EDM systematic uncertainties due to radial and longitudinal B-fields and pitch – spin tracking

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July 2, 2024

Continuous quad approximation
No nonlinearity

$$\frac{\mathrm{d}}{\mathrm{d}s}\mathbf{S} = \left\{ \frac{(1+\mathbf{r}_t\cdot\mathbf{g})}{c\,\beta_z}\,(\mathbf{\Omega}_{BMT}+\mathbf{\Omega}_{EDM}) - \mathbf{g}\times\hat{\mathbf{z}} \right\}\times\mathbf{S}$$

$$\mathbf{\Omega}_{BMT}(\mathbf{r},\mathbf{P},t) = -\frac{q}{m\,c}\left[\left(\frac{1}{\gamma}+a\right)c\,\mathbf{B} - \frac{a\,\gamma\,c}{1+\gamma}\,(\boldsymbol{\beta}\cdot\mathbf{B})\,\boldsymbol{\beta} - \left(a+\frac{1}{1+\gamma}\right)\boldsymbol{\beta}\times\mathbf{E}\right]$$

$$\mathbf{\Omega}_{EDM}(\mathbf{r},\mathbf{P},t) = -\frac{q\,\eta}{2\,m\,c}\left[\mathbf{E} - \frac{\gamma}{1+\gamma}\,(\boldsymbol{\beta}\cdot\mathbf{E})\,\boldsymbol{\beta} + c\,\boldsymbol{\beta}\times\mathbf{B}\right]$$

$$\mathbf{d} = \frac{\eta}{2}\,\frac{q}{m\,c}\,\mathbf{S}$$

$$\mathbf{d}[\mathrm{e-cm}] = 4.66\times 10^{-14}\eta$$

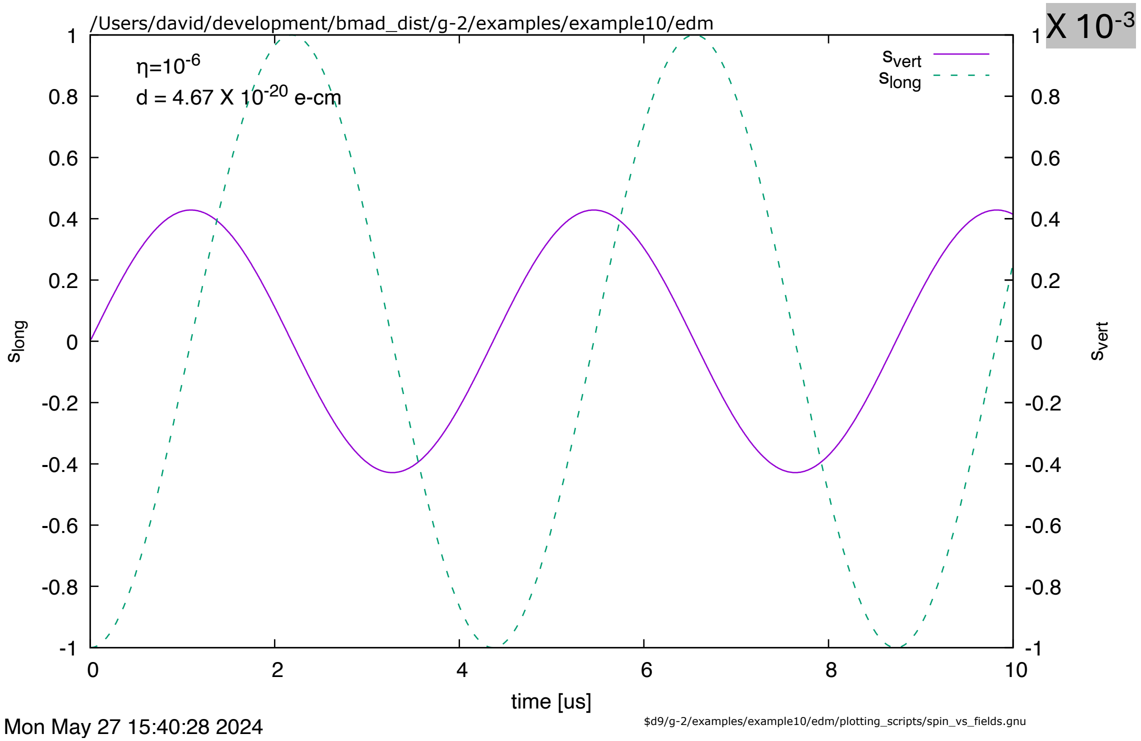
Single particle spin tracking

Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0,\ s_{long} = 1$$



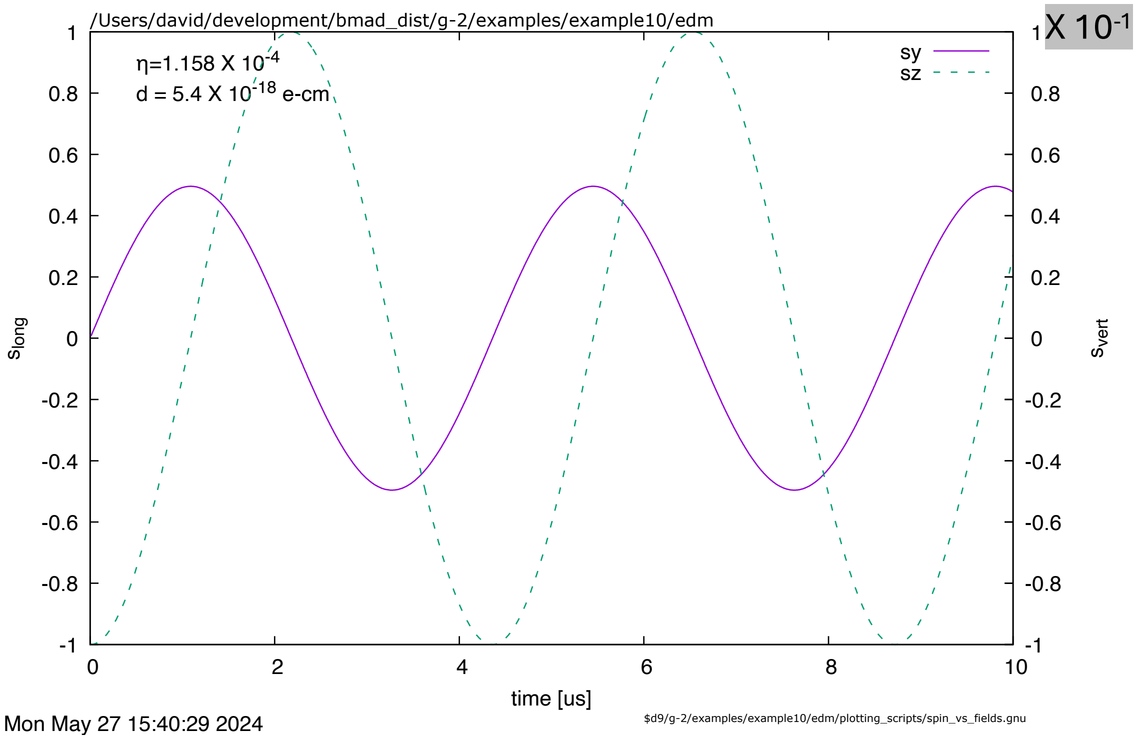
Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0,\ s_{long} = 1$$

