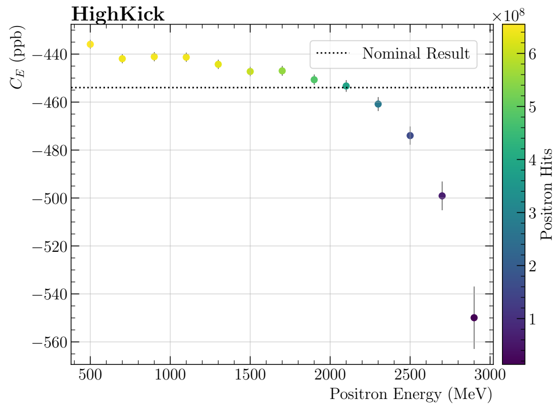
8 bunches

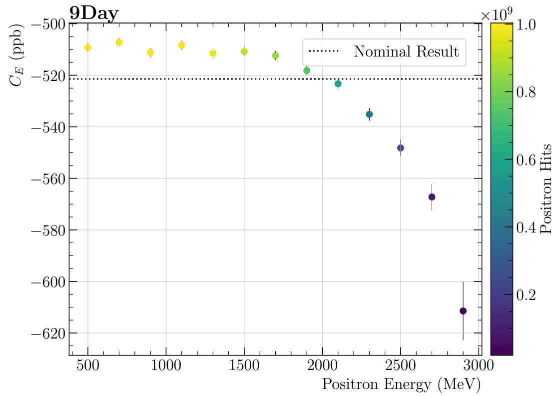
Dependence on positron energy



200 MeV energy bins

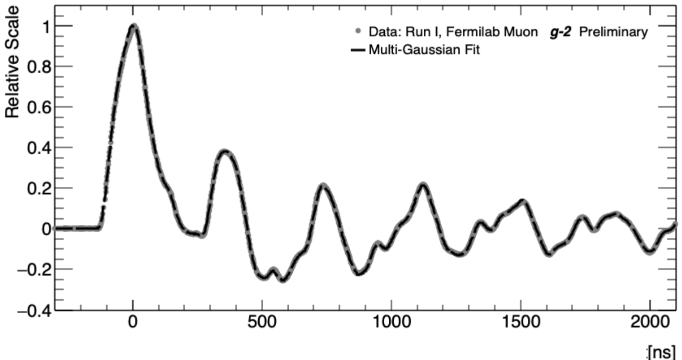


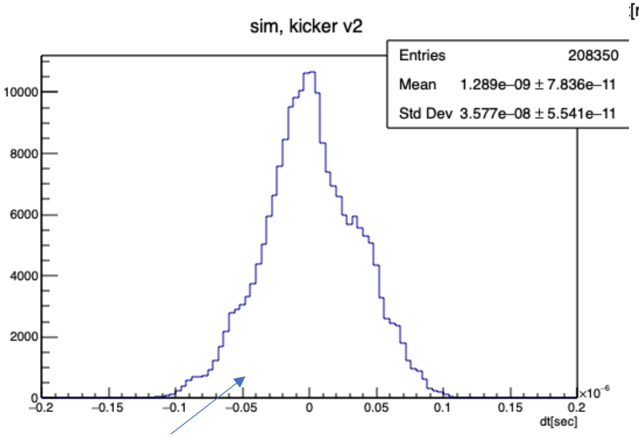




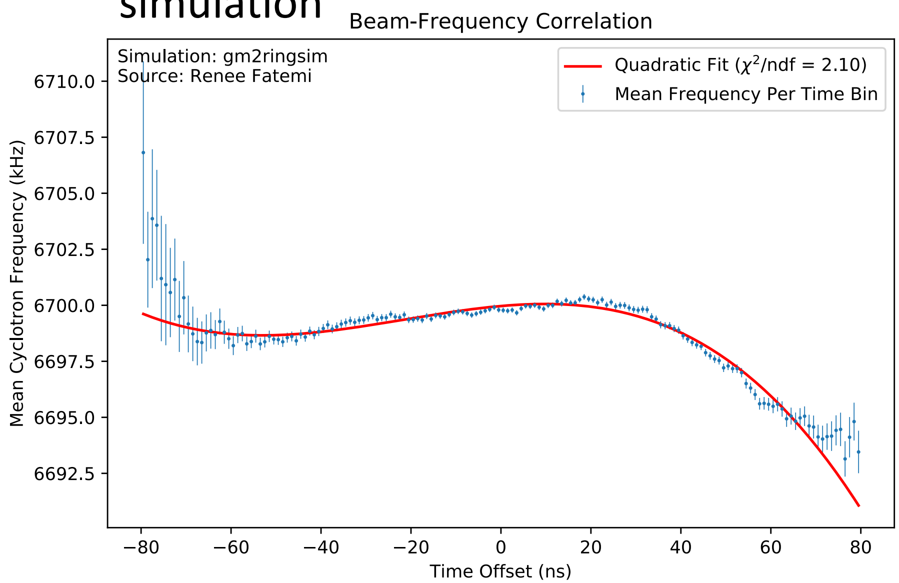
Momentum – time correlation

- The kicker field is not uniform across the the length of the muon pulse.
- Momentum acceptance scales inversely with kicker field.
- The result is a t=0 momentum-time correlation in the stored beam





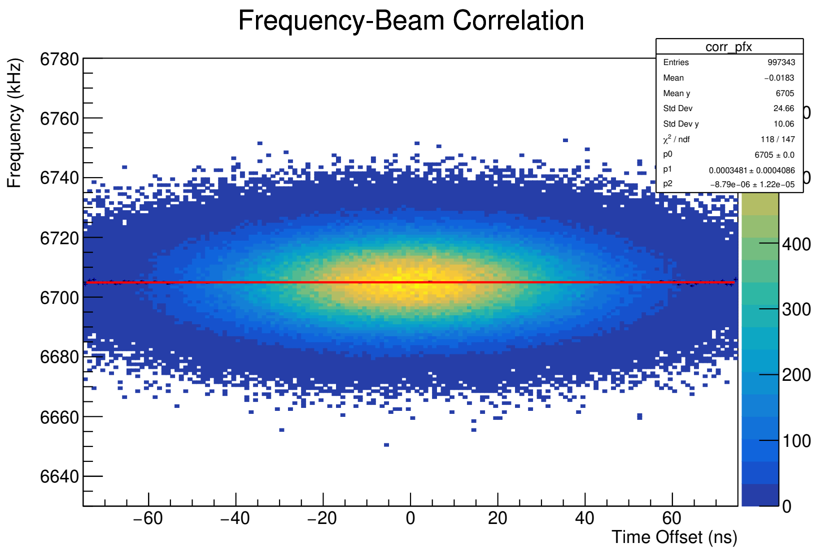
simulation

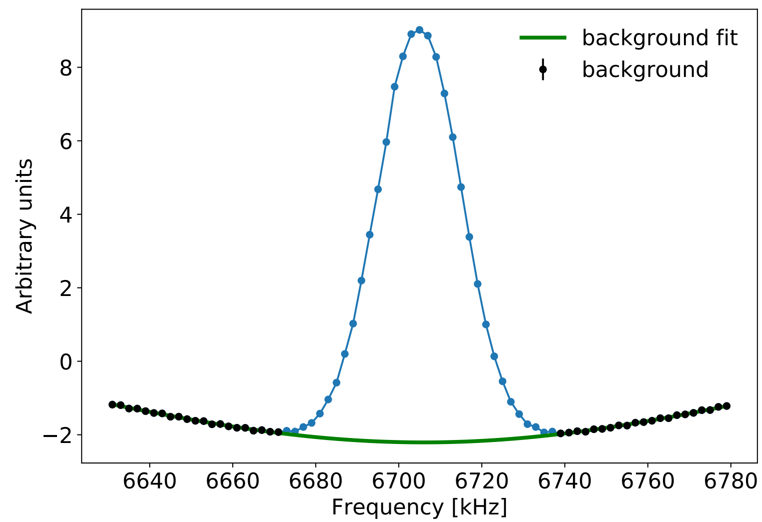


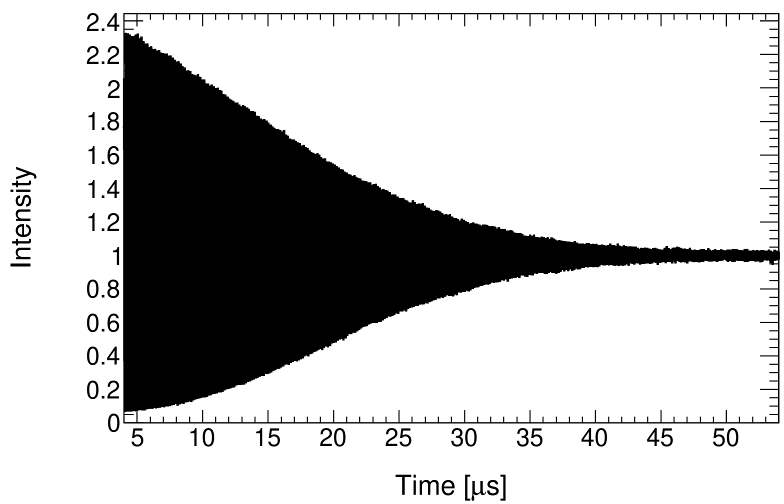
Debunching is no longer symmetric forward and backward in time.

