EDM systematic uncertainties due to radial and longitudinal B-fields and pitch – spin tracking

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Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0,\ \ s_{long} = 1$$

EDM $\quad d = 4.67 \times 10^{-20}\mathrm{e-cm}$

$$\frac{\mathrm{d}}{\mathrm{d}s}\mathbf{S} = \left\{ \frac{(1 + \mathbf{r}_t \cdot \mathbf{g})}{c\,\beta_z} \left(\mathbf{\Omega}_{BMT} + \mathbf{\Omega}_{EDM} \right) - \mathbf{g} \times \widehat{\mathbf{z}} \right\} \times \mathbf{S}$$

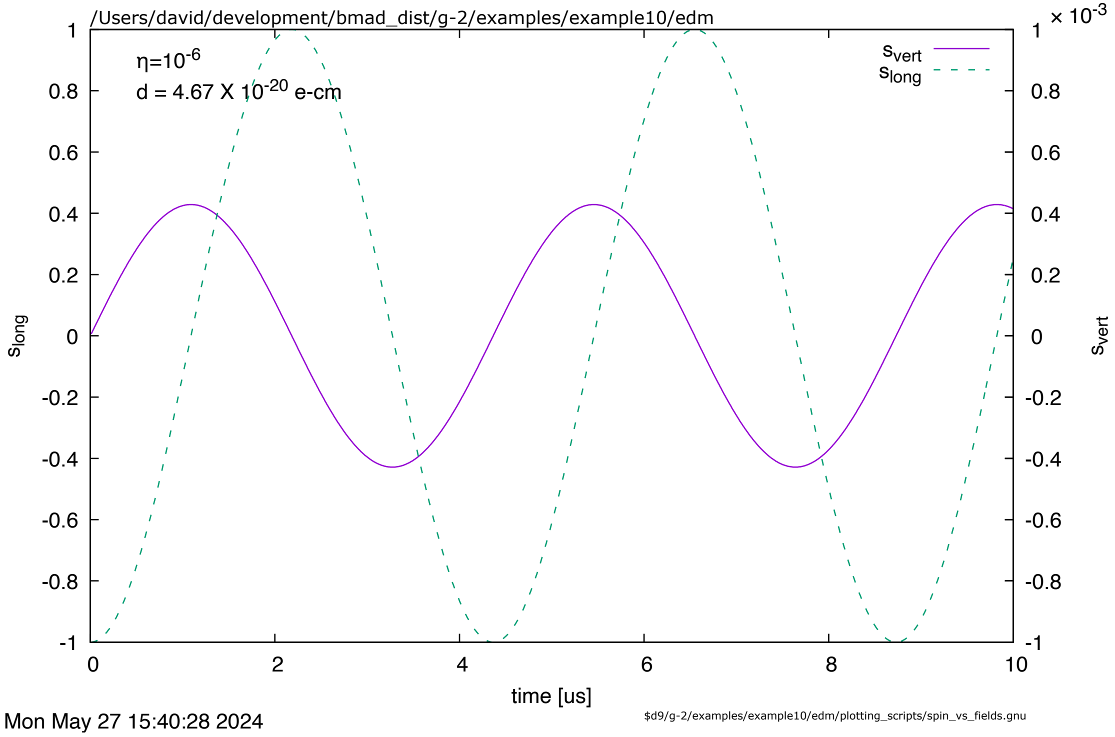
$$\mathbf{\Omega}_{BMT}(\mathbf{r}, \mathbf{P}, t) = -\frac{q}{m\,c} \left[\left(\frac{1}{\gamma} + a \right) c\,\mathbf{B} - \frac{a\,\gamma\,c}{1 + \gamma} (\boldsymbol{\beta} \cdot \mathbf{B})\,\boldsymbol{\beta} - \left(a + \frac{1}{1 + \gamma} \right) \boldsymbol{\beta} \times \mathbf{E} \right]$$

$$\mathbf{\Omega}_{EDM}(\mathbf{r}, \mathbf{P}, t) = -\frac{q\,\eta}{2\,m\,c} \left[\mathbf{E} - \frac{\gamma}{1 + \gamma} (\boldsymbol{\beta} \cdot \mathbf{E})\,\boldsymbol{\beta} + c\,\boldsymbol{\beta} \times \mathbf{B} \right]$$

