

EDM systematic uncertainties due to radial and longitudinal B-fields and pitch – spin tracking

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Initial phase space coordinates

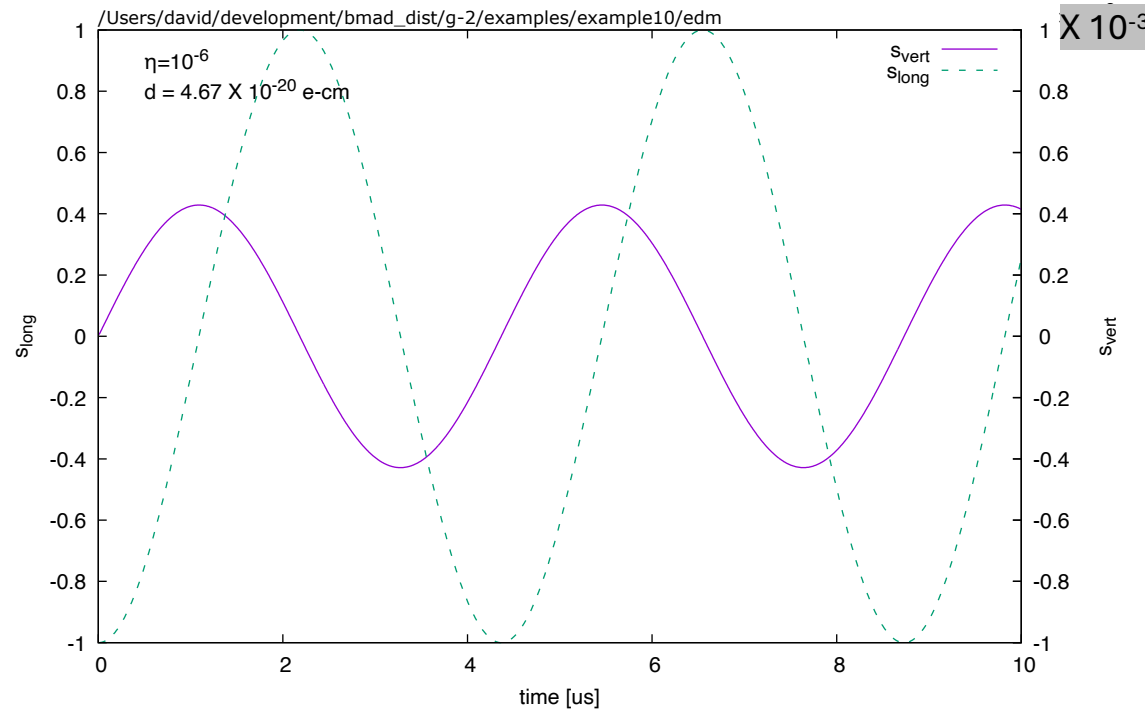
$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0, \quad s_{long} = 1$$

EDM $d = 4.67 \times 10^{-20} \text{ e-cm}$

Single particle spin tracking



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\$d9/g-2/examples/example10/edm/plotting_scripts/spin_vs_fields.gnu

Continuous quad approximation

No nonlinearity

$$\frac{d}{ds} \mathbf{S} = \left\{ \frac{(1 + \mathbf{r}_t \cdot \mathbf{g})}{c \beta_z} (\boldsymbol{\Omega}_{BMT} + \boldsymbol{\Omega}_{EDM}) - \mathbf{g} \times \hat{\mathbf{z}} \right\} \times \mathbf{S}$$

$$\boldsymbol{\Omega}_{BMT}(\mathbf{r}, \mathbf{P}, t) = -\frac{q}{mc} \left[\left(\frac{1}{\gamma} + a \right) c \mathbf{B} - \frac{a \gamma c}{1 + \gamma} (\boldsymbol{\beta} \cdot \mathbf{B}) \boldsymbol{\beta} - \left(a + \frac{1}{1 + \gamma} \right) \boldsymbol{\beta} \times \mathbf{E} \right]$$

$$\boldsymbol{\Omega}_{EDM}(\mathbf{r}, \mathbf{P}, t) = -\frac{q \eta}{2 m c} \left[\mathbf{E} - \frac{\gamma}{1 + \gamma} (\boldsymbol{\beta} \cdot \mathbf{E}) \boldsymbol{\beta} + c \boldsymbol{\beta} \times \mathbf{B} \right]$$

$$\mathbf{d} = \frac{\eta}{2} \frac{q}{m c} \mathbf{S}$$

$$d[\text{e-cm}] = 4.66 \times 10^{-14} \eta$$

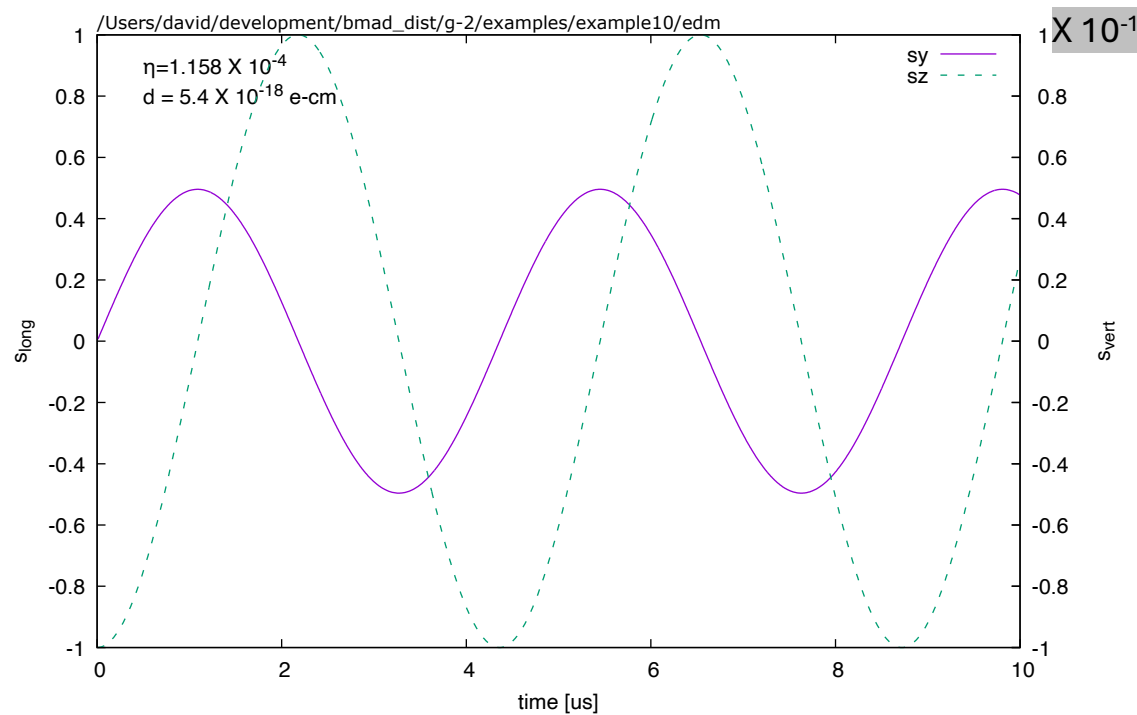
Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0, \quad s_{long} = 1$$

$$d = 5.4 \times 10^{-18} e \cdot \text{cm}$$



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\$d9/g-2/examples/example10/edm/plotting_scripts/spin_vs_fields.gnu

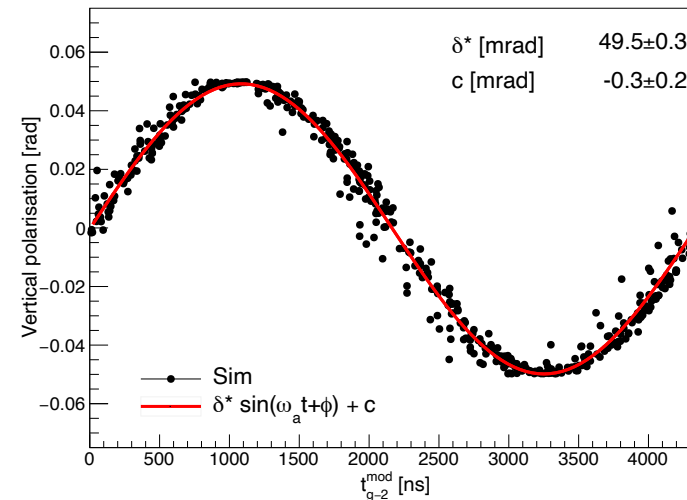


Figure 7: The time modulated average vertical component of the rest frame muon polarisation vector, with an injected EDM of $5.4 \times 10^{-18} e\cdot\text{cm}$ ($30 \times \text{BNL}$). The amplitude of the fit gives a rest frame tilt angle of 49.5 ± 0.3 mrad (1.69 ± 0.01 mrad in the laboratory frame), which is consistent with expectation.