$$d = 5.4 \times 10^{-18}\mathrm{e-cm}$$



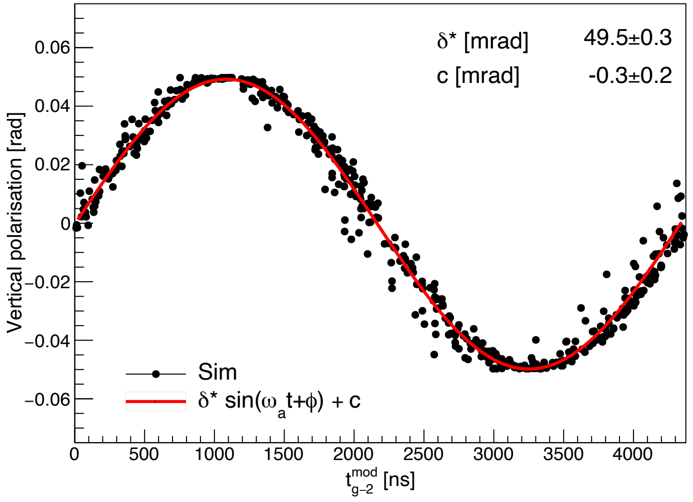


Figure 7: The time modulated average vertical component of the rest frame muon polarisation vector, with an injected EDM of 5.4×10^{-18} $e\cdot$cm (30× BNL). The amplitude of the fit gives a rest frame tilt angle of 49.5±0.3 mrad (1.69±0.01 mrad in the laboratory frame), which is consistent with expectation.

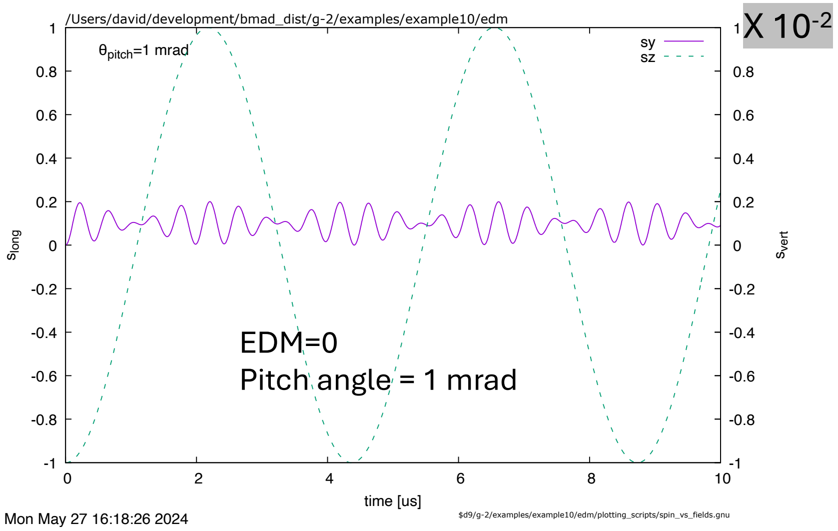
Pitch

Initial phase space coordinates

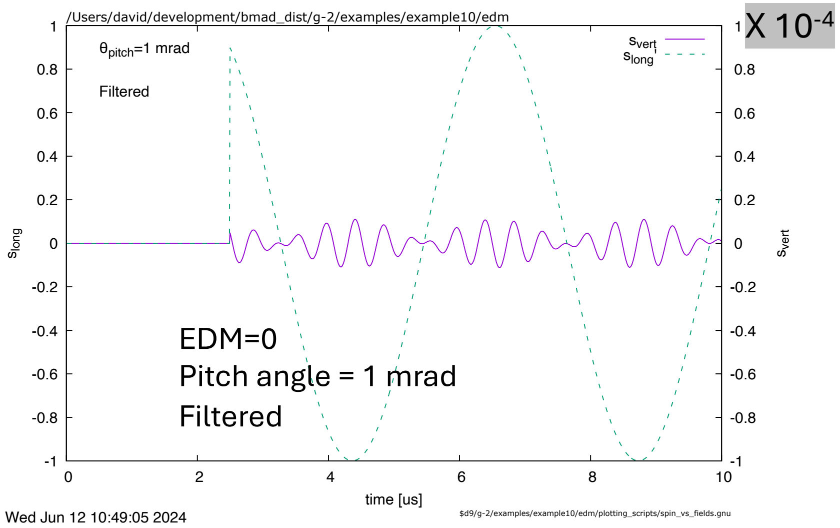
$$x = x' = y = 0, \; y' = 1 \text{ mrad}$$

Initial polarization

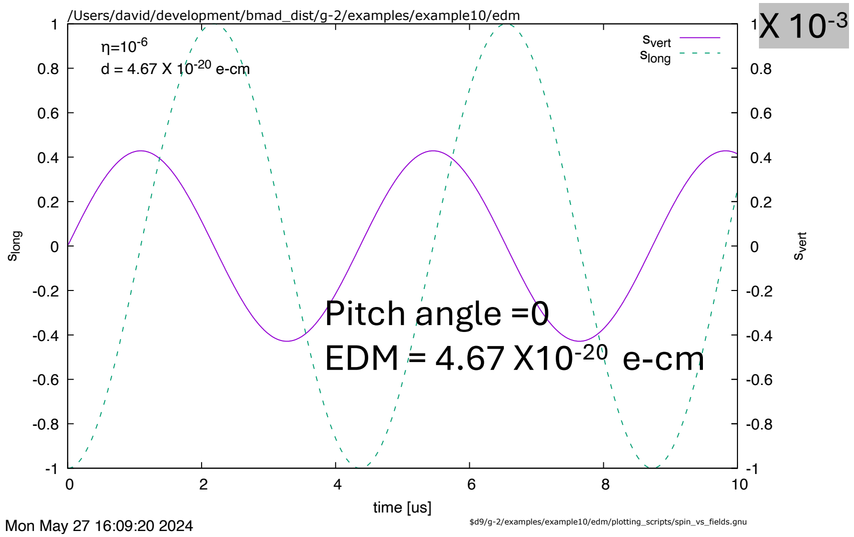
$$s_{rad} = s_{vert} = 0, \; s_{long} = 1$$