$$\mathbf{d} = \frac{\eta}{2} \frac{q}{m\,c} \mathbf{S}$$

$$\mathbf{d}[\mathrm{e-cm}] = 4.66 \times 10^{-14} \eta$$

Single particle spin tracking



Continuous quad approximation

No nonlinearity

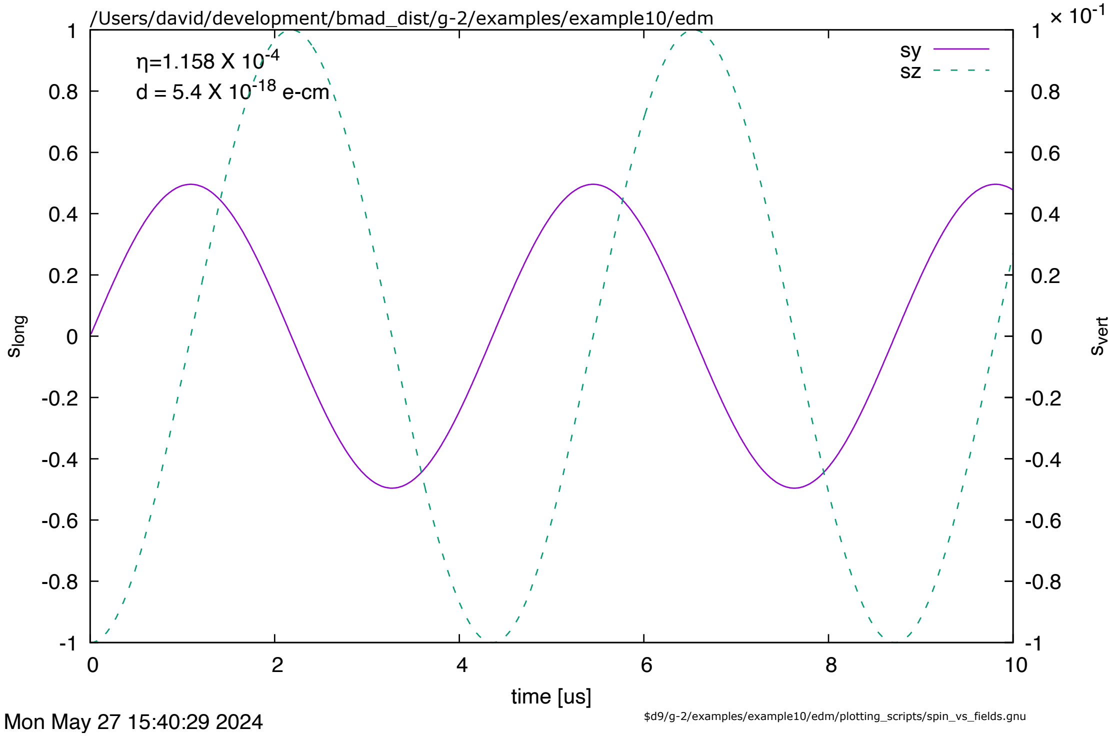
Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0,\ s_{long} = 1$$

$$d = 5.4 \times 10^{-18}\mathrm{e-cm}$$



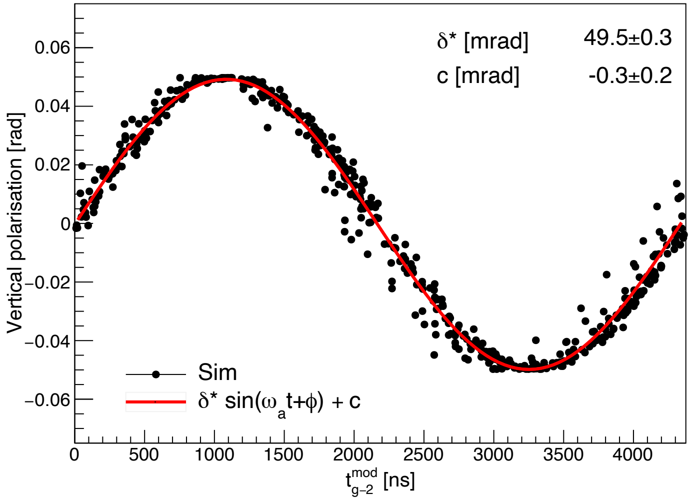


Figure 7: The time modulated average vertical component of the rest frame muon polarisation vector, with an injected EDM of 5.4×10^{-18} $e\cdot$cm (30× BNL). The amplitude of the fit gives a rest frame tilt angle of 49.5±0.3 mrad (1.69±0.01 mrad in the laboratory frame), which is consistent with expectation.