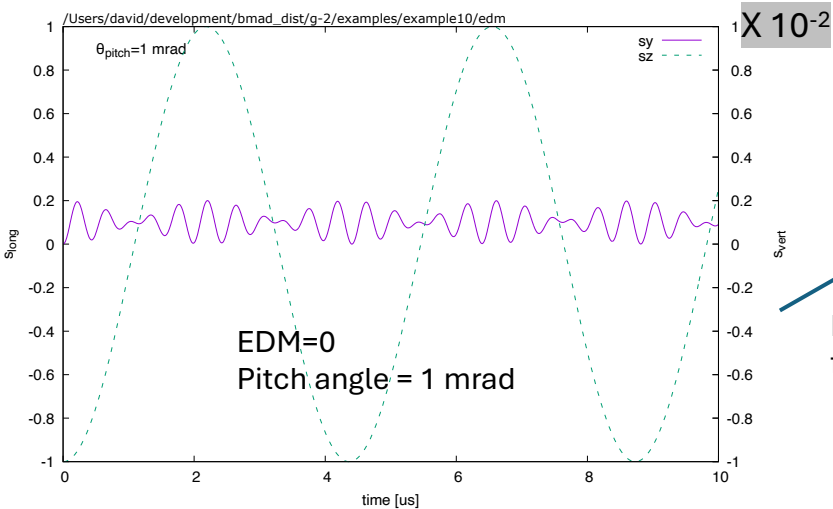
Pitch

Initial phase space coordinates

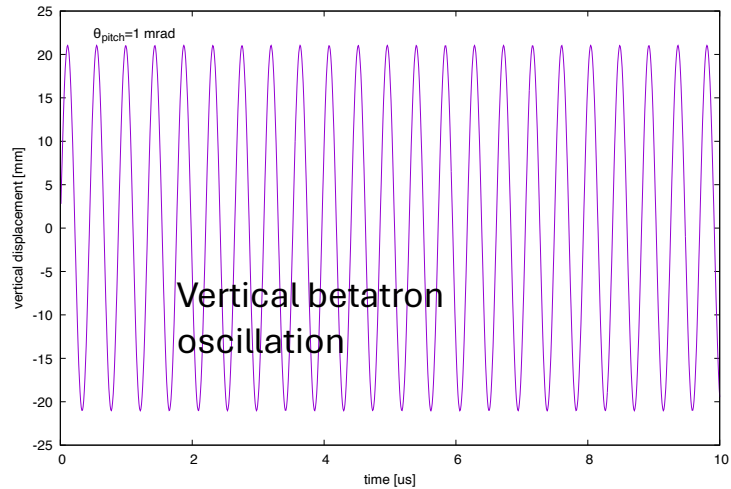
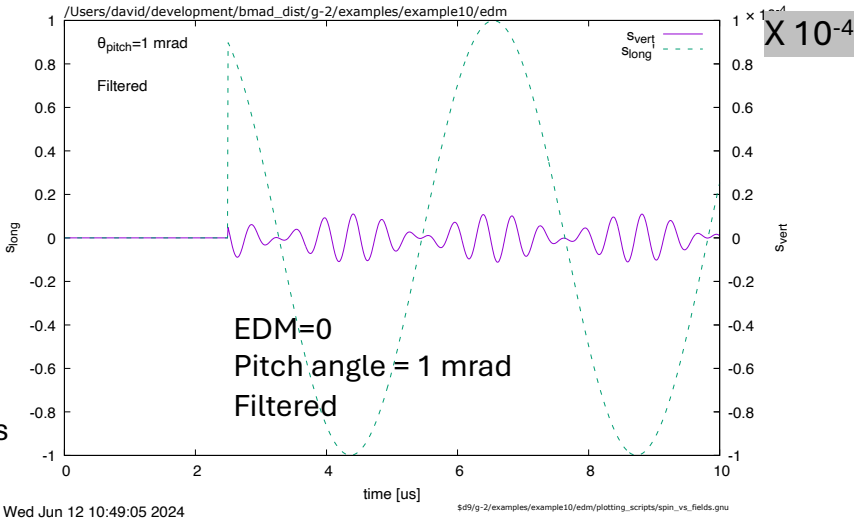
$x = x' = y = 0, y' = 1 \text{ mrad}$

Initial polarization

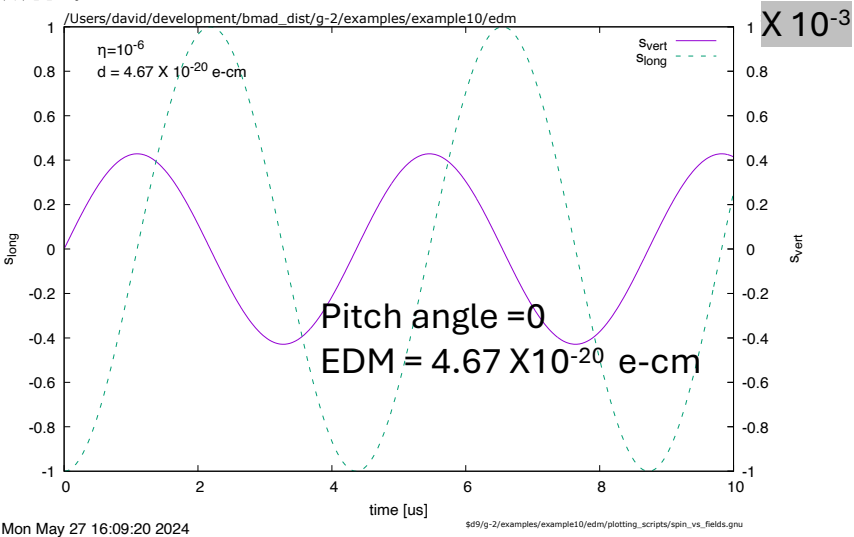
$s_{rad} = s_{vert} = 0, s_{long} = 1$



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(Longitudinal polarization refers to left hand axis labels. Vertical polarization refers to right hand axis. Note exponent.)

Longitudinal magnetic field

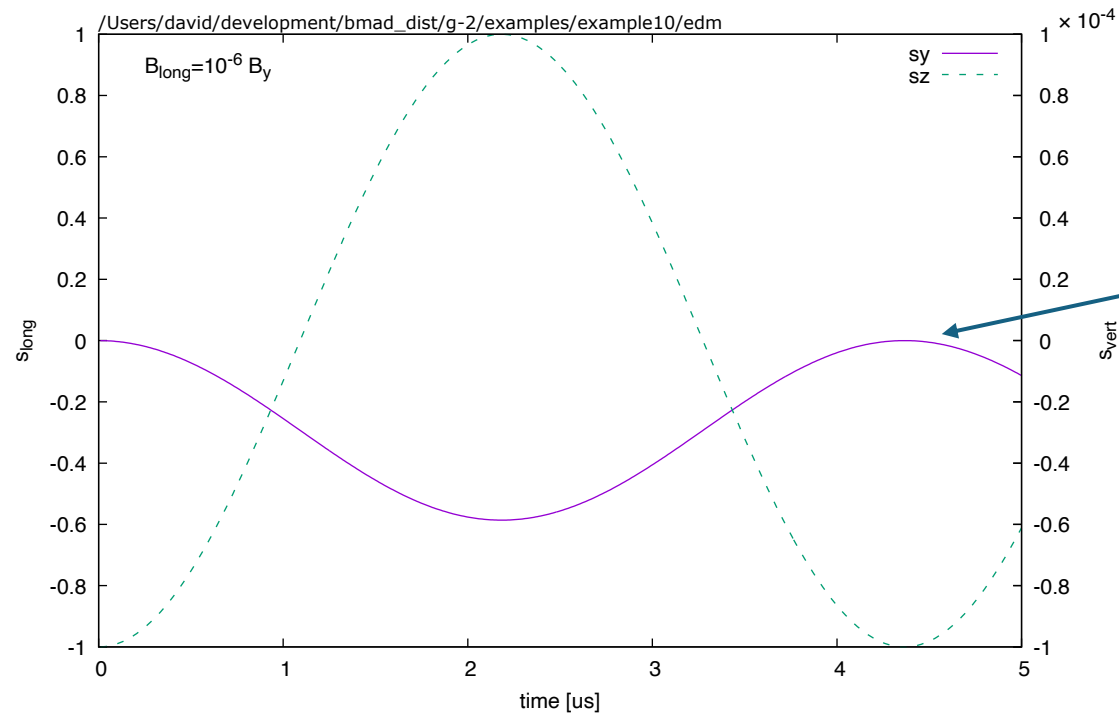
Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0, \quad s_{long} = 1$$