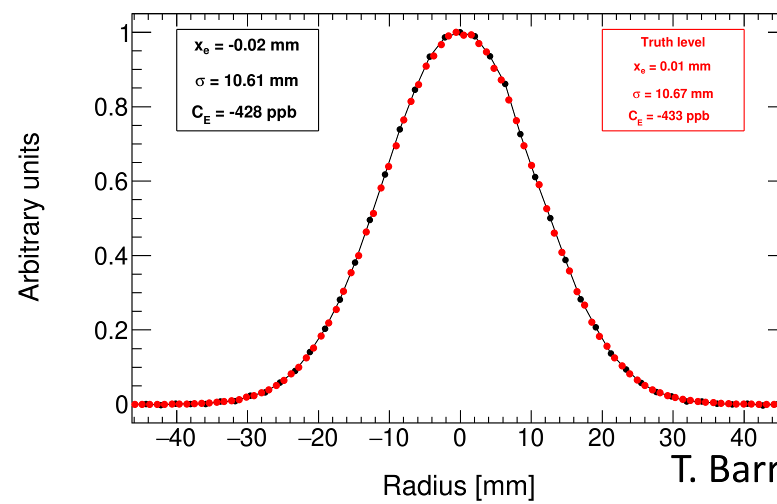
- the fundamental prerequisite for the Fourier method

Fourier method reproduces true frequency distribution with no correlation at least with Toy MC



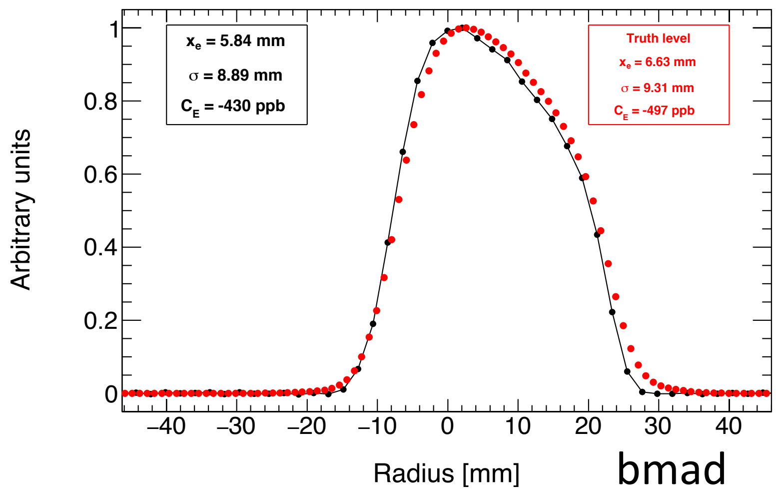




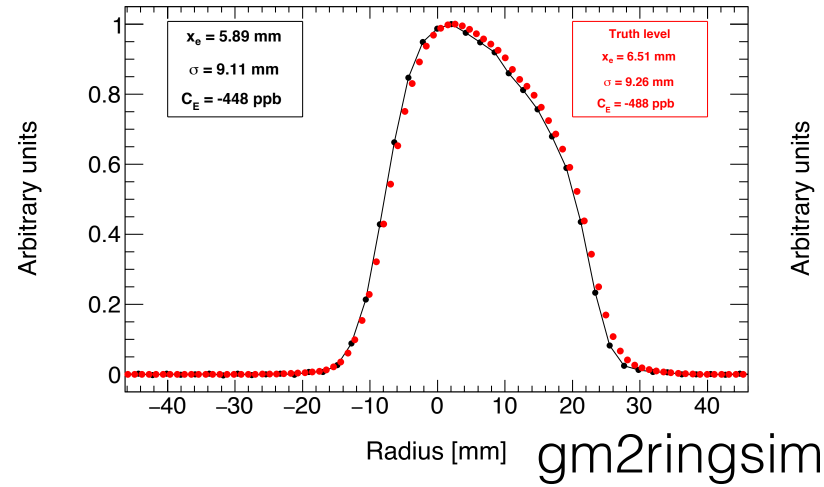


T. Barrett

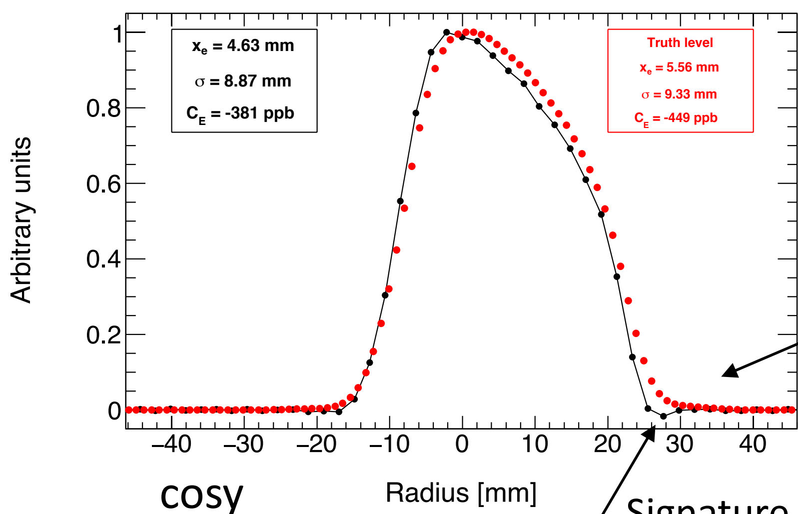
Initial distribution is generated with simulated correlation, and measured frequency and temporal distributions



ΔC_e = 67 ppb



ΔC_e = 40 ppb



There is no such feature in the Run 1 data
We conclude that the error due to correlation is limited to 50 ppb

More than that and we would see it.

Signature of correlation

Fourier method results

Fourier Method Systematics σ_{C_E} (ppb)

source	Run 1a	Run 1b	Run 1c	Run 1d
bkgd. fit	0.4	2.8	0.6	4.0
start time	2.1	3.8	0.8	3.1
end time	1.0	0.9	0.8	1.1
freq. bin	1.6	1.9	0.4	1.3
bfgd. def.	4.3	6.5	2.1	6.5
bmgd. rem.	1.9	3.8	0.5	2.2
quad. sum	6.1	9.1	2.5	8.7
linear sum	13.3	19.7	5.2	18.2
average	9.7	14.4	3.9	13.4

Correlated run to run

Run independent systematics σ_{C_E} (ppb)

source	Run 1a	Run 1b	Run 1c	Run 1d
Ring Model	8.6	5.6	8.6	8.6
Momentum-time correlation	50	50	50	50

Correlated run to run

Combined systematics σ_{C_E} (ppb)

source	Run 1a	Run 1b	rRun 1c	Run 1d
Fourier method parameters	9.7	14.4	3.9	13.4
ring model	8.7	8.7	8.7	8.7
momentum-time correlation	50	50	50	50
field index	1.7	1.7	4.0	1.5
quadruture sum	51.7	52.7	51.1	52.5

For detailed Run 1 analysis see

GM2-doc 21258

GM2-doc 21678

C_E (ppb)

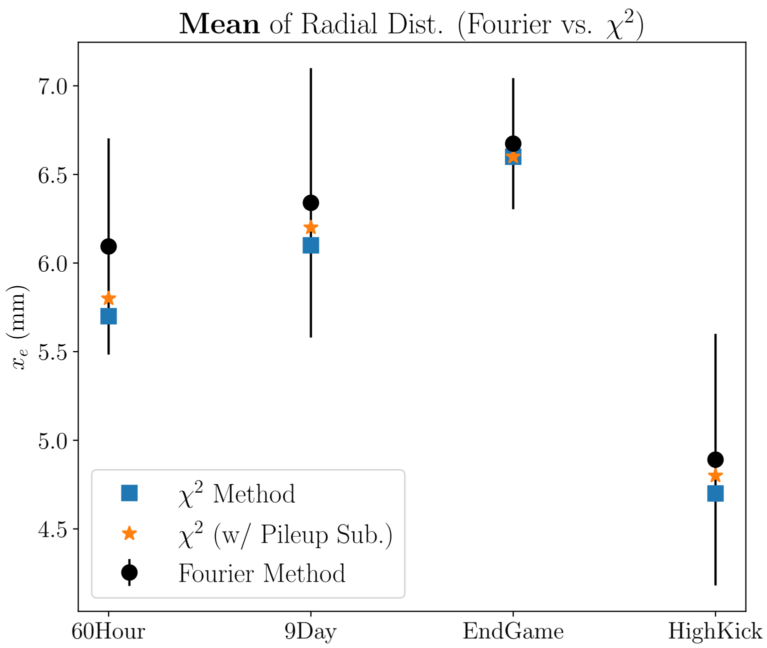
e^+ threshold	Run 1a	Run 1b	Run 1c	Run 1d
500 MeV	-459$\pm(0.60)_{\text{stat}}$	-515$\pm(0.54)_{\text{stat}}$	-461$\pm(0.23)_{\text{stat}}$	-446$\pm(0.64)_{\text{stat}}$
1700 MeV	-466$\pm(1.04)_{\text{stat}}$	-525 $\pm(0.94)_{\text{stat}}$	-469$\pm(0.39)_{\text{stat}}$	-456$\pm(1.10)_{\text{stat}}$
Asymmetry Weighted	-466$\pm(0.92)_{\text{stat}}$	-526$\pm(0.83)_{\text{stat}}$	-469$\pm(0.35)_{\text{stat}}$	-457$\pm(0.97)_{\text{stat}}$

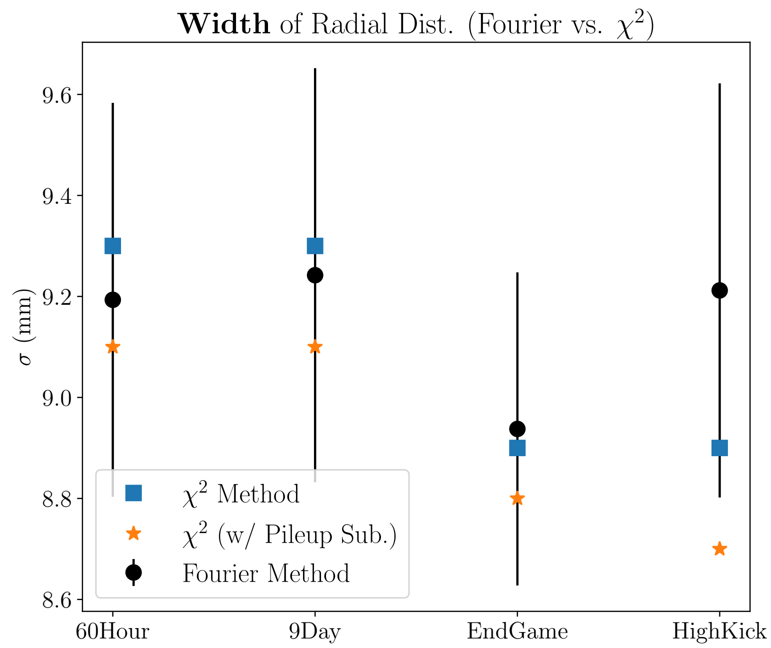
Efield summary

Momentum-time correlation uncertainty dominates everything else

In the works

- CERN/χ^2 method: Fit parameterized initial distribution to S(t). Correlation can be included in parameterization. Rob C. is extending the method with special functions to reduce the number of parameters to fit
- The fast rotation signal is number vs time. Trackers measure *radial position* **and** number vs time, allowing possibility of reconstructing correlation. (James M.)