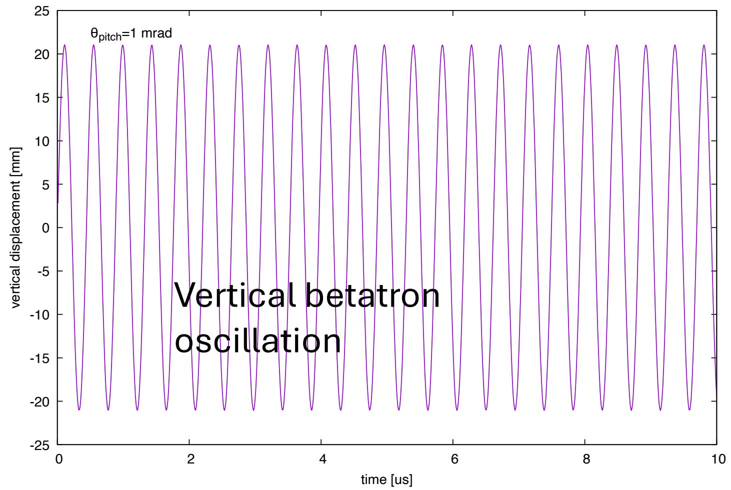


Lowpass filter



EDM reference





(Longitudinal polarization refers to left hand axis labels. Vertical polarization refers to right hand axis. Note exponent.)

Longitudinal magnetic field

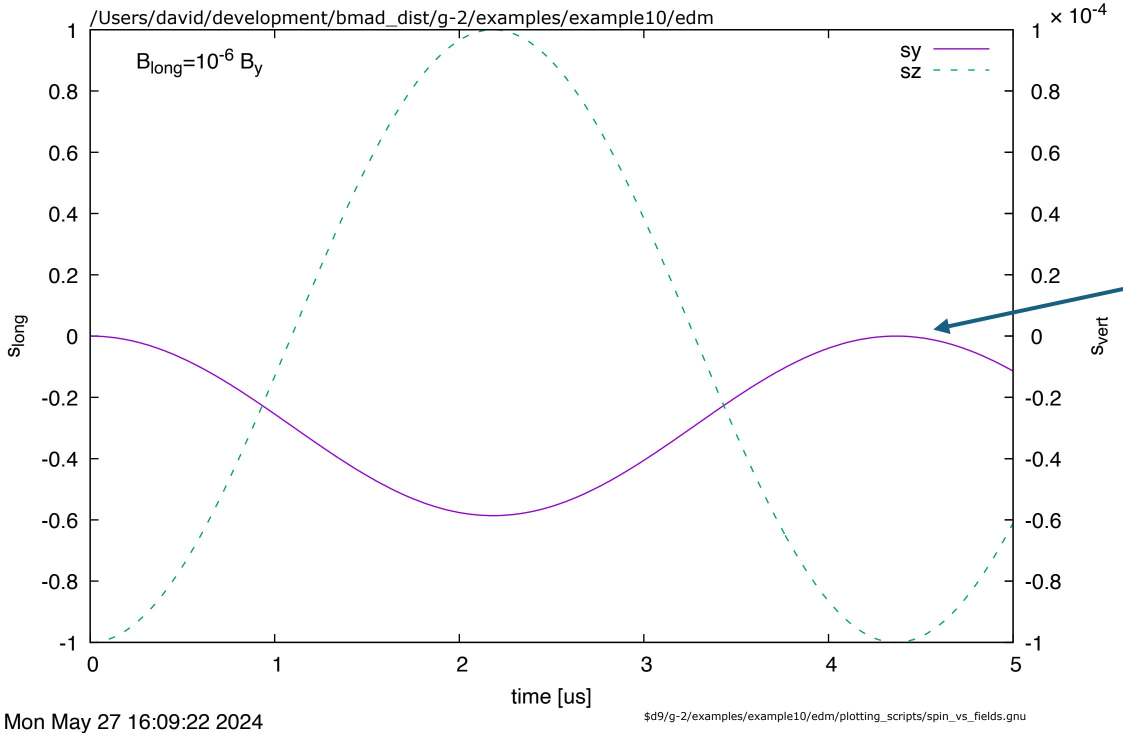
Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0,\ s_{long} = 1$$

$$B_{long} = 1 \text{ ppm}$$ uniform around ring



$$\langle\theta_y\rangle(t) = \frac{1}{N(t)}\left(A_{g-2}\cos(\omega_a t + \phi) + A_{\rm EDM}\sin(\omega_a t + \phi) + c\right)$$

Radial B-field - EDM equivalency

$$\mathbf{\Omega}_{BMT}(\mathbf{r},\mathbf{P},t) = -\frac{q}{m\,c}\left[\left(\frac{1}{\gamma}+a\right)c\,\mathbf{B} - \frac{a\,\gamma\,c}{1+\gamma}\,(\boldsymbol{\beta}\cdot\mathbf{B})\,\boldsymbol{\beta} - \left(a+\frac{1}{1+\gamma}\right)\boldsymbol{\beta}\times\mathbf{E}\right]$$

$$\mathbf{\Omega}_{EDM}(\mathbf{r},\mathbf{P},t) = -\frac{q\,\eta}{2\,m\,c}\left[\mathbf{E} - \frac{\gamma}{1+\gamma}\,(\boldsymbol{\beta}\cdot\mathbf{E})\,\boldsymbol{\beta} + c\,\boldsymbol{\beta}\times\mathbf{B}\right]$$

$$\left(\frac{1}{\gamma}+a\right)B_{radial} \leftrightarrow \frac{\eta}{2}(\beta\times\mathbf{B})$$

$$\mathbf{d}[\mathrm{e-cm}] = 4.66\times10^{-14}\eta$$

$$\left(\frac{1}{\gamma}+a\right)\frac{B_{radial}}{B} \quad\leftrightarrow\quad 0.215\times10^{14}\frac{d}{2}[\mathrm{e-cm}]$$

$$\frac{B_{radial}}{B} \quad\leftrightarrow\quad 6.29\times10^{14}\frac{d}{2}[\mathrm{e-cm}]$$

Dataset	$\langle B_r\rangle$ [ppm]	Equivalent d_μ [$\times10^{-20}$ e·cm]
1a	22 ± 7	7 ± 2
1b	23 ± 8	7 ± 3
1c	30 ± 8	9 ± 3
1d	34 ± 9	10 ± 3

Table 15: Estimates for $\langle B_r\rangle$ in ppm, as well as the equivalent fake EDM signal in e·cm, for various E989 datasets [33][2].