$B_{long} = 1$ ppm uniform around ring



$$\langle \theta_y \rangle(t) = \frac{1}{N(t)} (A_{g-2} \cos(\omega_a t + \phi) + A_{EDM} \sin(\omega_a t + \phi) + c)$$

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sd9/g-2/examples/example10/edm/plotting_scripts/spin_vs_fields.gnu

Radial B-field - EDM equivalency

$$\mathbf{\Omega}_{BMT}(\mathbf{r}, \mathbf{P}, t) = -\frac{q}{m c} \left[\left(\frac{1}{\gamma} + a \right) c \mathbf{B} - \frac{a \gamma c}{1 + \gamma} (\boldsymbol{\beta} \cdot \mathbf{B}) \boldsymbol{\beta} - \left(a + \frac{1}{1 + \gamma} \right) \boldsymbol{\beta} \times \mathbf{E} \right]$$

$$\mathbf{\Omega}_{EDM}(\mathbf{r}, \mathbf{P}, t) = -\frac{q \eta}{2 m c} \left[\mathbf{E} - \frac{\gamma}{1 + \gamma} (\boldsymbol{\beta} \cdot \mathbf{E}) \boldsymbol{\beta} + c \boldsymbol{\beta} \times \mathbf{B} \right]$$

$$\left(\frac{1}{\gamma} + a \right) B_{radial} \leftrightarrow \frac{\eta}{2} (\boldsymbol{\beta} \times \mathbf{B})$$

$$\mathbf{d}[\text{e} - \text{cm}] = 4.66 \times 10^{-14} \eta$$

$$\begin{aligned} \left(\frac{1}{\gamma} + a \right) \frac{B_{radial}}{B} &\leftrightarrow 0.215 \times 10^{14} \frac{d}{2} [\text{e} - \text{cm}] \\ \frac{B_{radial}}{B} &\leftrightarrow 6.29 \times 10^{14} \frac{d}{2} [\text{e} - \text{cm}] \end{aligned}$$

Dataset	$\langle B_r \rangle$ [ppm]	Equivalent d_μ [$\times 10^{-20}$ e·cm]
1a	22 ± 7	7 ± 2
1b	23 ± 8	7 ± 3
1c	30 ± 8	9 ± 3
1d	34 ± 9	10 ± 3

Table 15: Estimates for $\langle B_r \rangle$ in ppm, as well as the equivalent fake EDM signal in e·cm, for various E989 datasets [33][2].

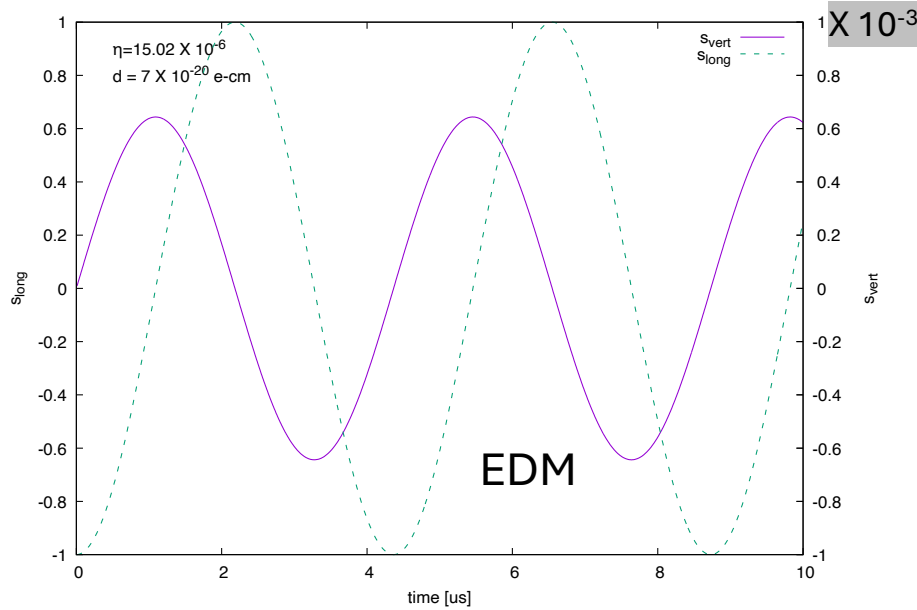
Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0, s_{long} = 1$$

$$d = 7 \times 10^{-20} \text{ e-cm}, B_{radial} = 0$$



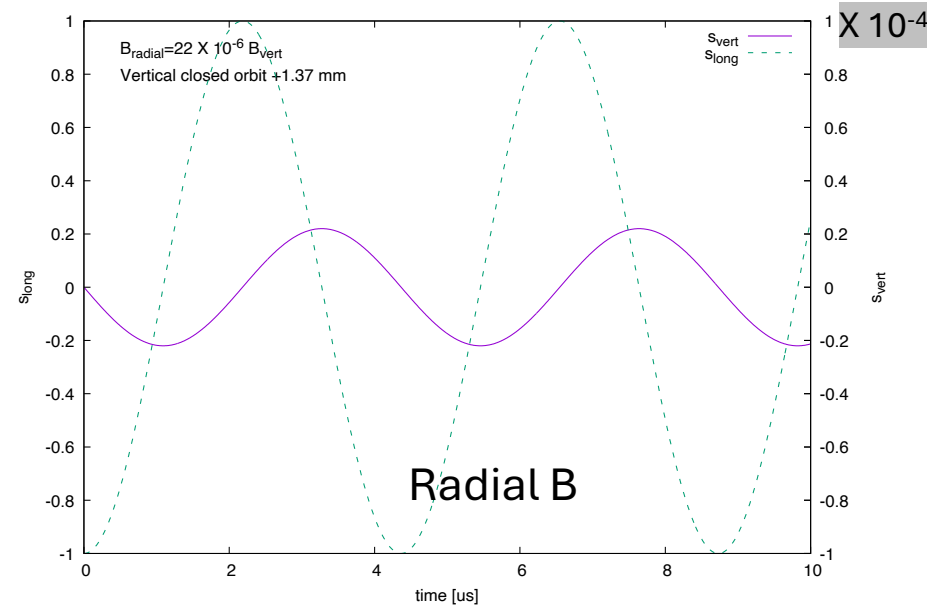
Initial phase space coordinates

$$y = 1.37 \text{ mm}, x = x' = y' = 0 \text{ Closed orbit displaced vertically by radial B field}$$