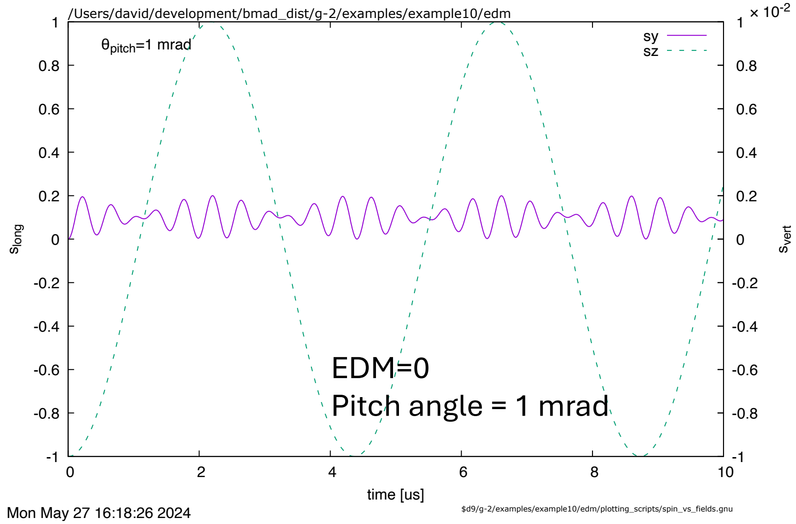
Pitch

Initial phase space coordinates

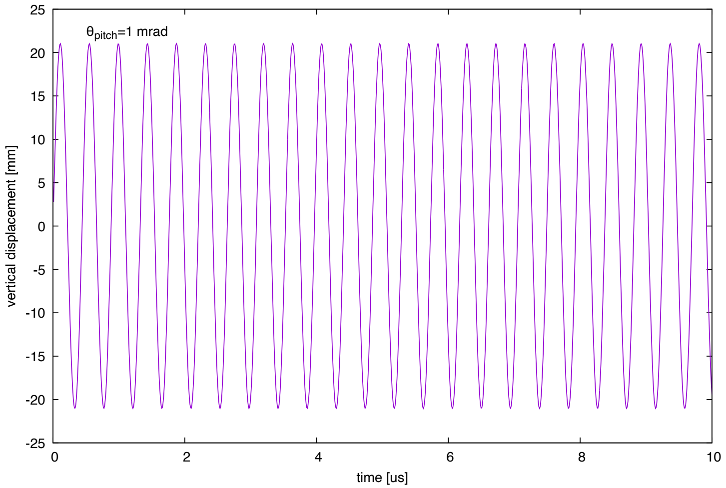
$$x = x' = y = 0,\ y' = 1 \text{ mrad}$$