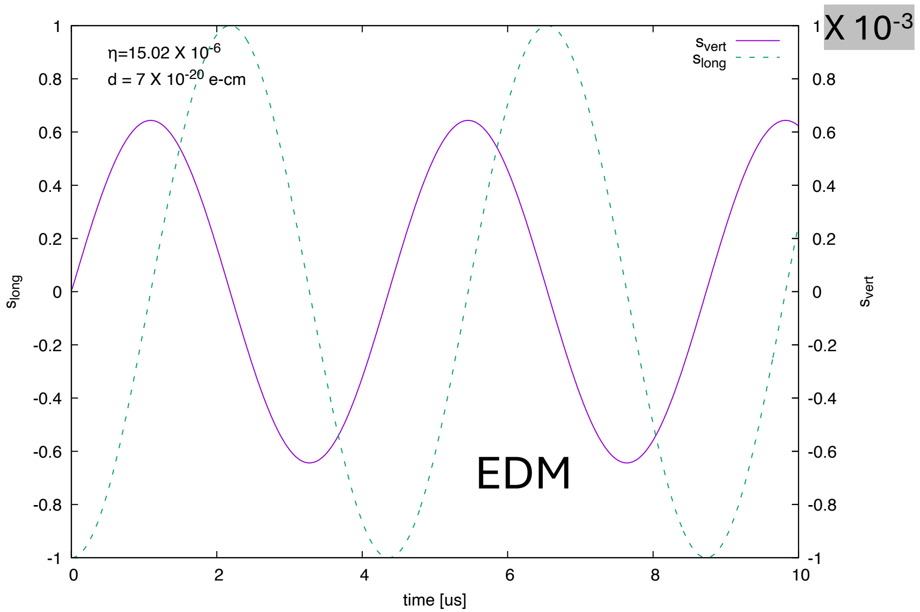
Initial phase space coordinates

$$x = x' = y = y' = 0$$

Initial polarization

$$s_{rad} = s_{vert} = 0,\ s_{long} = 1$$

$$d = 7 \times 10^{-20}\mathrm{e-cm},\ B_{radial} = 0$$



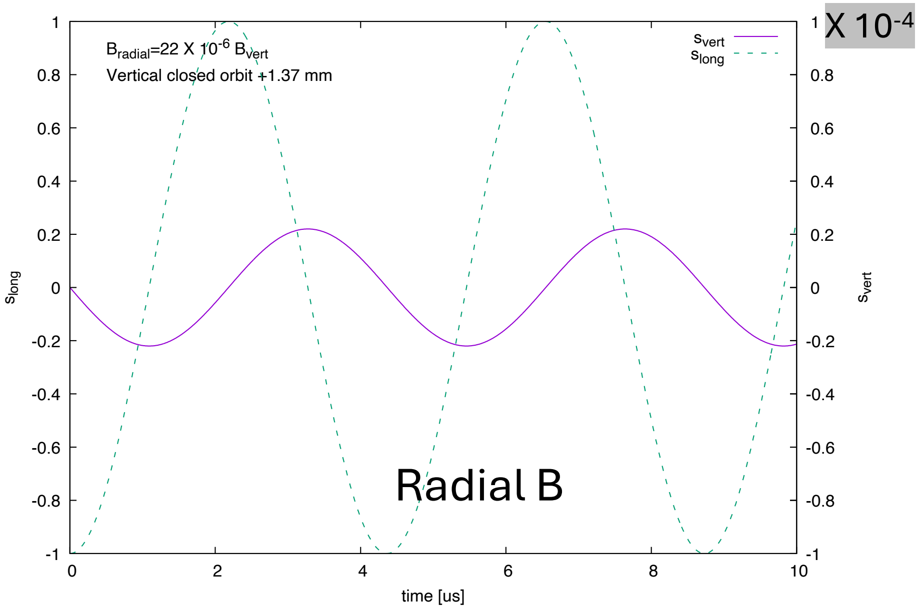
Initial phase space coordinates

$$y = \ 1.37\mathrm{mm},\ x = x' = y' = 0$$ *Closed orbit displaced vertically by radial B field*

Initial polarization

$$s_{rad} = s_{vert} = 0,\ s_{long} = 1$$

$$B_{radial} = 22\ \mathrm{ppm},\ d = 0$$



$$\mathbf{\Omega}_{BMT}(\mathbf{r},\mathbf{P},t) = -\frac{q}{m\,c}\left[\left(\frac{1}{\gamma}+a\right)c\,\mathbf{B} - \frac{a\,\gamma\,c}{1+\gamma}(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta} - \left(a+\frac{1}{1+\gamma}\right)\boldsymbol{\beta}\times\mathbf{E}\right]$$

$$F_e = qE$$
$$F_b = qc(\beta \times \mathbf{B})$$
$$E = c(\beta B_{rad})$$

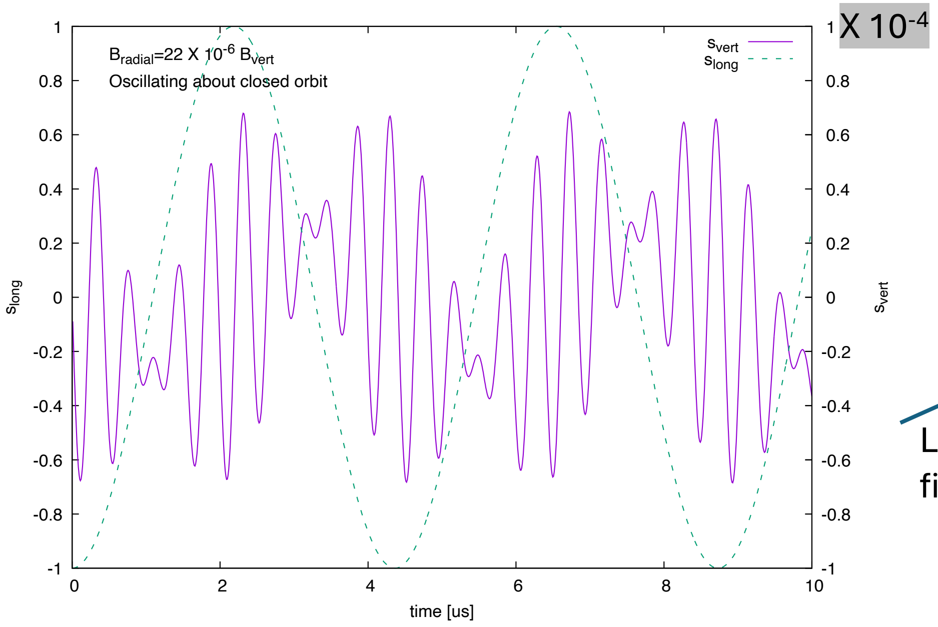
Net contribution from B_{rad} and compensating E_{vert}

$$\frac{\left(\frac{1}{\gamma}+a\right)cB_{rad} - \left(a+\frac{1}{1+\gamma}\right)\beta E_{vert}}{\left(\frac{1}{\gamma}+a\right)cB_{rad}} \sim \frac{1}{\gamma}$$