Initial polarization

$$s_{rad} = s_{vert} = 0, s_{long} = 1$$

$$B_{radial} = 22 \text{ ppm}, d = 0$$



$$\Omega_{BMT}(\mathbf{r}, \mathbf{P}, t) = -\frac{q}{mc} \left[\left(\frac{1}{\gamma} + a \right) c \mathbf{B} - \frac{a \gamma c}{1 + \gamma} (\boldsymbol{\beta} \cdot \mathbf{B}) \boldsymbol{\beta} - \left(a + \frac{1}{1 + \gamma} \right) \boldsymbol{\beta} \times \mathbf{E} \right]$$

$$F_e = qE$$

$$F_b = qc(\boldsymbol{\beta} \times \mathbf{B})$$

$$E = c(\boldsymbol{\beta} B_{rad})$$

Net contribution from B_{rad} and compensating E_{vert}

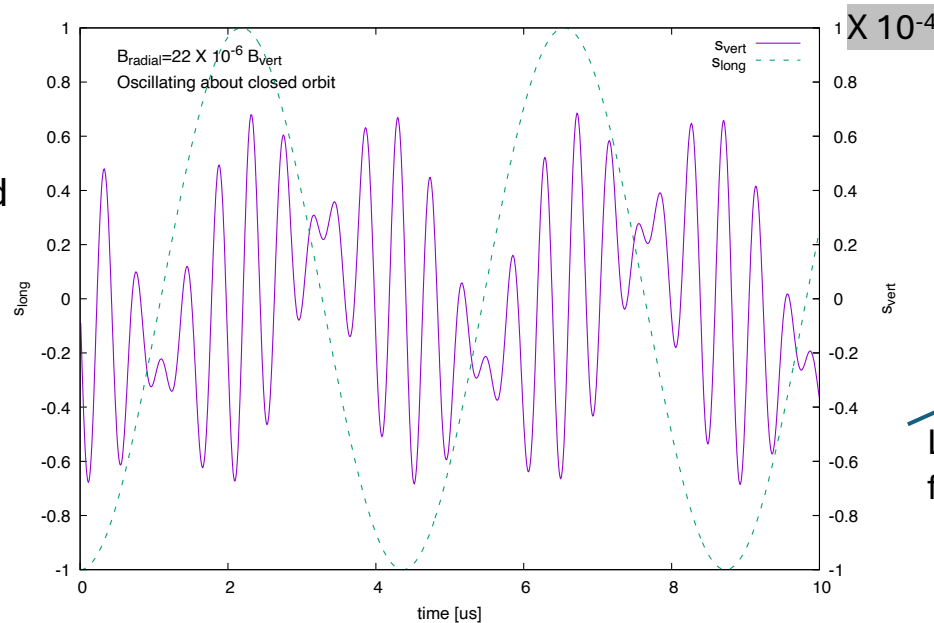
$$\frac{\left(\frac{1}{\gamma} + a \right) c B_{rad} - \left(a + \frac{1}{1 + \gamma} \right) \beta E_{vert}}{\left(\frac{1}{\gamma} + a \right) c B_{rad}} \sim \frac{1}{\gamma}$$

$$B_{\text{radial}} = 22 \text{ ppm, EDM}=0$$

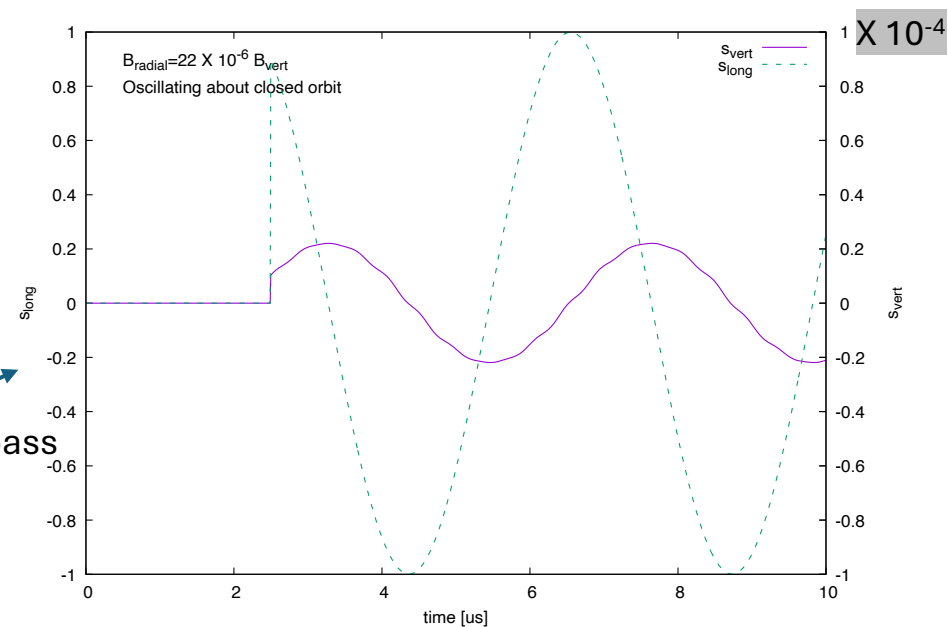
Oscillation about the displaced
vertical closed orbit