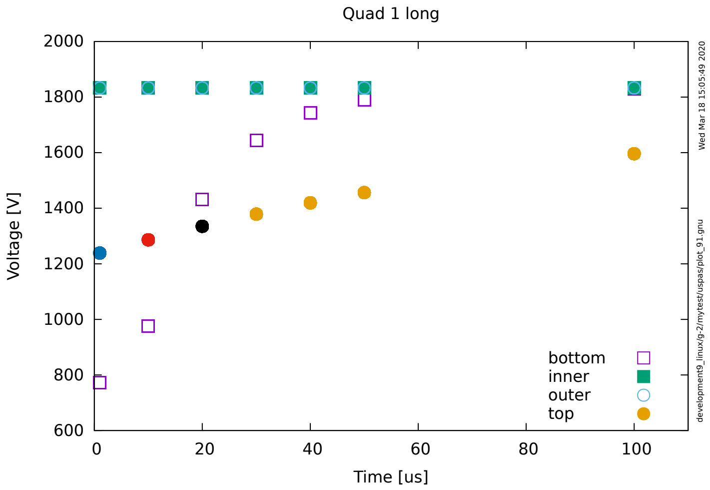




Latest CERN/χ^2 results from Rob Carey (GM2-doc-21672).

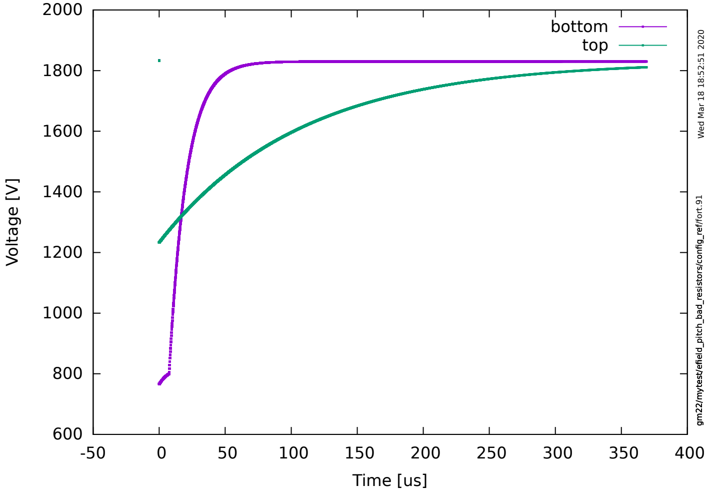
Thanks to everyone who participated in Efield/pitch discussions and contributed to this work.

END



All 4 plates

Q1 long Top and bottom

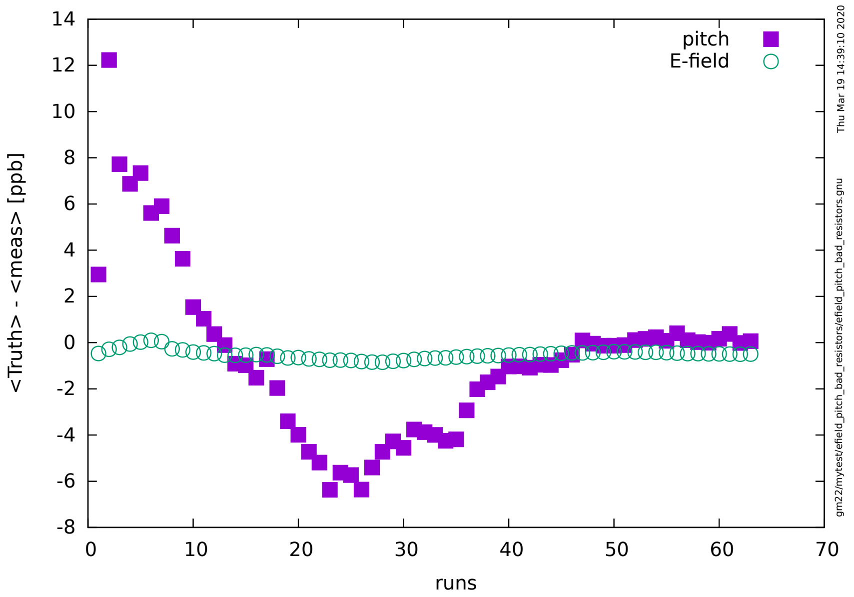


$$C_p(\text{meas}) = -\frac{n_y \langle y^2 \rangle}{2R_0^2}$$

$$C_e(\text{meas}) = -2\beta^2 n_x (1 - n_x) \frac{\langle x_e^2 \rangle}{R_0^2}$$

Index from measured tunes

Running average



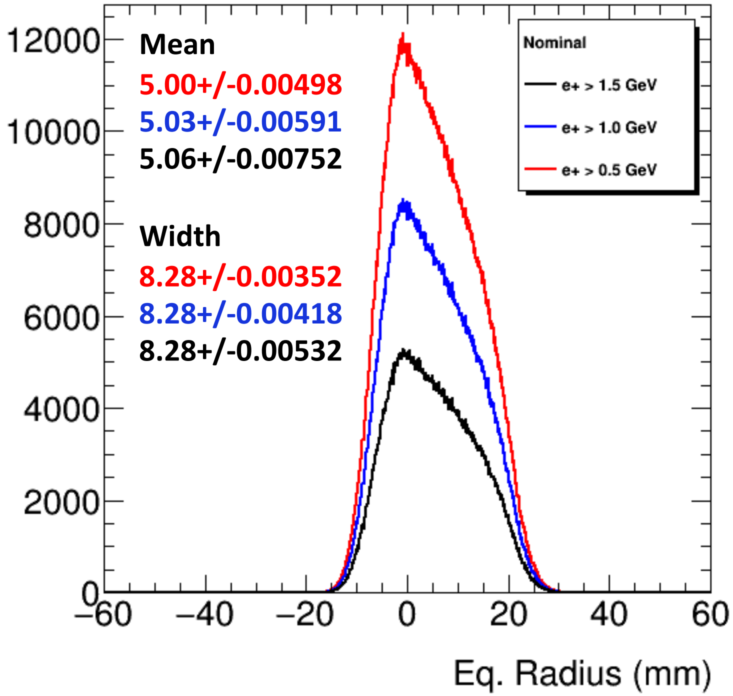
~ 150 muon decays / run

Simulation with gm2ringsim

- 10^9 muons thrown at ring
- Equilibrium radius (truth) measured at tracking planes
- Fast rotation signal is calo hits

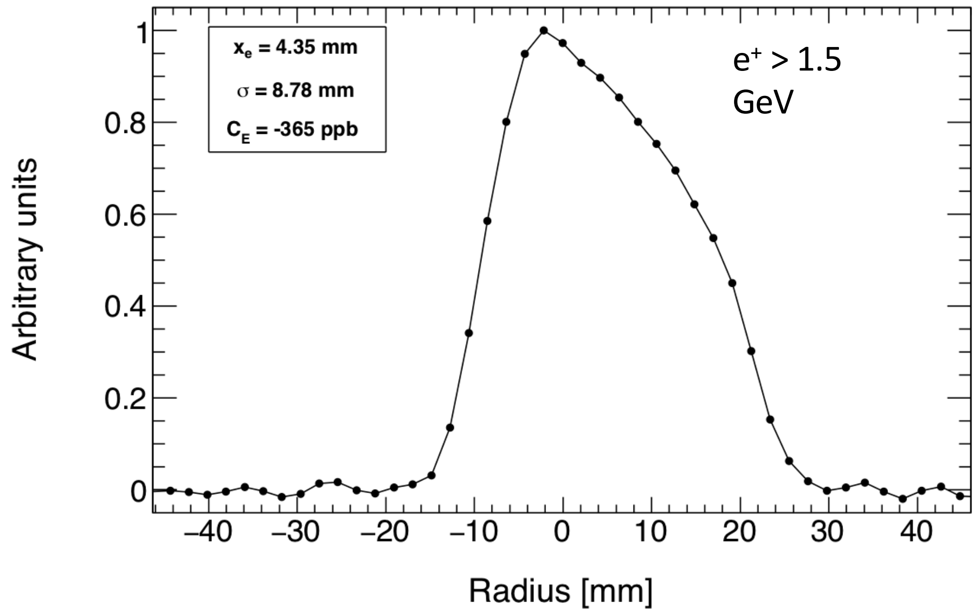
χ_e from Tracking planes at t > 30 **μ**s

$$\chi_e = R_{magic}\frac{\Delta p}{p_{magic}(1-n)}$$



Cornell FR reconstruction from decay positrons in calorimeters (4< t < 150 **μ**s)

Means are different 5.060 → 4.35. but shape comparison looks good.