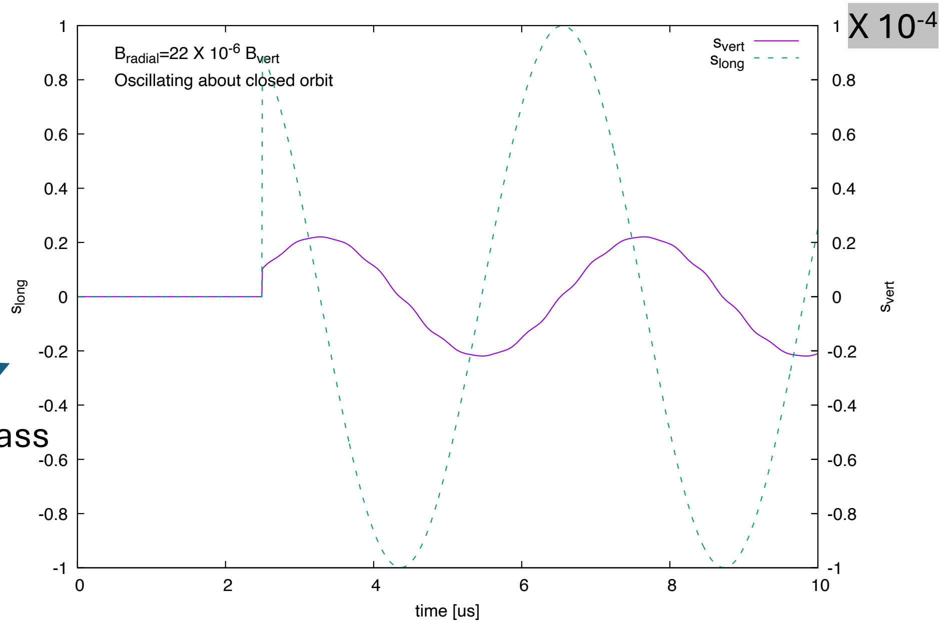
$B_{radial} = 22$ ppm, EDM=0

Oscillation about the displaced vertical closed orbit

Initial phase space coordinates
$x = x' = y = y' = 0$

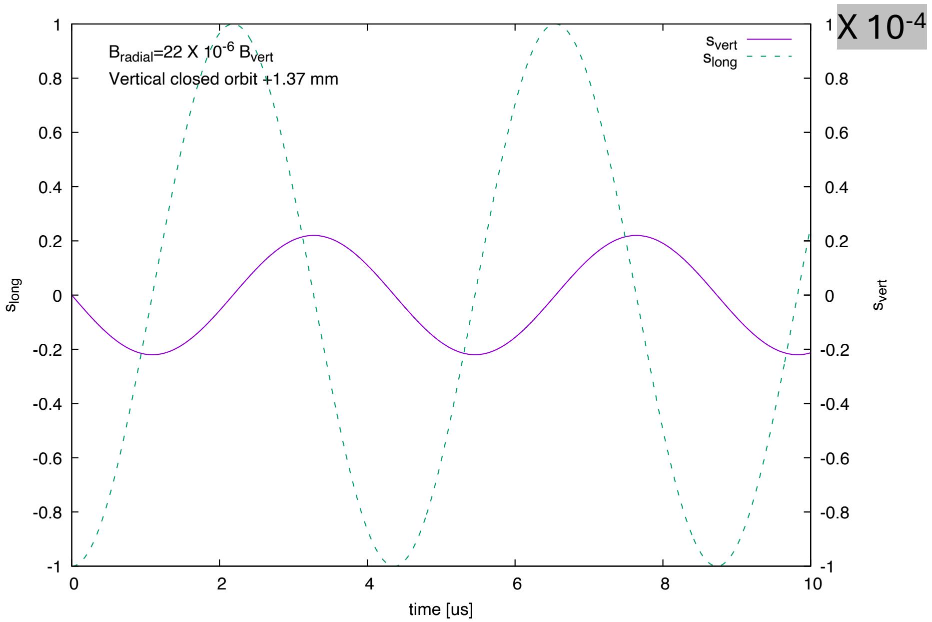


Lowpass filter



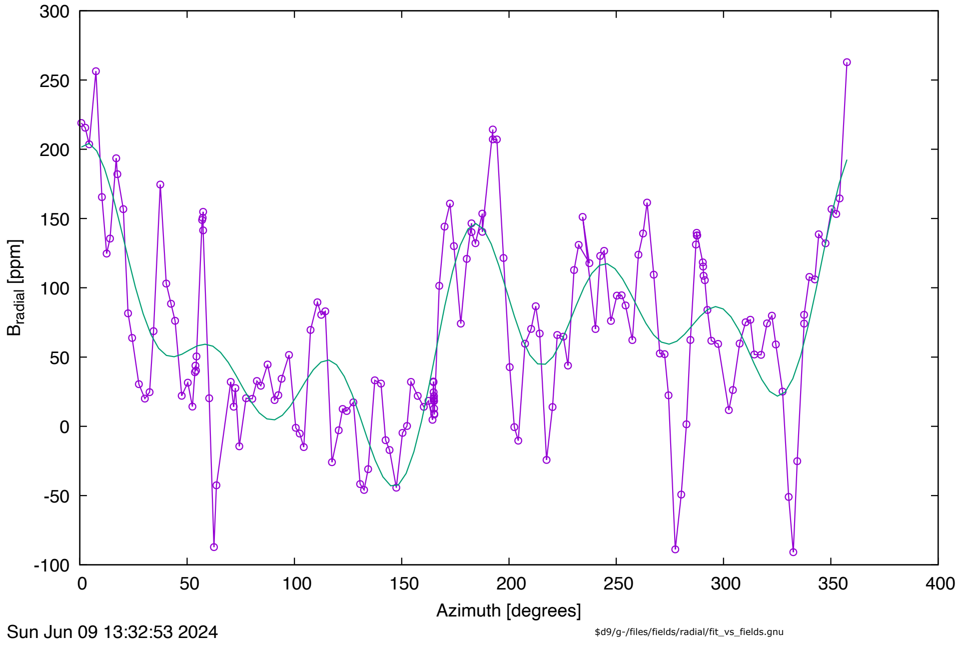
On the displaced closed orbit

Initial phase space coordinates