Initial phase space coordinates

$$x = x' = y = y' = 0$$



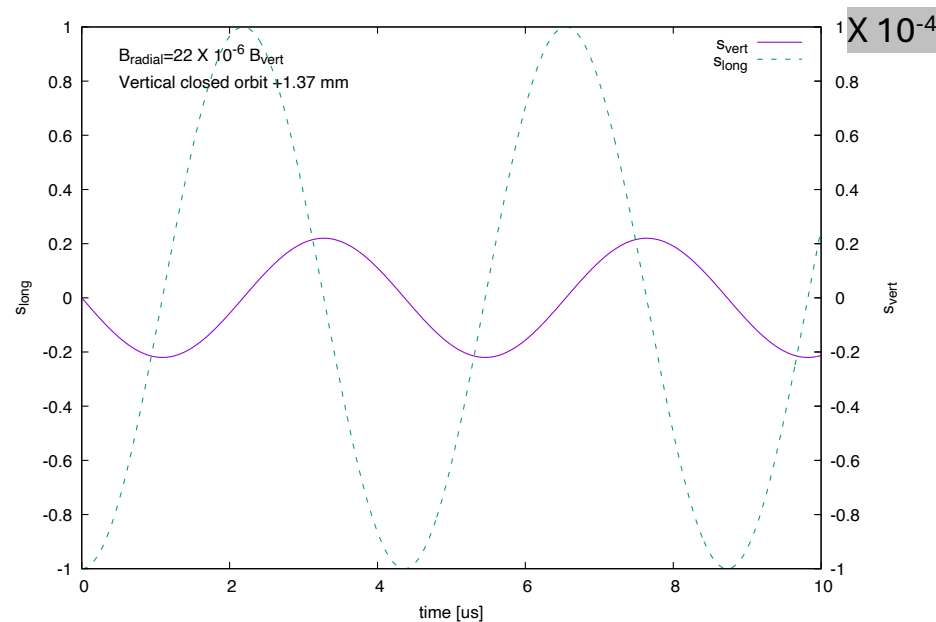
Lowpass
filter



On the displaced closed orbit

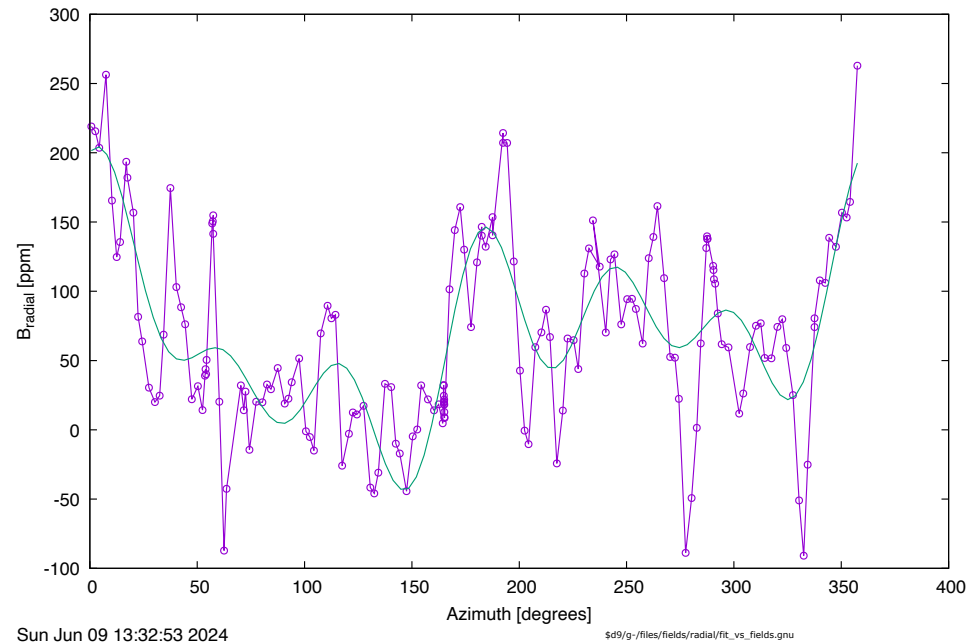
Initial phase space coordinates

$$y = 1.37\text{mm}, x = x' = y' = 0$$



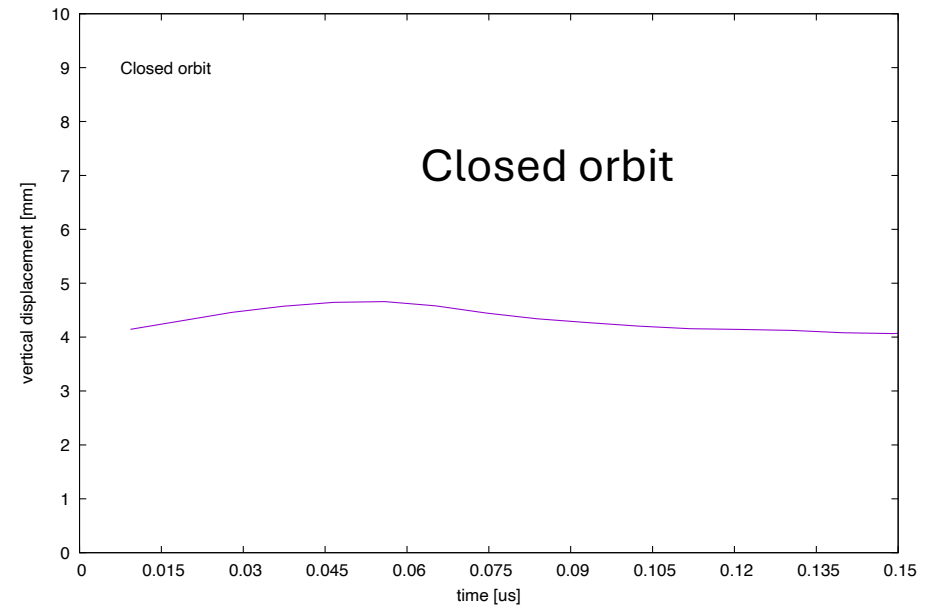
Measured radial field

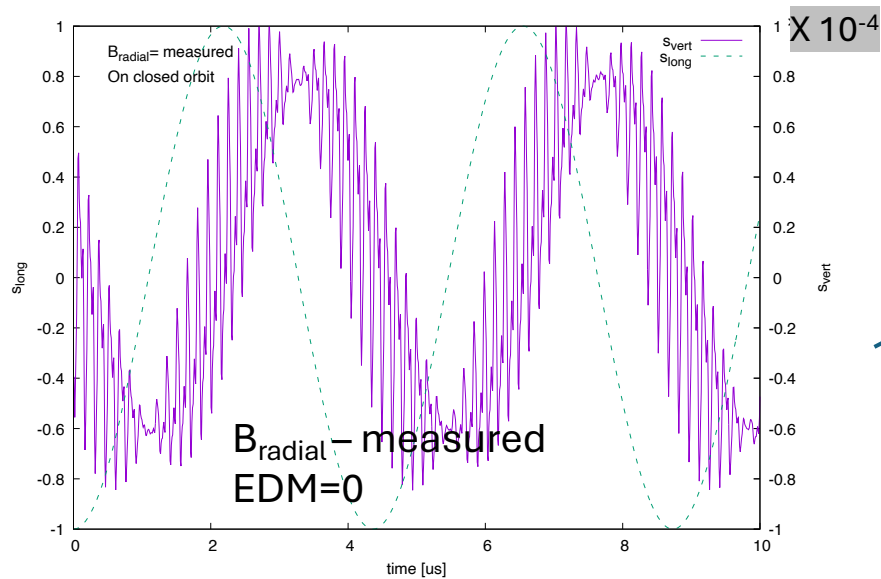
$$\langle B_{radial} \rangle = 69.6 \text{ ppb}$$



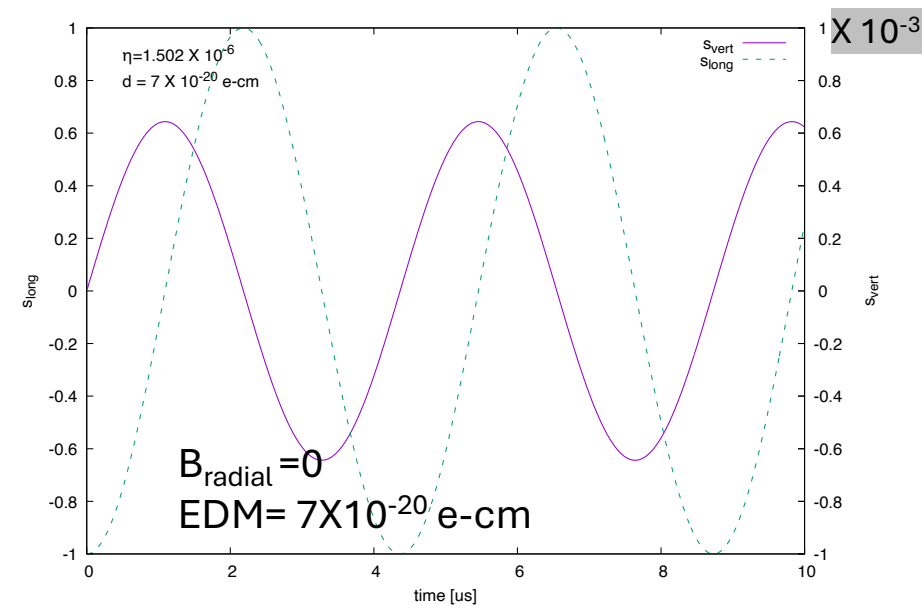
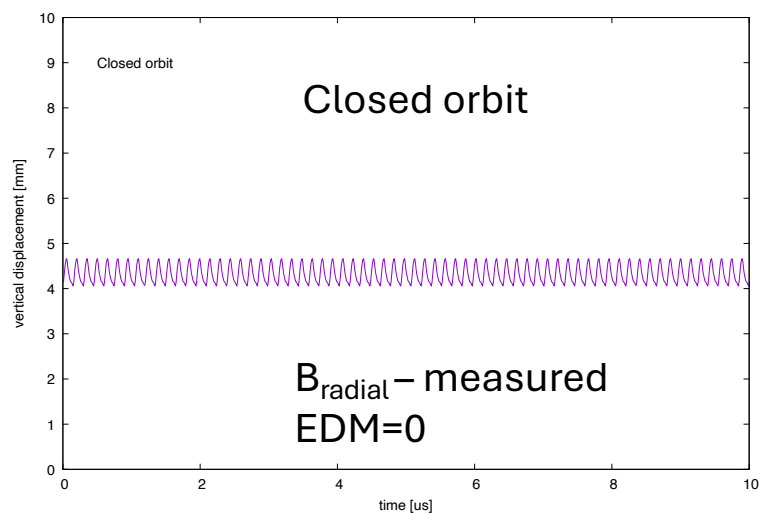
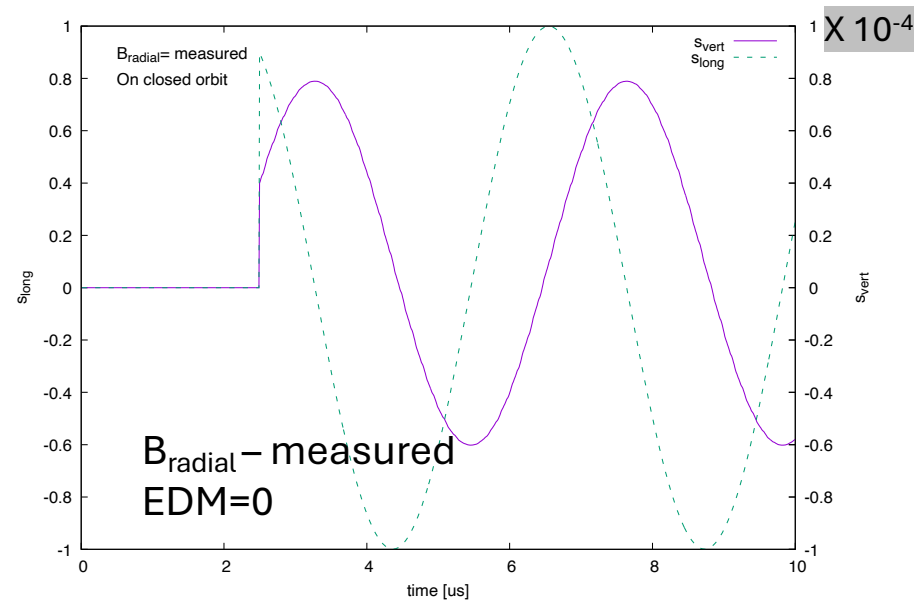
Smooth curve is fit to

$$B_{radial}(\theta) = \sum_{i=0}^5 A_i \cos\left(2\pi i \frac{\theta}{360} + \phi_i\right)$$