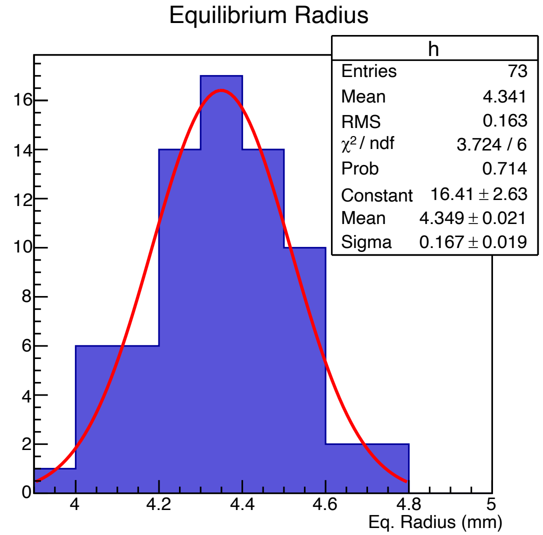
``Statistical Error” on FR extraction

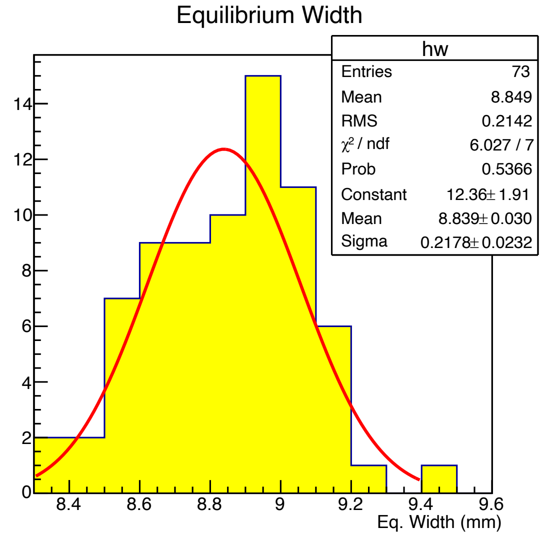
73 randcom variations over same input data

Average Mean = 4.35 +/- 0.021
Width of Mean = 0.17 +/- 0.019

Average Width = 8.84 +/- 0.030
Width of Width = 0.22 +/- 0.023

Difference between truth and FR reconstruction is significant.

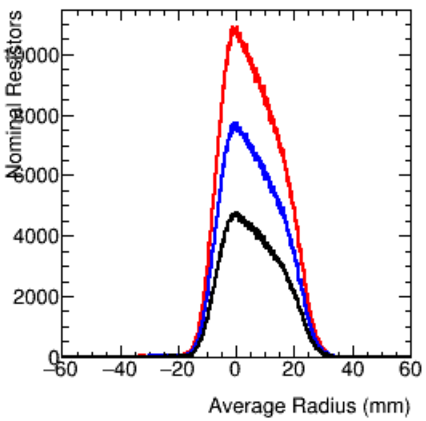




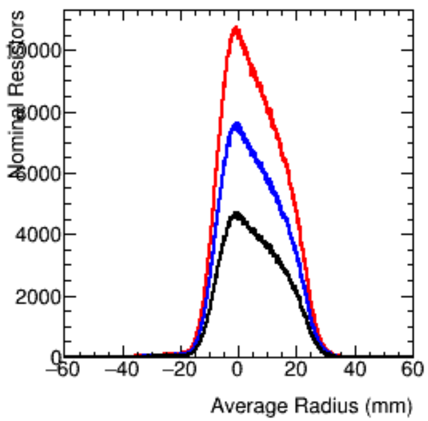
Average Radius <R> from Tracking planes for t > 30 μs.

Averaged over all planes <R> ~ 5.6 mm

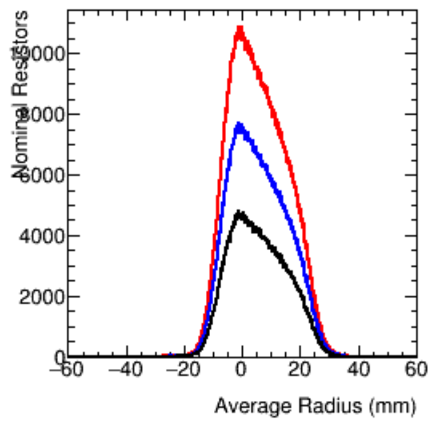
Plane 0 Mean
5.73 +/- 0.0057
5.76 +/- 0.0068
5.80 +/- 0.0086



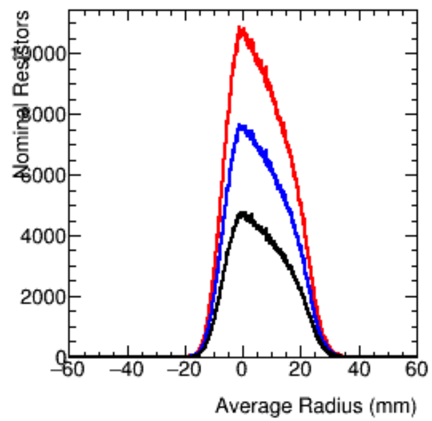
Plane 1 Mean
5.25+/-0.0059
5.29+/-0.0070
5.34+/-0.0089



Plane 2 Mean
5.57 +/- 0.0058
5.59 +/- 0.0069
5.62 +/- 0.0088

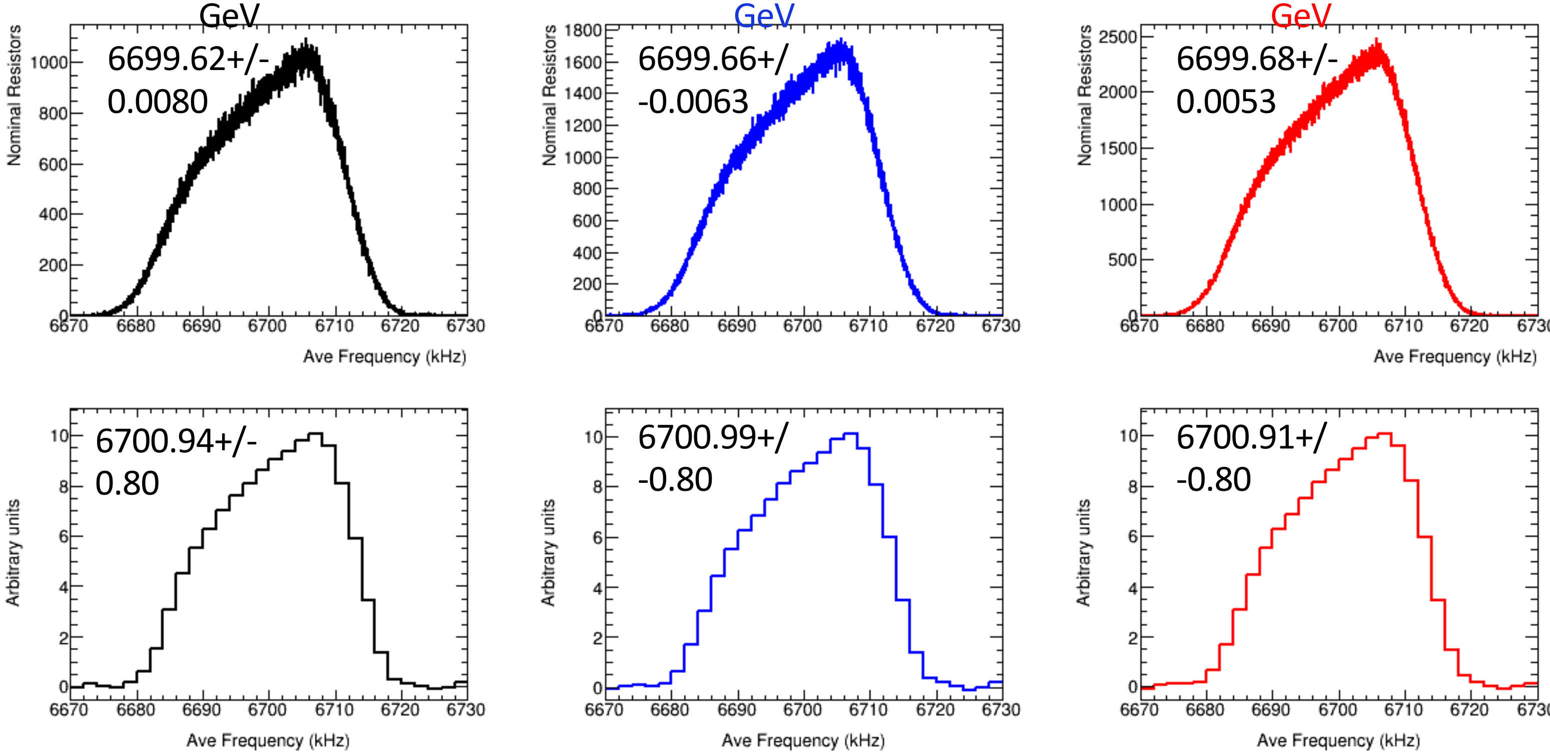


Plane 3 Mean
6.039 +/- 0.0057
6.049 +/- 0.0067
6.071 +/- 0.0085



Compare Frequency Spectrum

$e^+ > 1.5$ GeV | $e^+ > 1.0$ GeV | $e^+ > 0.5$ GeV



Tracking Plane 0 (all look the same)