$y = 1.37\text{mm},\ x = x' = y' = 0$



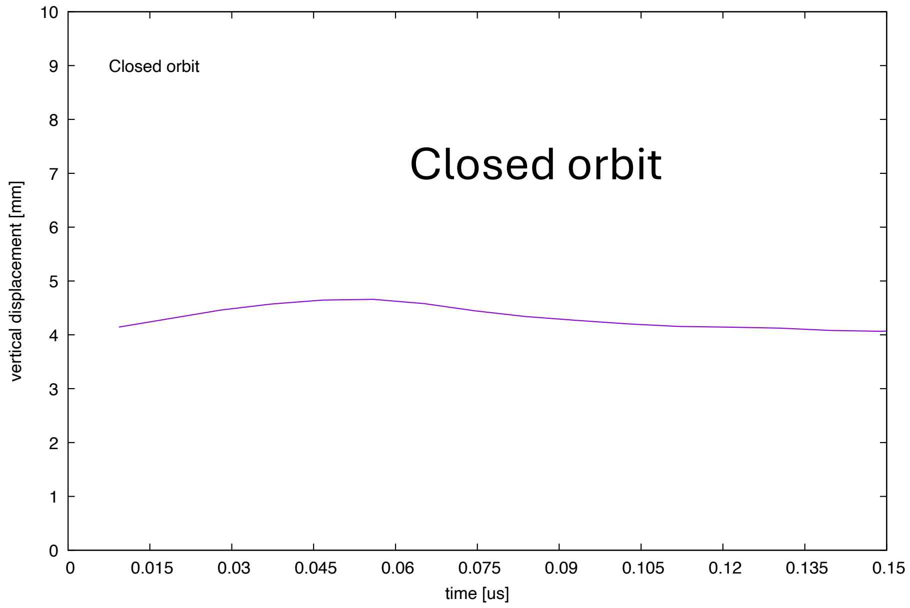
Measured radial field

$$\langle B_{radial} \rangle = 69.6 \text{ ppb}$$

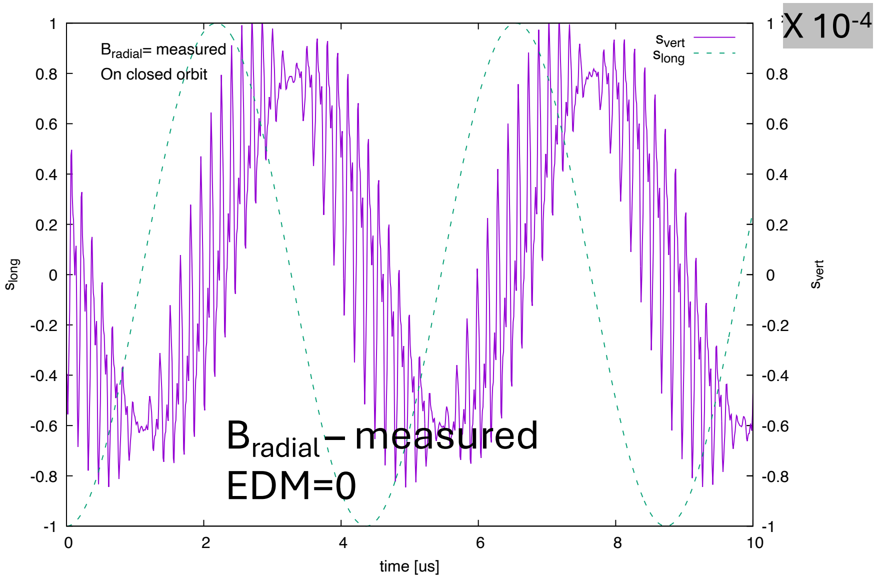


Smooth curve is fit to

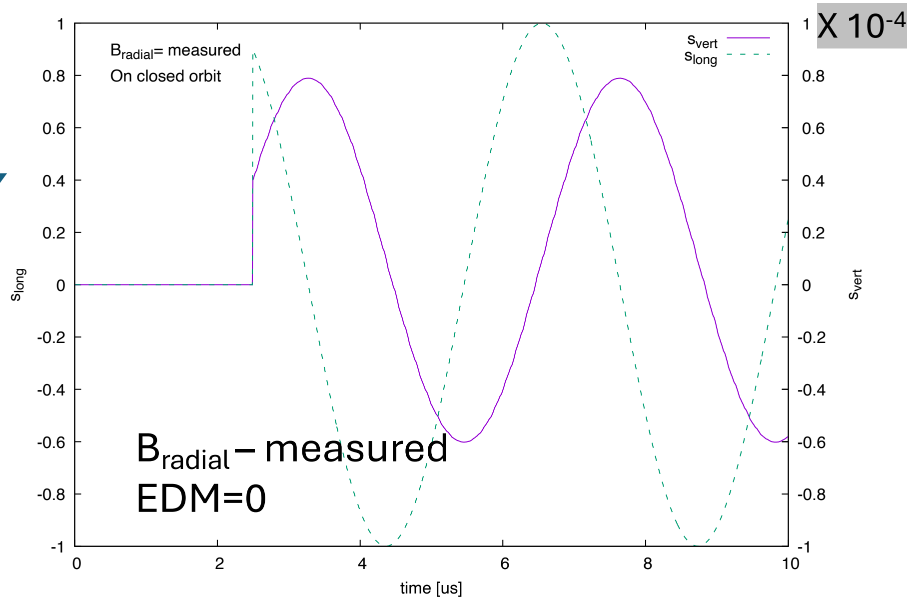
$$B_{radial}(\theta) = \sum_{i=0}^{5} A_i \cos(2\pi i \frac{\theta}{360} + \phi_i)$$