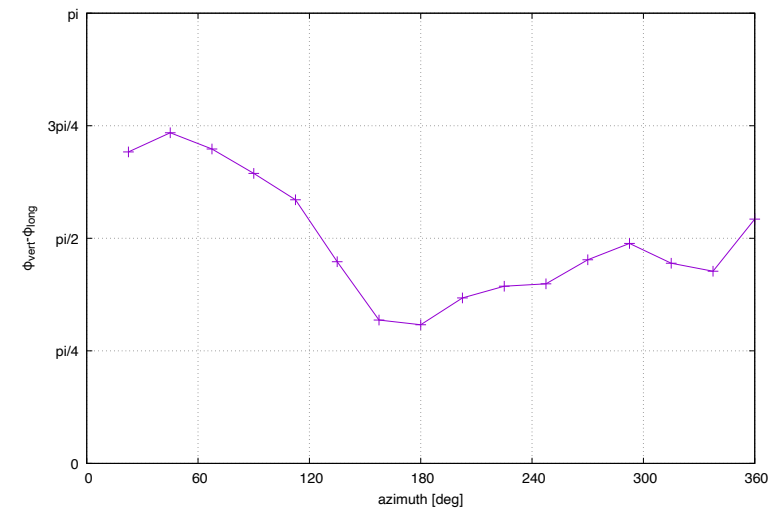




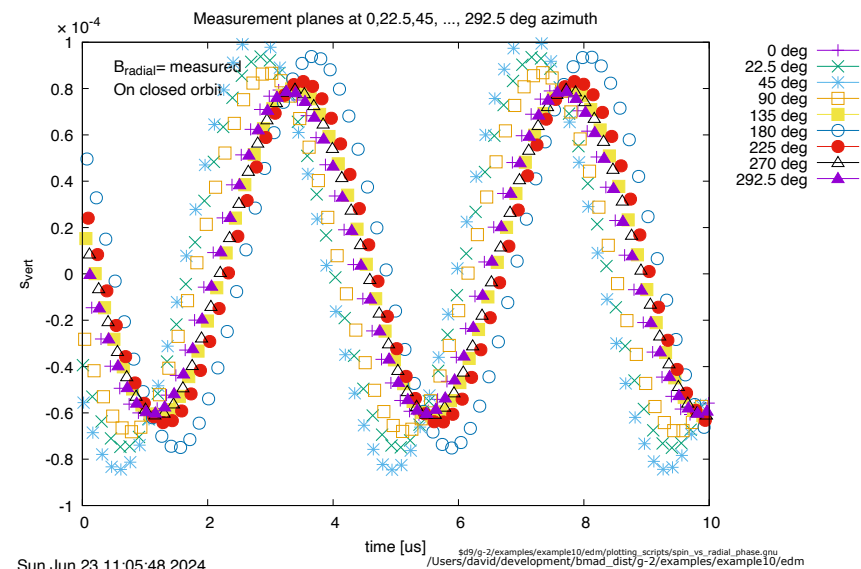
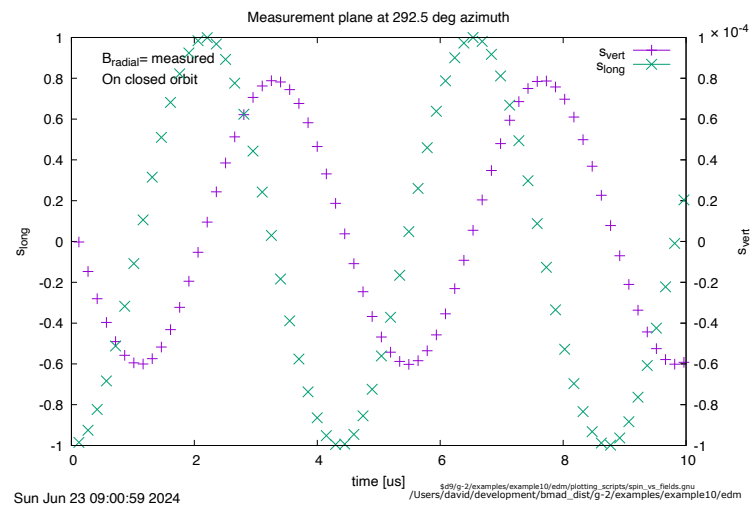
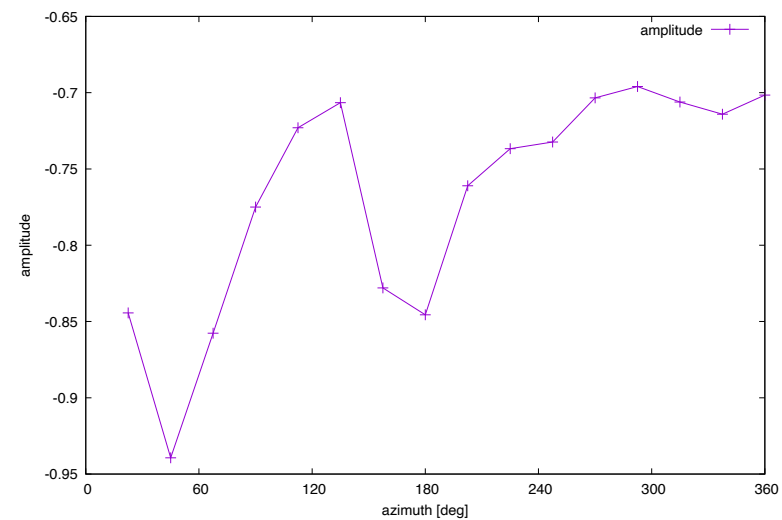
Lowpass
filter



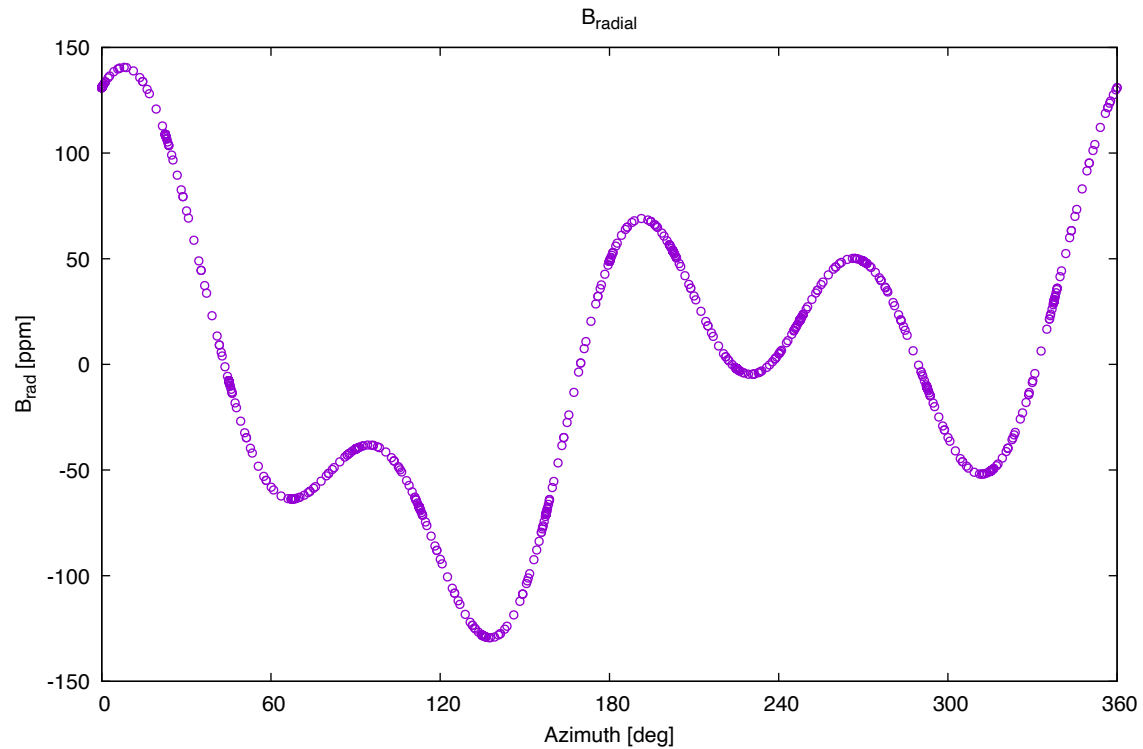
Phase of vertical polarization vs azimuthal 'measurement' plane