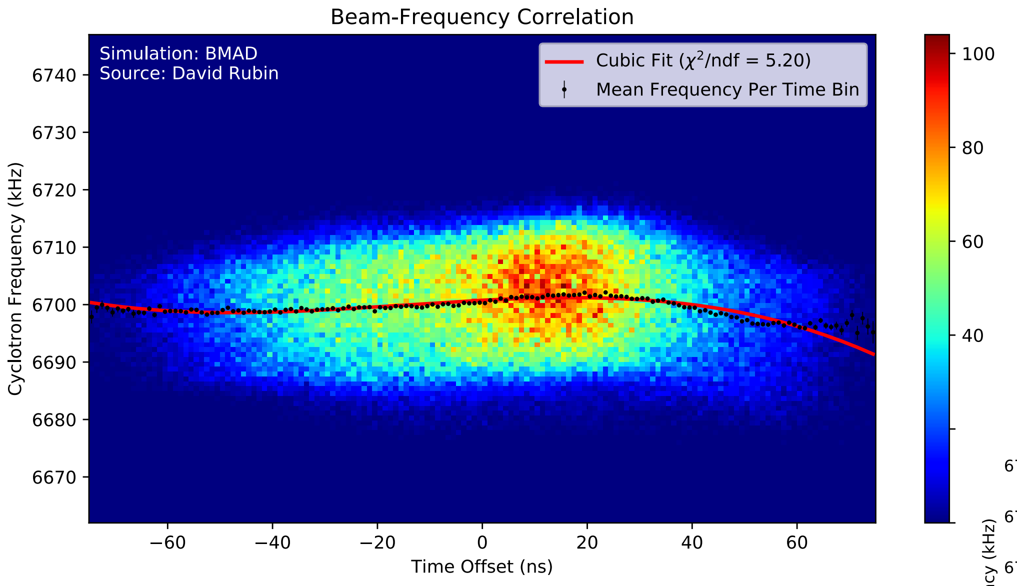
Fast Rotation Extraction

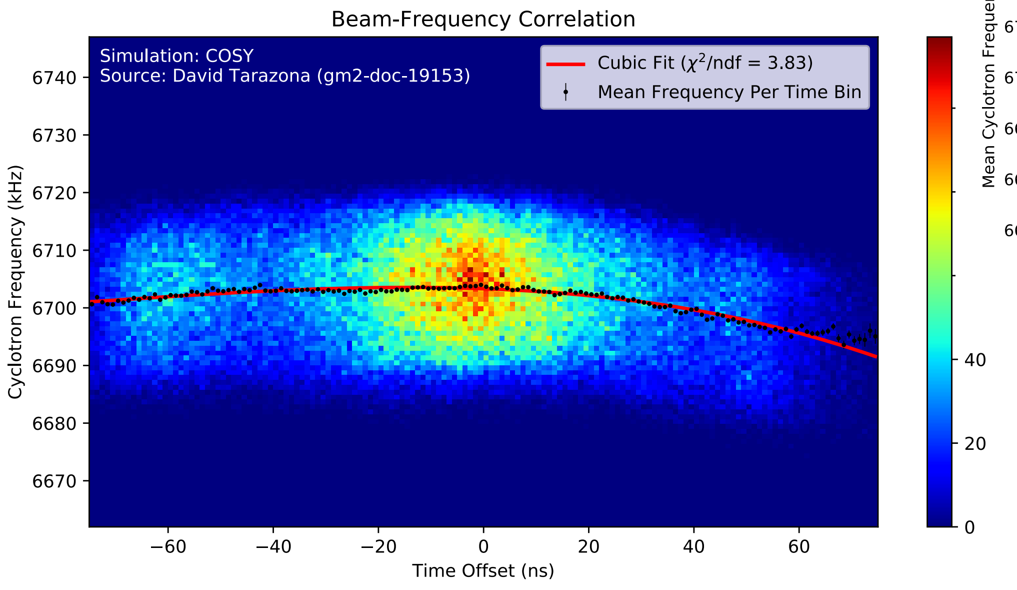
Maximal Error?

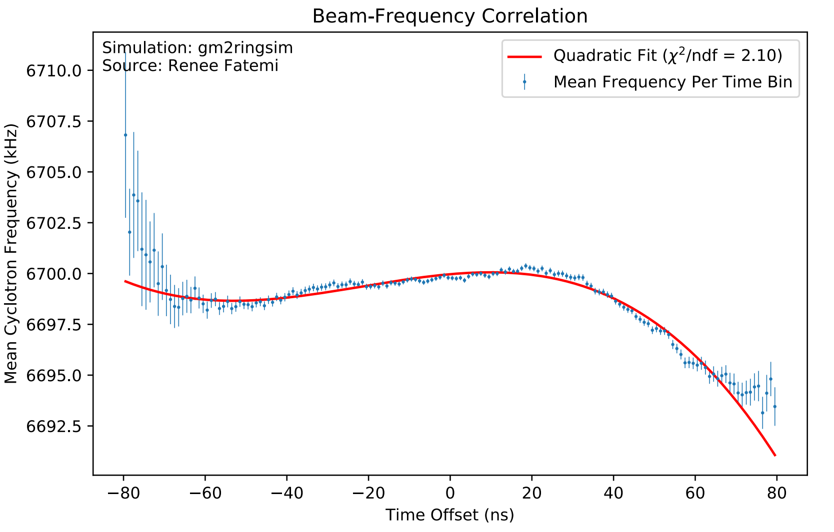
How does the deviation between the fast rotation analysis and the "truth" from the tracking planes bias the E-field correction? (n = 0.108)

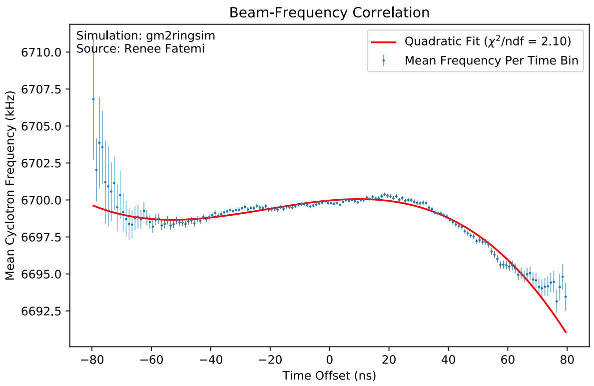
FR ➔ 365 ppb -Mean 4.35 -Width 8.78	$C_E = \frac{-2n(n-1)\beta^2}{R_{magic}^2}\langle\chi_{eq}^2\rangle$	TRUTH ➔ 358 ppb -Mean 5.06 -Width 8.28

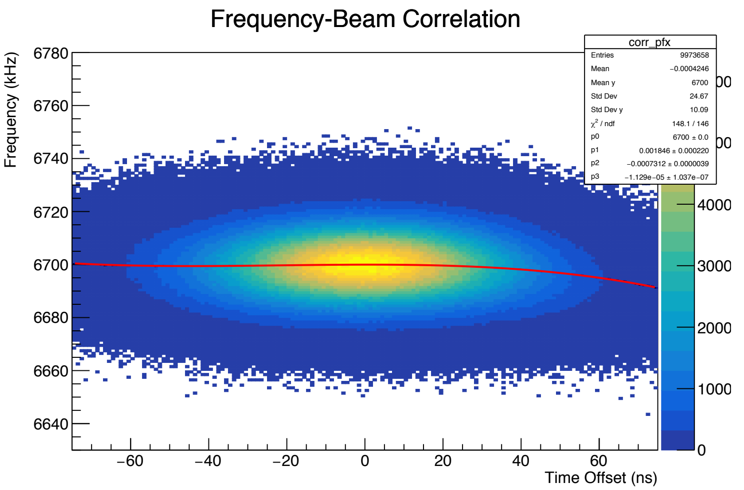
Deviation is 7 ppb

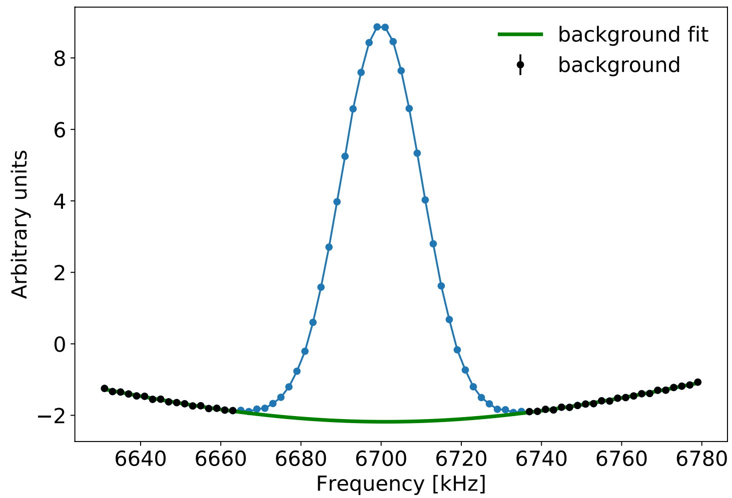

Beam-Frequency Correlation
Simulation: BMAD
Source: David Rubin
Cubic Fit (χ^2/ndf = 5.20)
Mean Frequency Per Time Bin
Cyclotron Frequency (kHz)
Time Offset (ns)

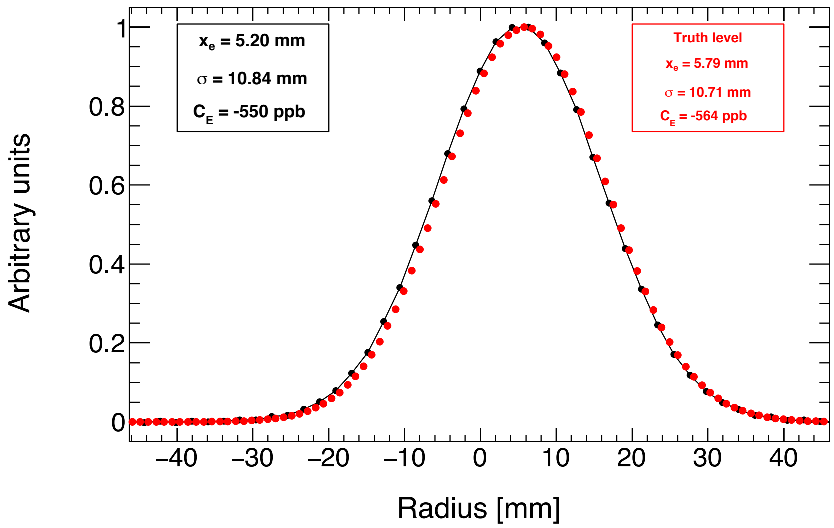

Beam-Frequency Correlation
Simulation: COSY
Source: David Tarazona (gm2-doc-19153)
Cubic Fit (χ^2/ndf = 3.83)
Mean Frequency Per Time Bin
Cyclotron Frequency (kHz)
Time Offset (ns)