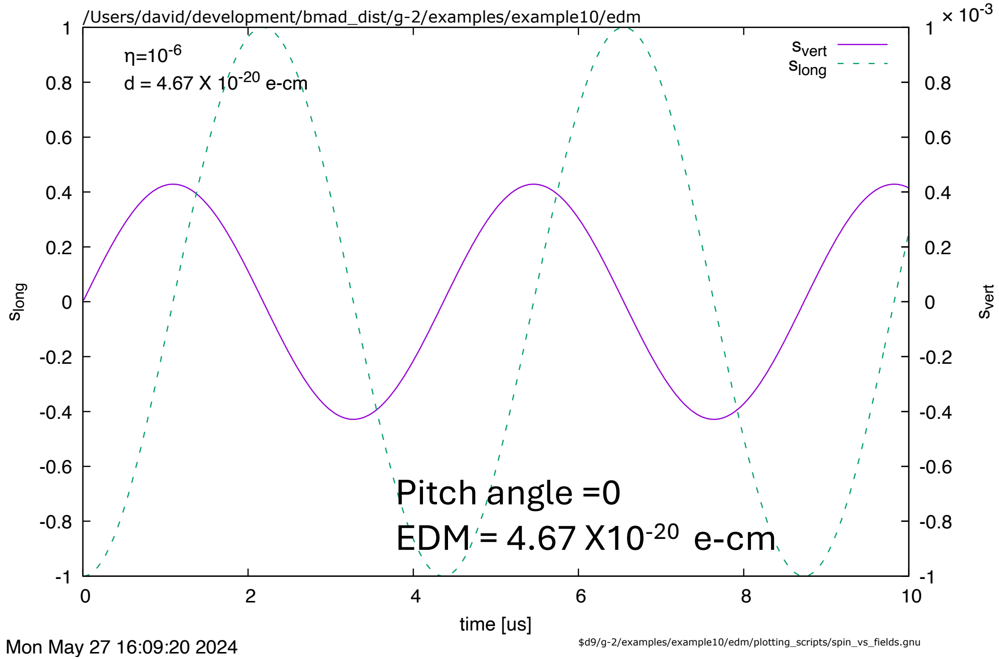
Initial polarization

$$s_{rad} = s_{vert} = 0,\ s_{long} = 1$$



Vertical betatron oscillation





(Longitudinal polarization refers to left hand axis labels. Vertical polarization refers to right hand axis. Note exponent.)

Longitudinal magnetic field

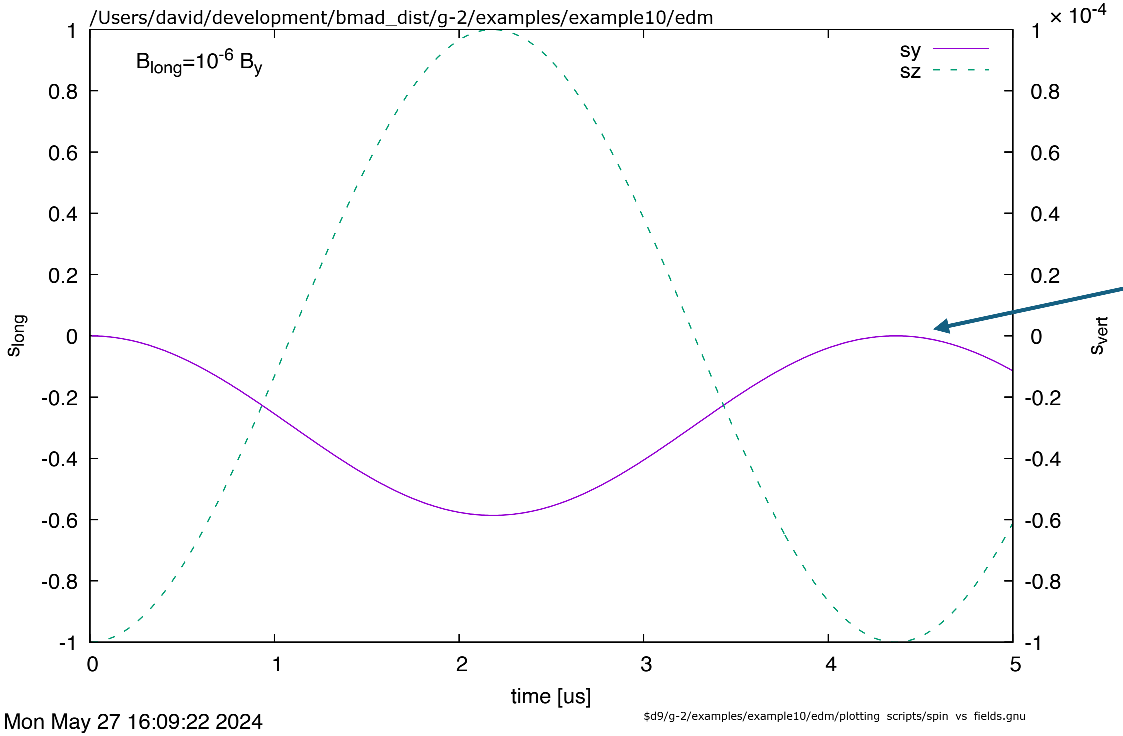
Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0,\ s_{long} = 1$$

$B_{long} = 1\ \mathrm{ppm}$ uniform around ring



$$\langle\theta_y\rangle(t) = \frac{1}{N(t)}\left(A_{g-2}\cos(\omega_a t + \phi) + A_{\mathrm{EDM}}\sin(\omega_a t + \phi) + c\right)$$