Amplitude of vertical polarization vs azimuthal 'measurement' plane

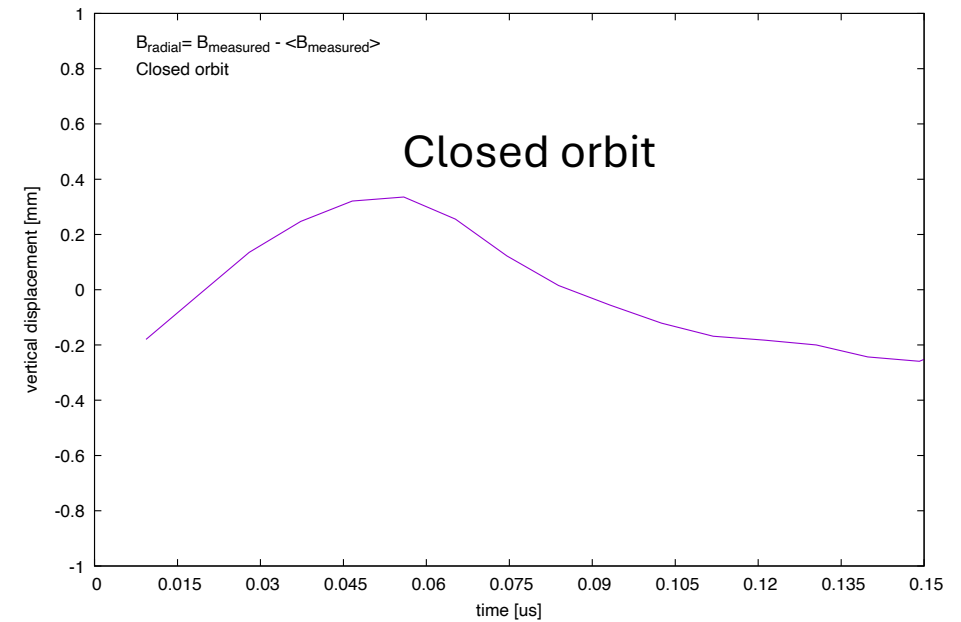


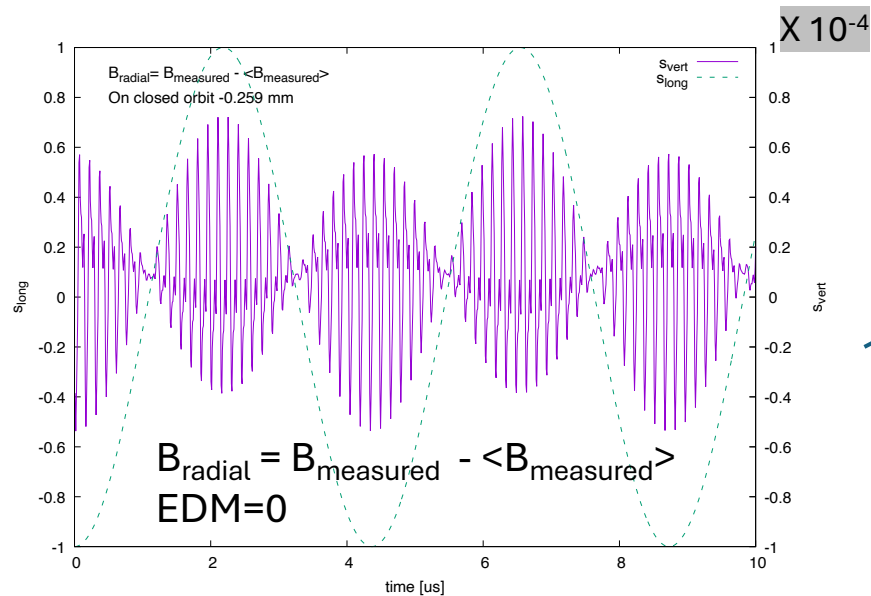
$$B_{radial} = B_{measured} - \langle B_{measured} \rangle$$



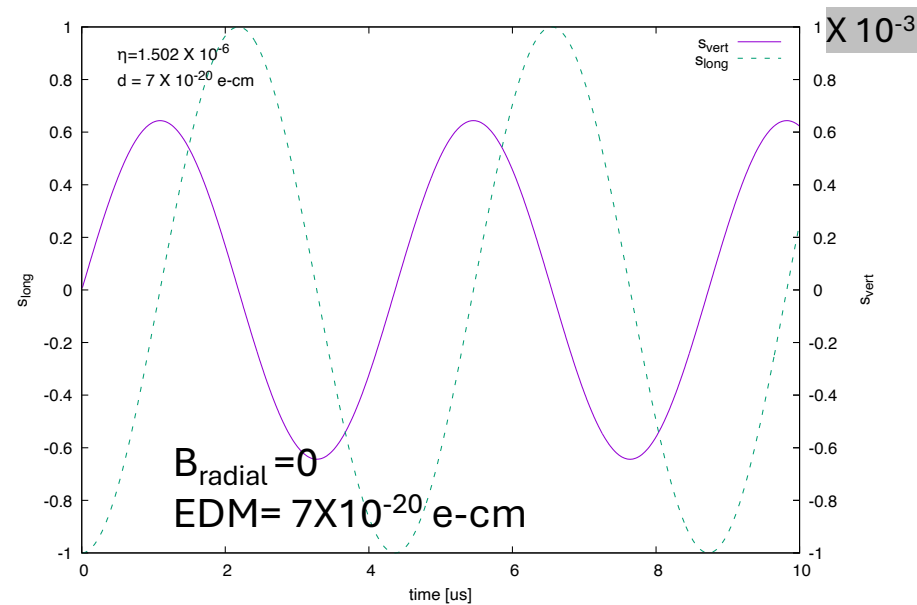
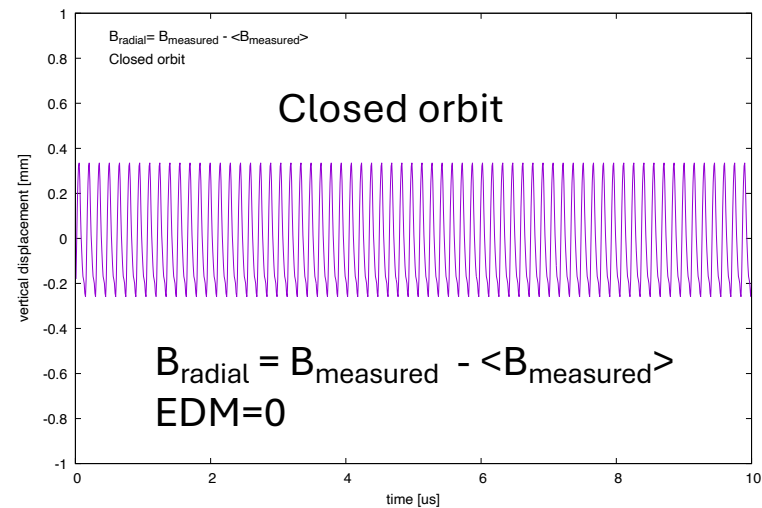
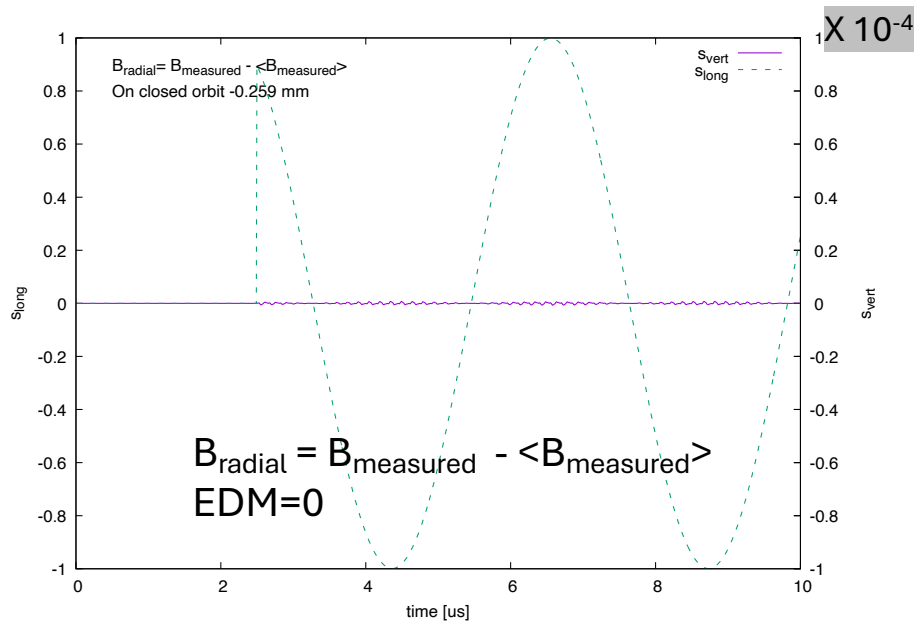
$$B_{radial}(\theta) = \sum_{i=0}^5 A_i \cos\left(2\pi i \frac{\theta}{360} + \phi_i\right)$$