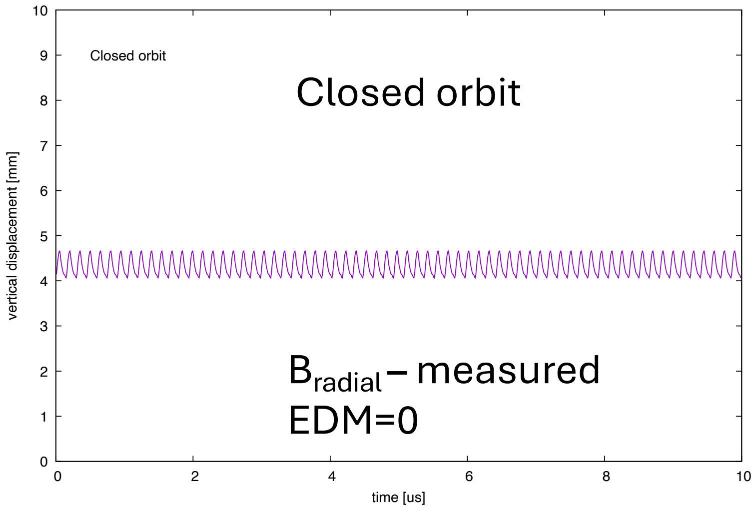


Radial magnetic field $\langle B_{radial} \rangle = 69.6 \text{ ppb}$

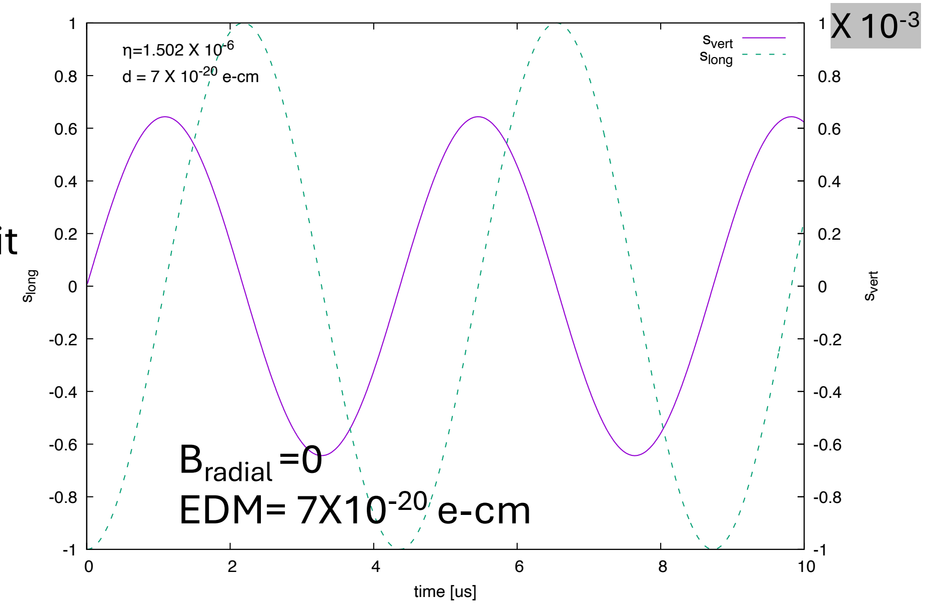


Lowpass filter





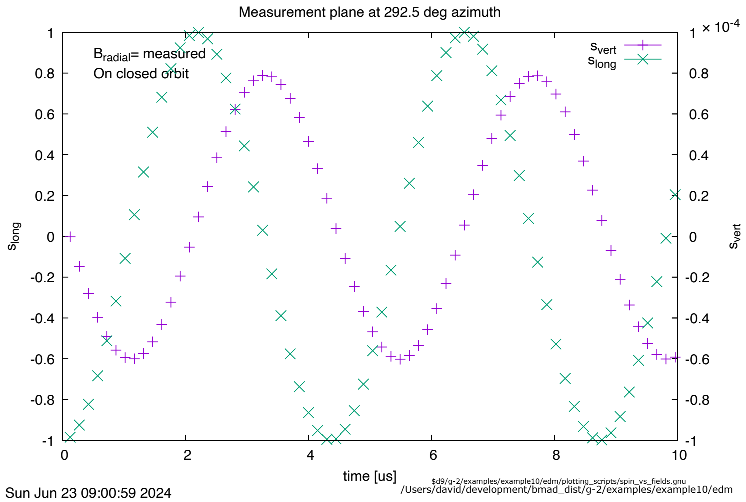
Target EDM limit

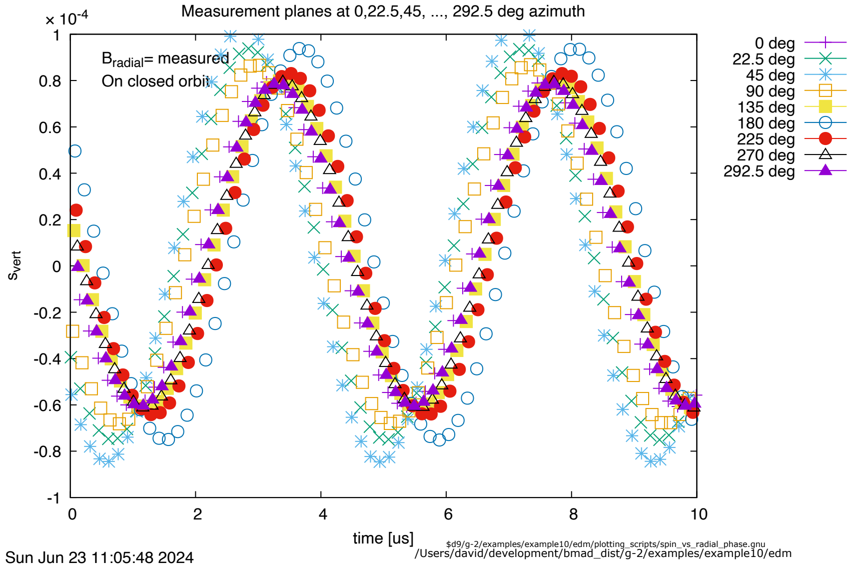


Radial magnetic field

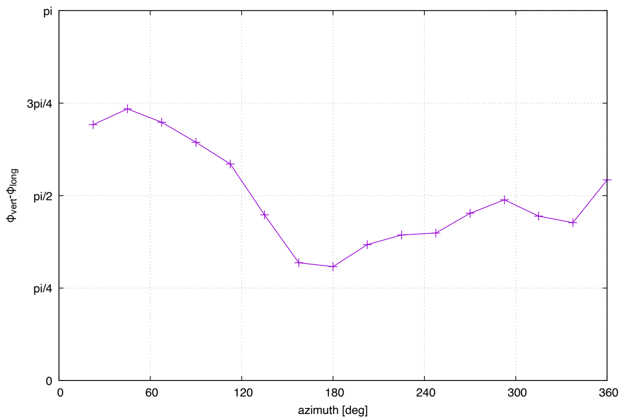
$$\langle B_{radial} \rangle = 69.6 \text{ ppb}$$