Radial B-field - EDM equivalency

$$\boldsymbol{\Omega}_{BMT}(\mathbf{r},\mathbf{P},t) = -\frac{q}{m\,c}\left[\left(\frac{1}{\gamma}+a\right)c\,\mathbf{B} - \frac{a\,\gamma\,c}{1+\gamma}(\boldsymbol{\beta}\cdot\mathbf{B})\,\boldsymbol{\beta} - \left(a+\frac{1}{1+\gamma}\right)\boldsymbol{\beta}\times\mathbf{E}\right]$$

$$\boldsymbol{\Omega}_{EDM}(\mathbf{r},\mathbf{P},t) = -\frac{q\,\eta}{2\,m\,c}\left[\mathbf{E} - \frac{\gamma}{1+\gamma}(\boldsymbol{\beta}\cdot\mathbf{E})\,\boldsymbol{\beta} + c\,\boldsymbol{\beta}\times\mathbf{B}\right]$$

$$\left(\frac{1}{\gamma}+a\right)B_{radial} \leftrightarrow \frac{\eta}{2}(\beta\times\mathbf{B})$$

$$\mathbf{d}[\mathrm{e-cm}] = 4.66\times10^{-14}\eta$$

$$\left(\frac{1}{\gamma}+a\right)\frac{B_{radial}}{B} \quad \leftrightarrow \quad 0.215\times10^{14}\frac{d}{2}[\mathrm{e-cm}]$$

$$\frac{B_{radial}}{B} \quad \leftrightarrow \quad 6.29\times10^{14}\frac{d}{2}[\mathrm{e-cm}]$$

Dataset	$\langle B_r\rangle$ [ppm]	Equivalent d_μ [$\times10^{-20}$ $e{\cdot}$cm]
1a	22 ± 7	7 ± 2
1b	23 ± 8	7 ± 3
1c	30 ± 8	9 ± 3
1d	34 ± 9	10 ± 3

Table 15: Estimates for $\langle B_r\rangle$ in ppm, as well as the equivalent fake EDM signal in $e{\cdot}$cm, for various E989 datasets [33][2].

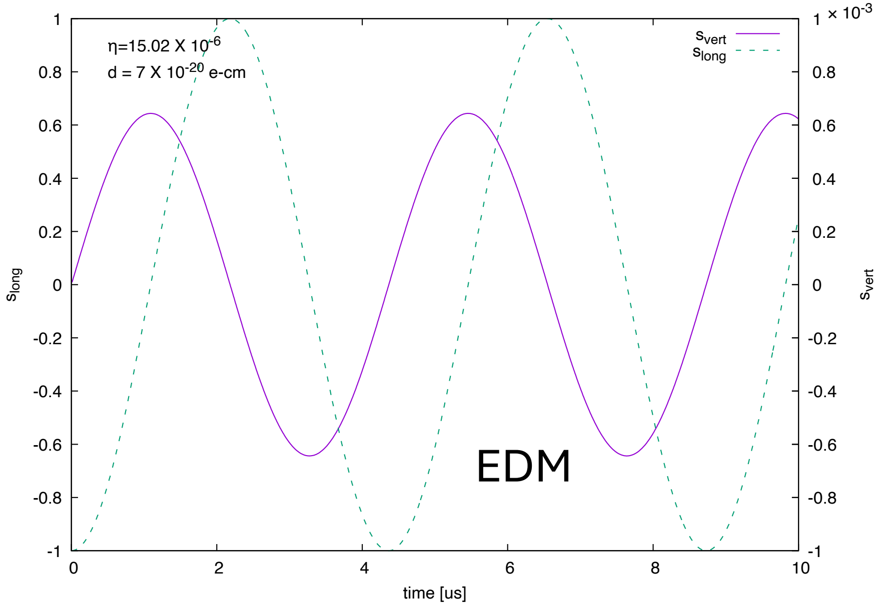
Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0,\ s_{long} = 1$$