$$\text{Set } A_0 = 0 \Rightarrow \langle B_{rad} \rangle = 0$$

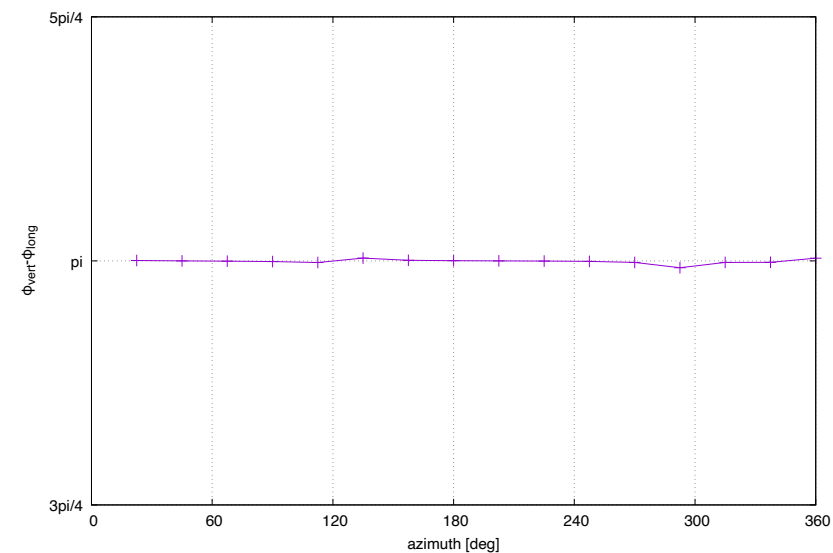




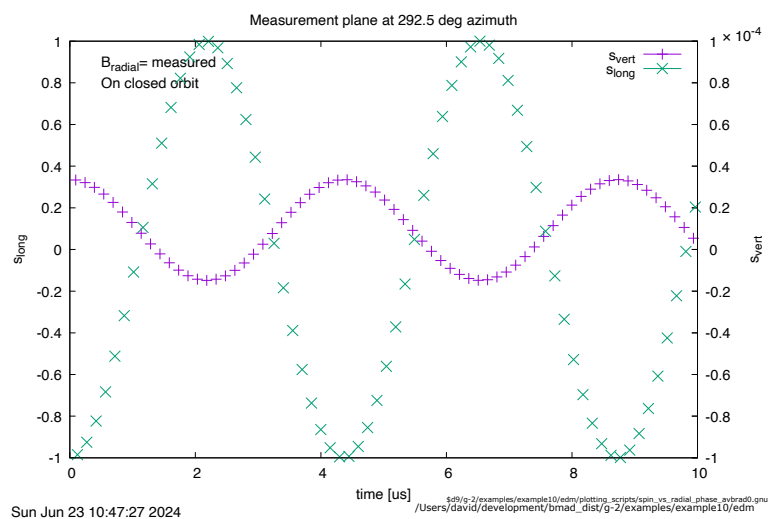
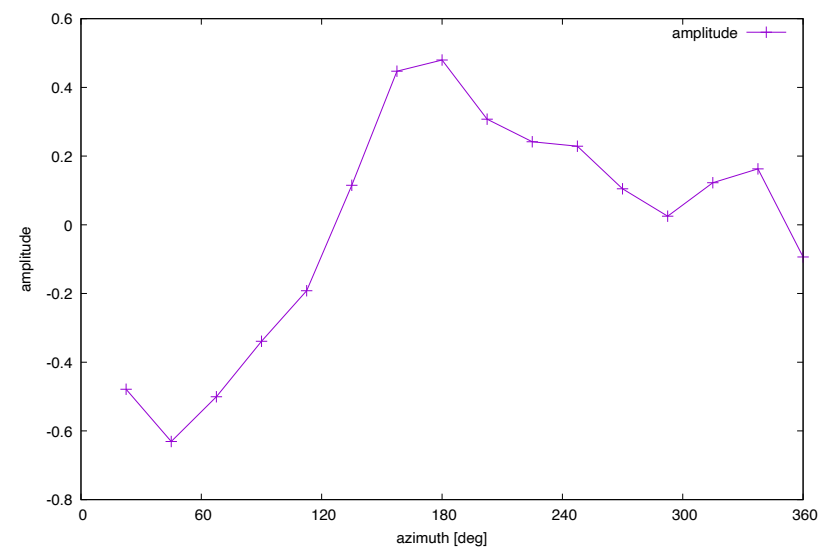
Lowpass
filter



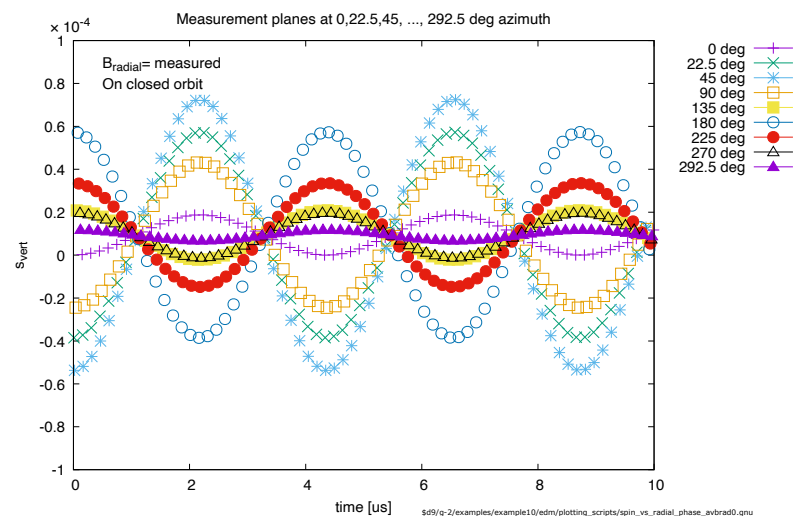
Phase of vertical polarization vs azimuthal 'measurement' plane



Amplitude of vertical polarization vs azimuthal 'measurement' plane



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Sun Jun 23 10:58:13 2024