Beam-Frequency Correlation
Simulation: gm2ringsim
Source: Renee Fatemi
Quadratic Fit (χ^2/ndf = 2.10)
Mean Frequency Per Time Bin
Mean Cyclotron Frequency (kHz)
Time Offset (ns)

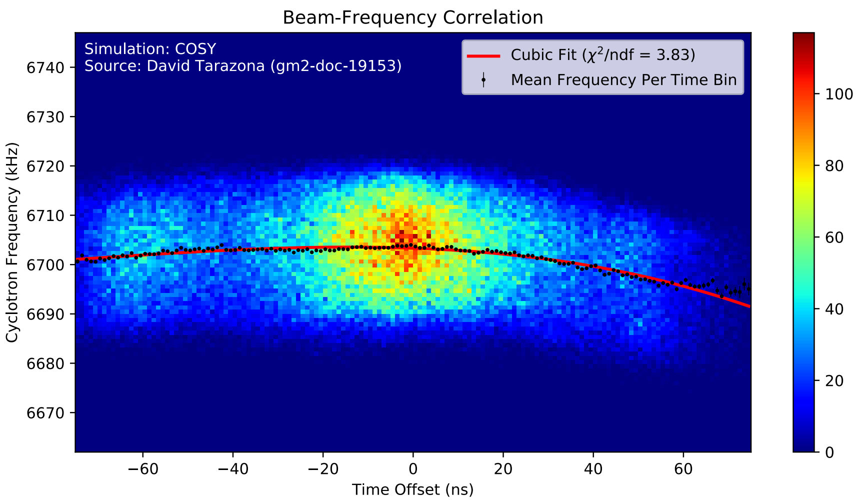

Beam-Frequency Correlation
Simulation: gm2ringsim
Source: Renee Fatemi
Quadratic Fit (χ^2/ndf = 2.10)
Mean Frequency Per Time Bin
Mean Cyclotron Frequency (kHz)
6710.0
6707.5
6705.0
6702.5
6700.0
6697.5
6695.0
6692.5
−80
−60
−40
−20
0
20
40
60
80
Time Offset (ns)

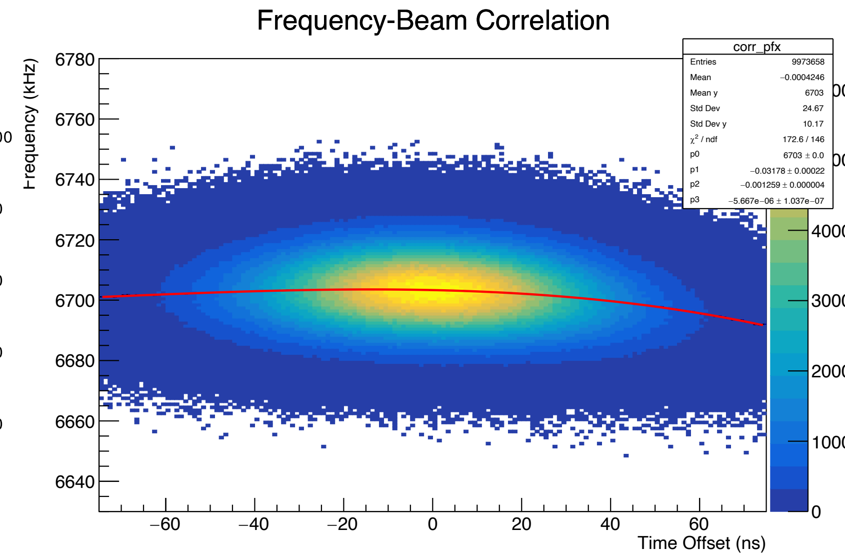

Frequency-Beam Correlation
corr_pfx
Entries 9973658
Mean −0.0004246
Mean y 6700
Std Dev 24.67
Std Dev y 10.09
χ^2 / ndf 148.1 / 146
p0 6700 ± 0.0
p1 0.001846 ± 0.000220
p2 −0.0007312 ± 0.0000039
p3 −1.129e−05 ± 1.037e−07
Frequency (kHz)
6780
6760
6740
6720
6700
6680
6660
6640
−60
−40
−20
0
20
40
60
Time Offset (ns)
4000
3000
2000
1000
0

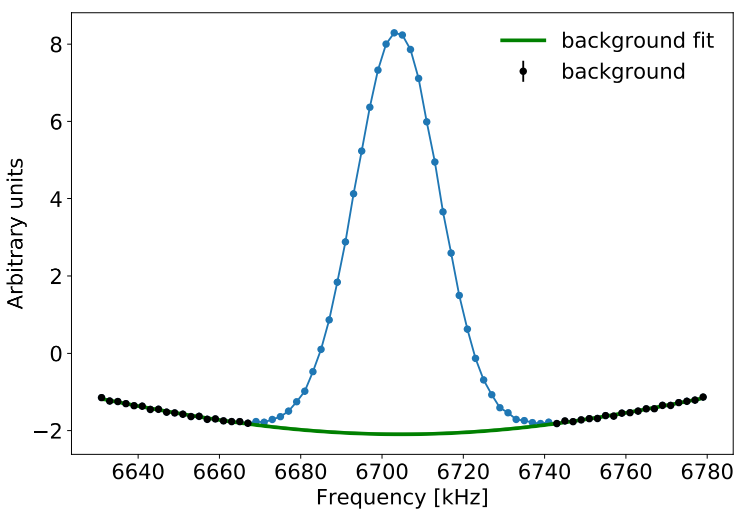

background fit
background
Arbitrary units
8
6
4
2
0
−2
6640
6660
6680
6700
6720
6740
6760
6780
Frequency [kHz]

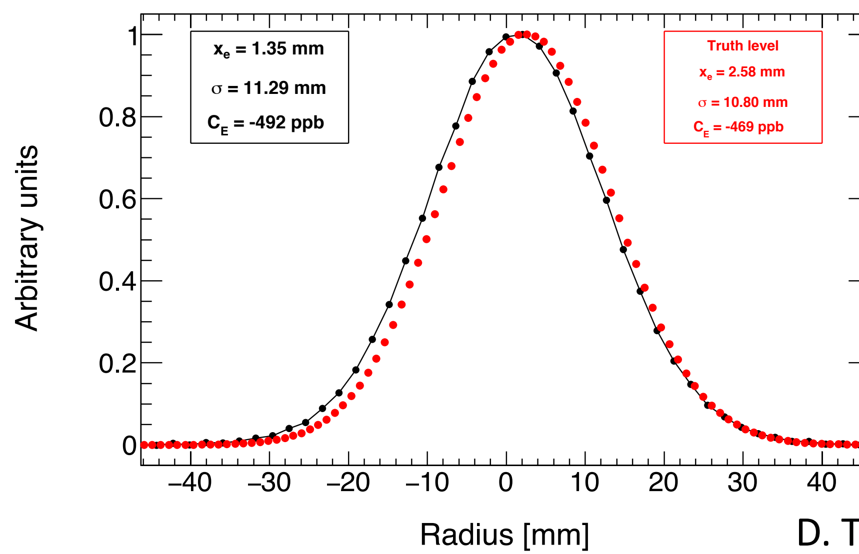

x_e = 5.20 mm
σ = 10.84 mm
C_E = -550 ppb
Truth level
x_e = 5.79 mm
σ = 10.71 mm
C_E = -564 ppb
Arbitrary units
1
0.8
0.6
0.4
0.2
0
−40
−30
−20
−10
0
10
20
30
40
Radius [mm]