$$d = 7 \times 10^{-20}\mathrm{e-cm},\ B_{radial} = 0$$



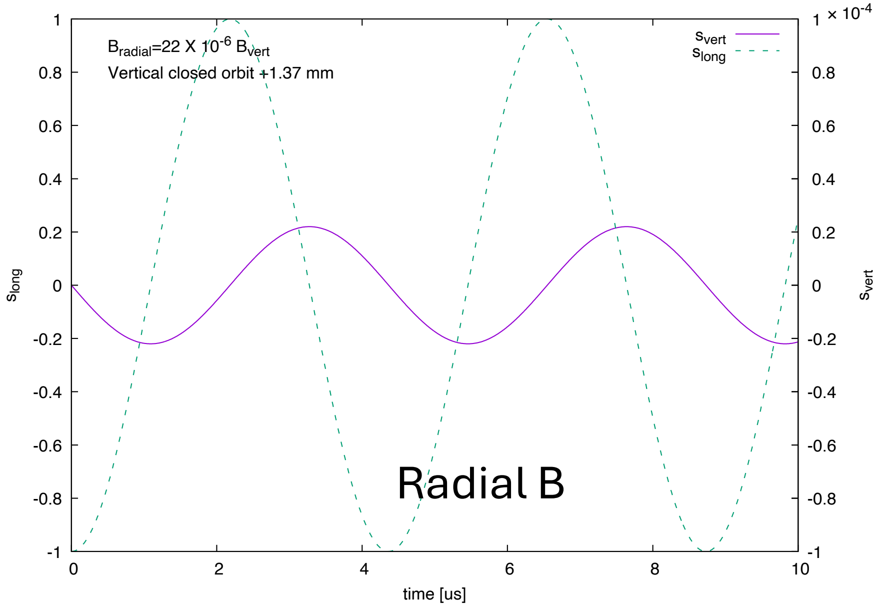
Initial phase space coordinates

$$y = \ 1.37\mathrm{mm},\ x = x' = y' = 0$$ *Closed orbit displaced vertically by radial B field*

Initial polarization

$$s_{rad} = s_{vert} = 0,\ s_{long} = 1$$

$$B_{radial} = 22\ \mathrm{ppm},\ d = 0$$



$$\mathbf{\Omega}_{BMT}(\mathbf{r},\mathbf{P},t) = -\frac{q}{m\,c}\left[\left(\frac{1}{\gamma}+a\right)c\,\mathbf{B} - \frac{a\,\gamma\,c}{1+\gamma}\,(\boldsymbol{\beta}\cdot\mathbf{B})\,\boldsymbol{\beta} - \left(a+\frac{1}{1+\gamma}\right)\boldsymbol{\beta}\times\mathbf{E}\right]$$

$$\begin{aligned} F_e &= qE \\ F_b &= qc(\beta \times \mathbf{B}) \\ E &= c(\beta B_{rad}) \end{aligned}$$

Net contribution from B_{rad} and compensating E_{vert}

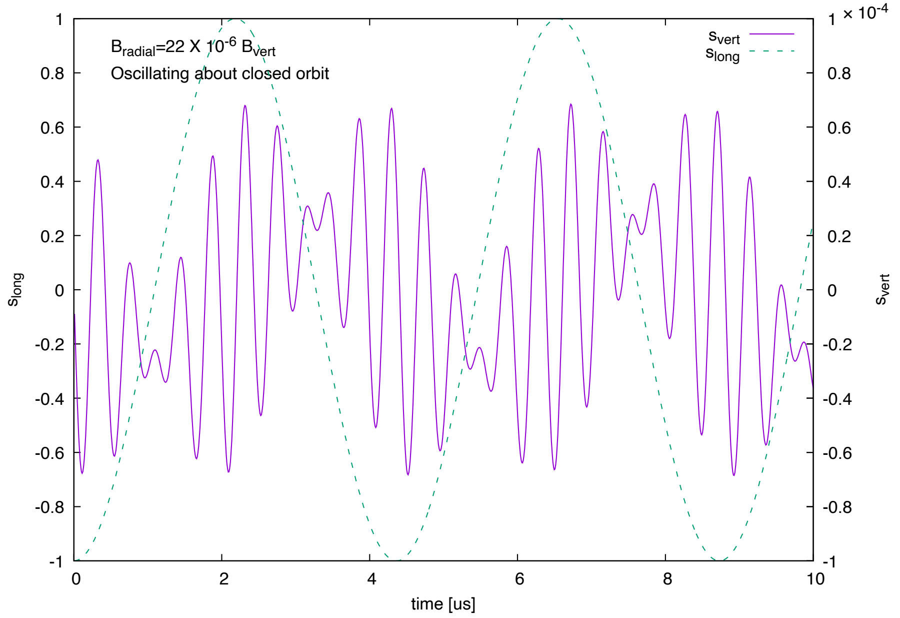
$$\frac{\left(\frac{1}{\gamma}+a\right)cB_{rad} - \left(a+\frac{1}{1+\gamma}\right)\beta E_{vert}}{\left(\frac{1}{\gamma}+a\right)cB_{rad}} \sim \frac{1}{\gamma}$$

$B_{radial} = 22$ ppm, EDM=0

Oscillation about the displaced vertical closed orbit

Initial phase space coordinates

$$x = x' = y = y' = 0$$



On the displaced closed orbit

Initial phase space coordinates

$$y = \ 1.37\text{mm},\ x = x' = y' = 0$$