T. Barrett

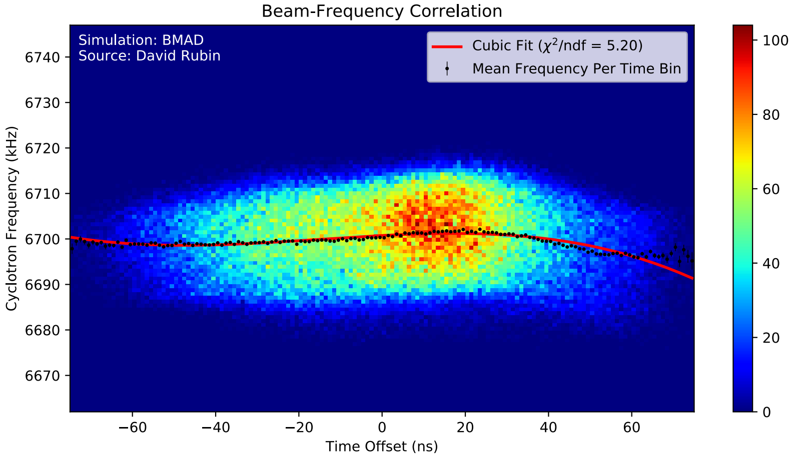


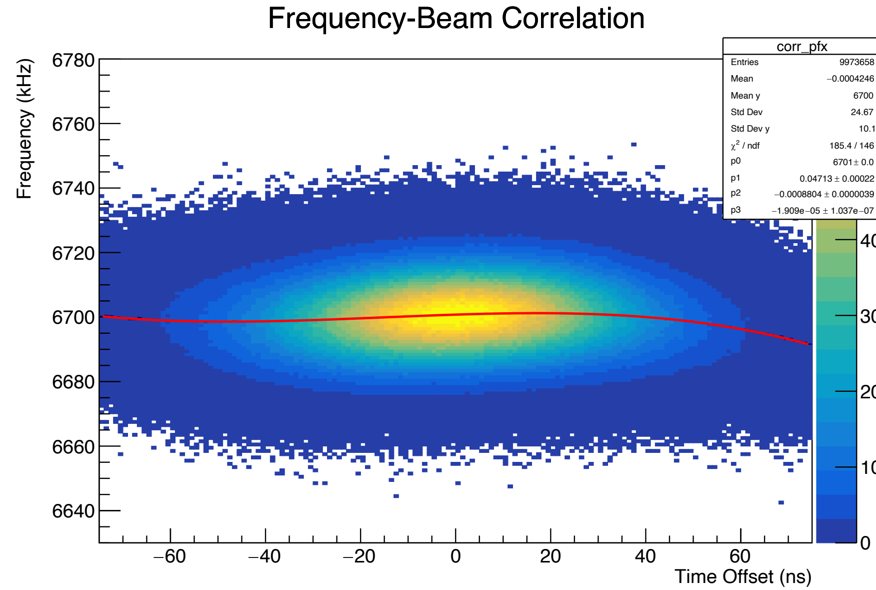


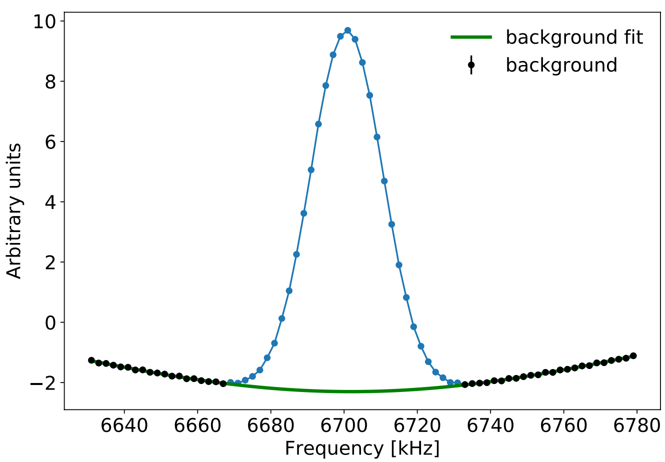


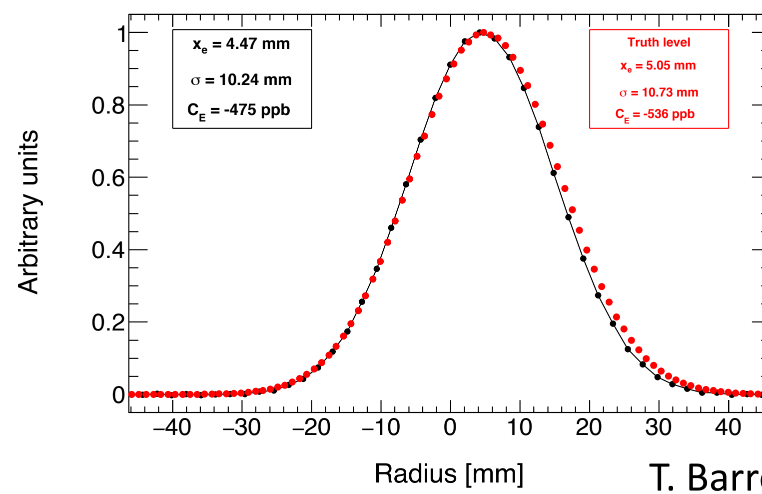


D. Tarazona
T. Barrett

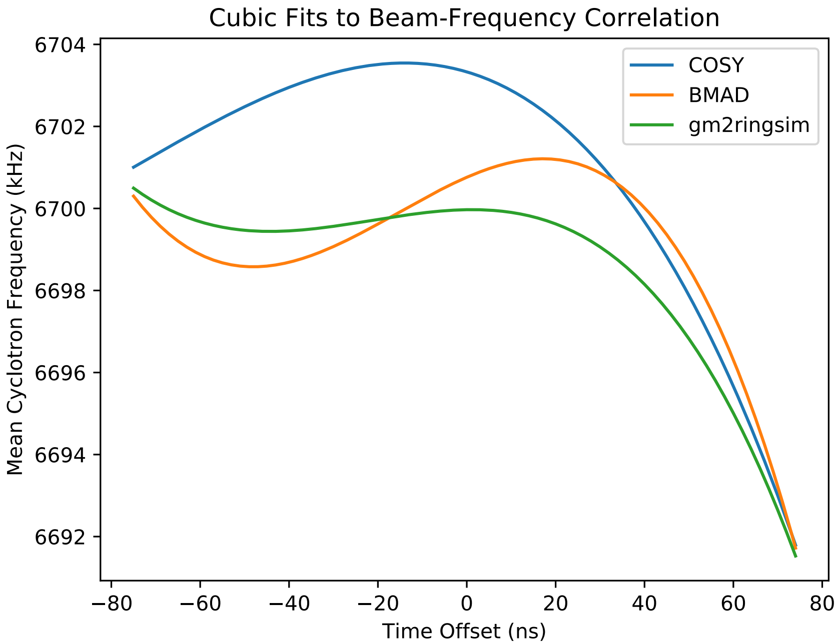

Beam-Frequency Correlation
Simulation: BMAD
Source: David Rubin
Cubic Fit (χ^2/ndf = 5.20)
Mean Frequency Per Time Bin
Cyclotron Frequency (kHz)
Time Offset (ns)


Frequency-Beam Correlation
corr_pfx
Entries 9973658
Mean −0.0004246
Mean y 6700
Std Dev 24.67
Std Dev y 10.1
χ^2 / ndf 185.4 / 146
p0 6701 ± 0.0
p1 0.04713 ± 0.00022
p2 −0.0008804 ± 0.0000039
p3 −1.909e−05 ± 1.037e−07
Frequency (kHz)
Time Offset (ns)


background fit
background
Arbitrary units
Frequency [kHz]


x_e = 4.47 mm
σ = 10.24 mm
C_E = -475 ppb
Truth level
x_e = 5.05 mm
σ = 10.73 mm
C_E = -536 ppb
Arbitrary units
Radius [mm]

T. Barrett



$$f(t) = p_0 + p_1 t + p_2 t^2 + p_3 t^3$$

Coefficient	COSY	BMAD	gm2ringsim
p_3 (kHz/ns^3)	$-5.6(9) \times 10^{-6}$	$-1.9(1) \times 10^{-5}$	$-1.13(6) \times 10^{-5}$
p_2 (kHz/ns^2)	$-1.26(3) \times 10^{-3}$	$-8.8(4) \times 10^{-4}$	$-7.3(2) \times 10^{-4}$
p_1 (kHz/ns)	$-3.2(3) \times 10^{-2}$	$+4.7(3) \times 10^{-2}$	$+1.8(1) \times 10^{-3}$
p_0 (kHz)	$+6703.33(5)$	$+6700.76(6)$	$+6699.97(3)$

- Time-momentum correlation introduces systematic error in Fourier method
- The correlation will depend on kicker parameters
- Modeling the correlation may be problematic

Summary

E-field

- Correlation systematic dominates uncertainty of E-field contribution
- We are exploring refinements to Fourier method to mitigate effects of correlation
- Redefine t_0 to peak of fast rotation signal (maximal bunching point) ?

- χ^2 method? Parameterize initial distribution and fit to S(t).
  Correlation can be included in that parameterization

Or perhaps a hybrid of Fourier and χ^2 method?

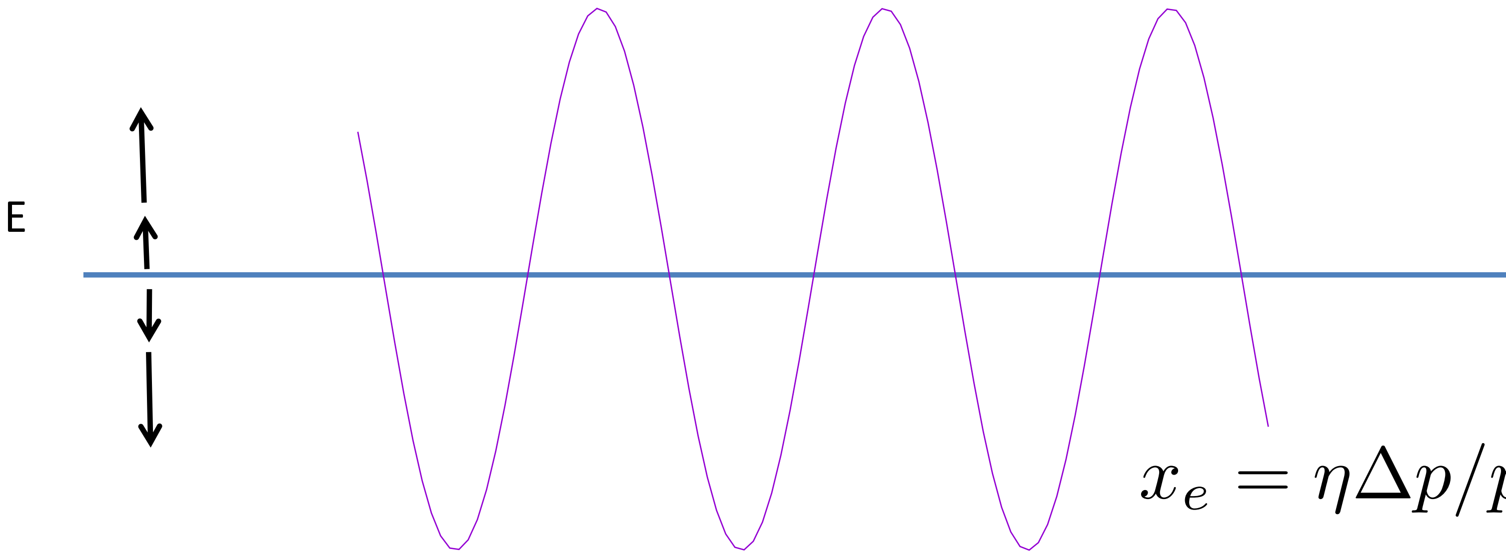
Pitch

- Effects of misalignments are small
- Remaining uncertainty dominated by relative acceptance
  In progress

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{m^2}{p^2}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

E-field contribution
Quad linearity

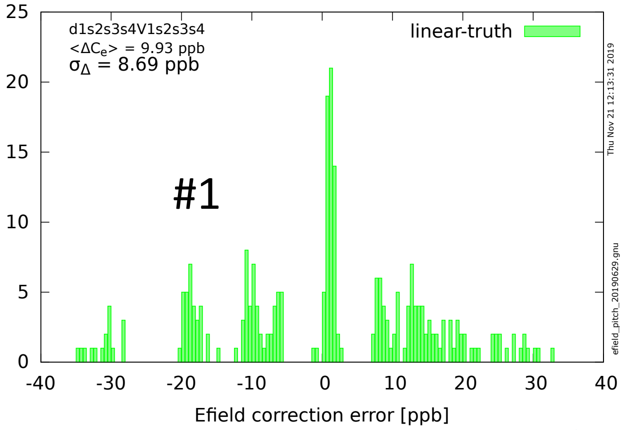
In an ideal cartesian geometry and quadrupole where the horizontal field is antisymmetric about the closed orbit, the E-field correction is independent of betatron amplitude

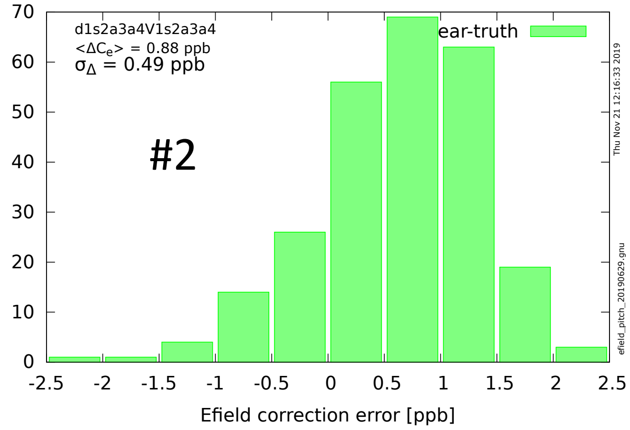


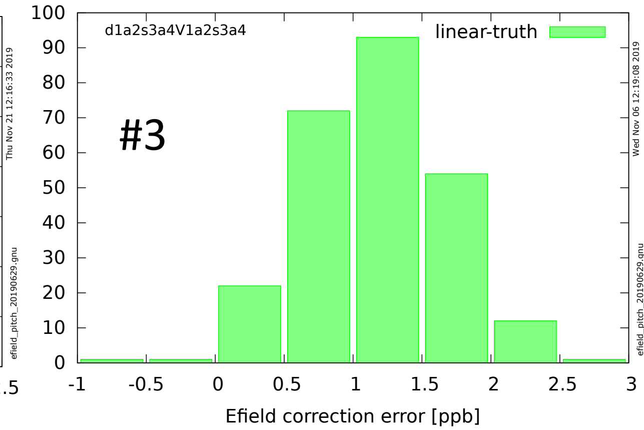
$$x_e = \eta \Delta p / p$$

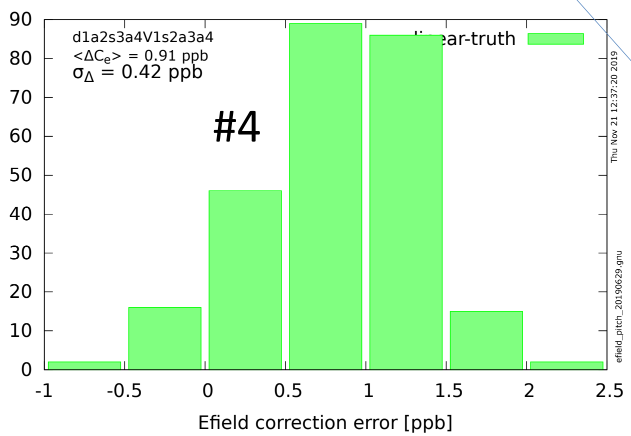
In a curved geometry, the integrated E-field along the trajectory depends on betatron amplitude in two ways

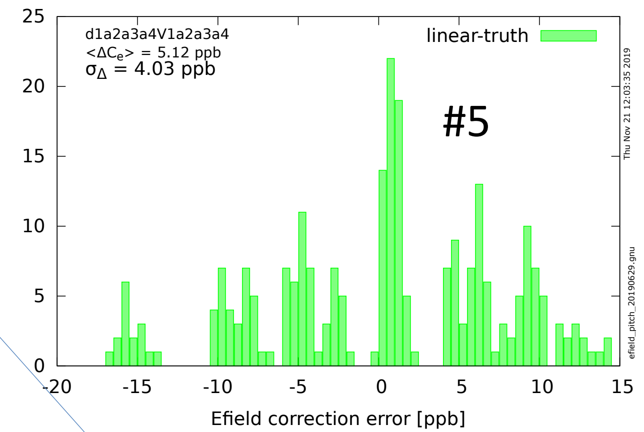
1. Sextupole (quadratic) component of the quads
   - Sextupole component is symmetric about magic radius
   - Shifts the 'closed orbit'
2. Path length (asymmetric about magic radius)









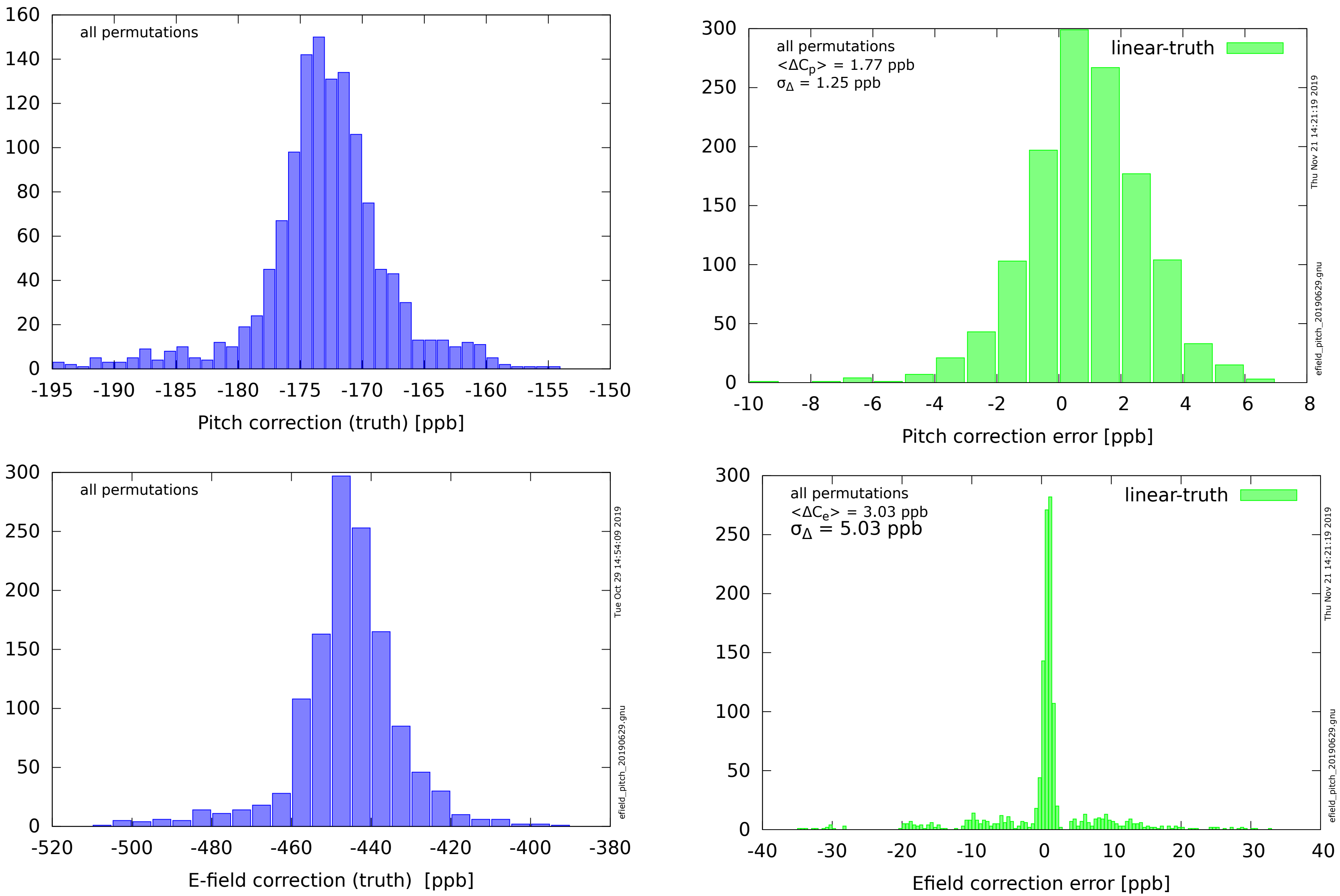


$$C_e(\text{meas}) = -2\beta^2 n_x(1-n_x)\frac{\langle x_e^2\rangle}{R_0^2}$$

$$|C_e(\text{meas}) - C_e(\text{truth})| < 35 \text{ ppb}$$

$$\sigma_\Delta \leq 8.69 \text{ ppb}$$

All 1280 configurations



truth

measured - truth

Systematic uncertainty due to quad misalignment/voltage errors for 1280 configurations

Efield

$$C_e(\text{meas}) = -2\beta^2 n_x(1-n_x)\frac{\langle x_e^2\rangle}{R_0^2}$$

$$|C_e(\text{meas}) - C_e(\text{truth})| < 35\ \text{ppb}$$

$$\sigma_\Delta \leq 8.69\ \text{ppb}$$

Where $n_x = 1 - Q_x^2$ and $x_e = \frac{\beta c}{2\pi f_{cyc}} - R_0$

Pitch

$$C_p(\text{meas}) = -\frac{n_y\langle y^2\rangle}{2R_0^2}$$

$$|C_p(\text{meas}) - C_p(\text{truth})| < 10\ \text{ppb}$$

$$\sigma_\Delta \leq 1.3\ \text{ppb}$$

Where $n_y = Q_y^2$