Detector location

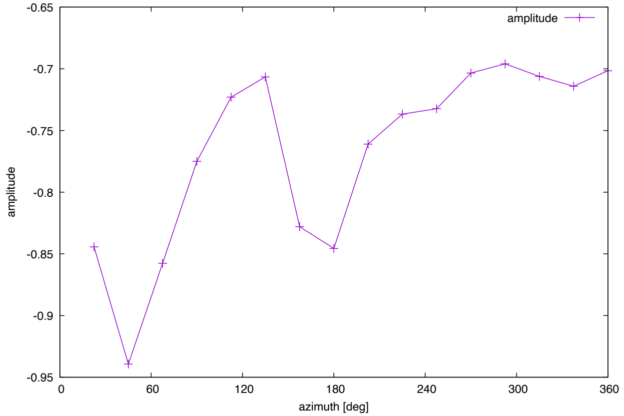




Phase of vertical polarization vs azimuthal 'measurement' plane



Amplitude of vertical polarization vs azimuthal 'measurement' plane

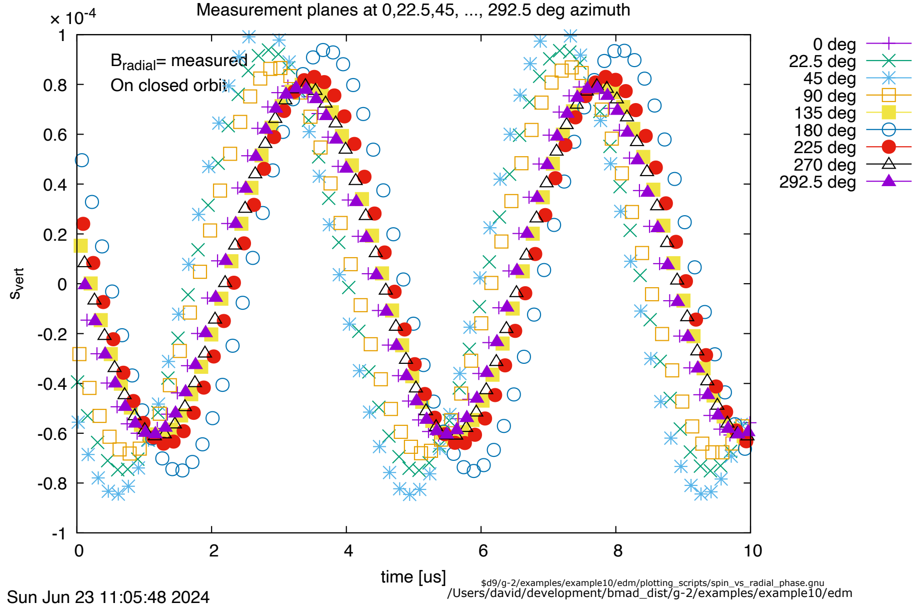


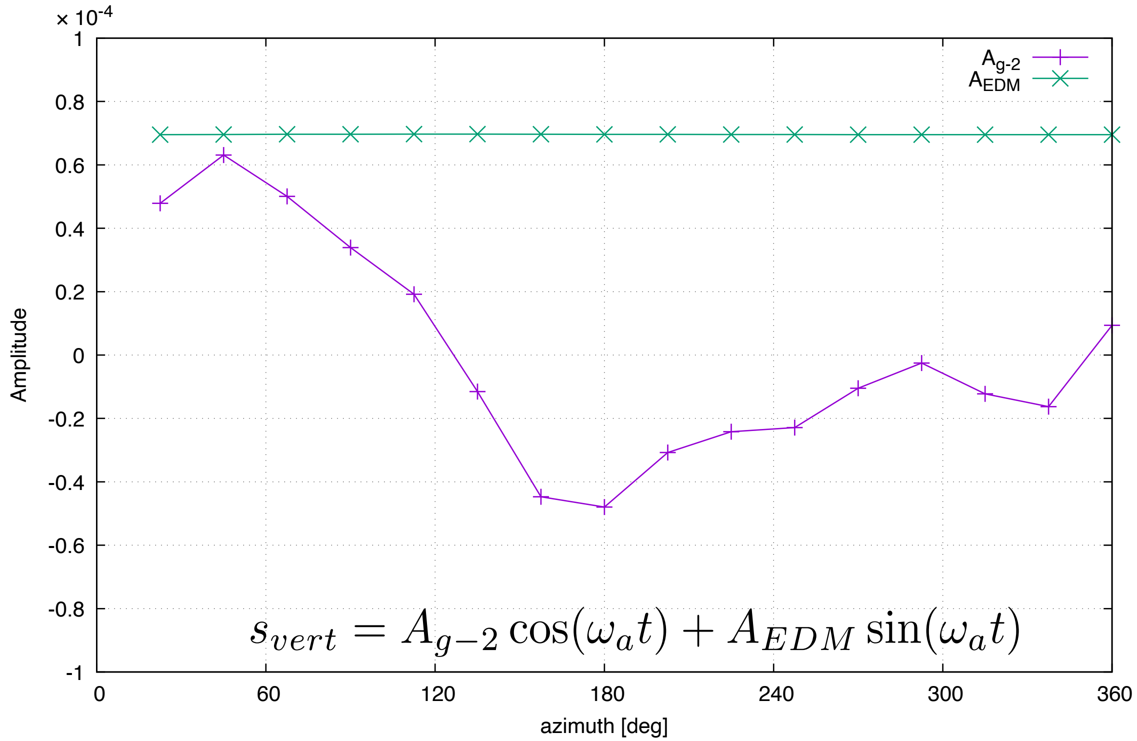
Detector location

Radial magnetic field

$$\langle B_{radial} \rangle = 69.6 \text{ ppb}$$

$$\langle \theta_y \rangle(t) = \frac{1}{N(t)}(A_{g-2}\cos(\omega_a t + \phi) + A_{\rm EDM}\sin(\omega_a t + \phi) + c). \quad (24)$$



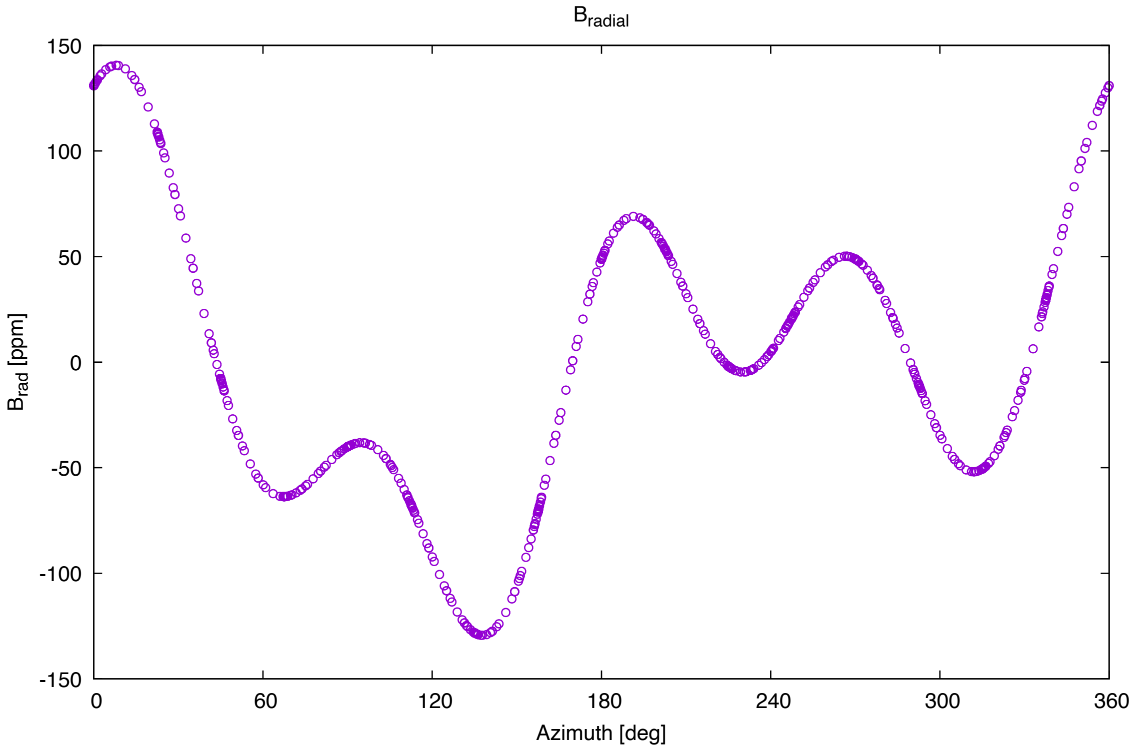


$$s_{vert} = A_{g-2}\cos(\omega_a t) + A_{EDM}\sin(\omega_a t)$$

EDM-like signal from radial field depends only on <B_{radial}> and is independent of detector location

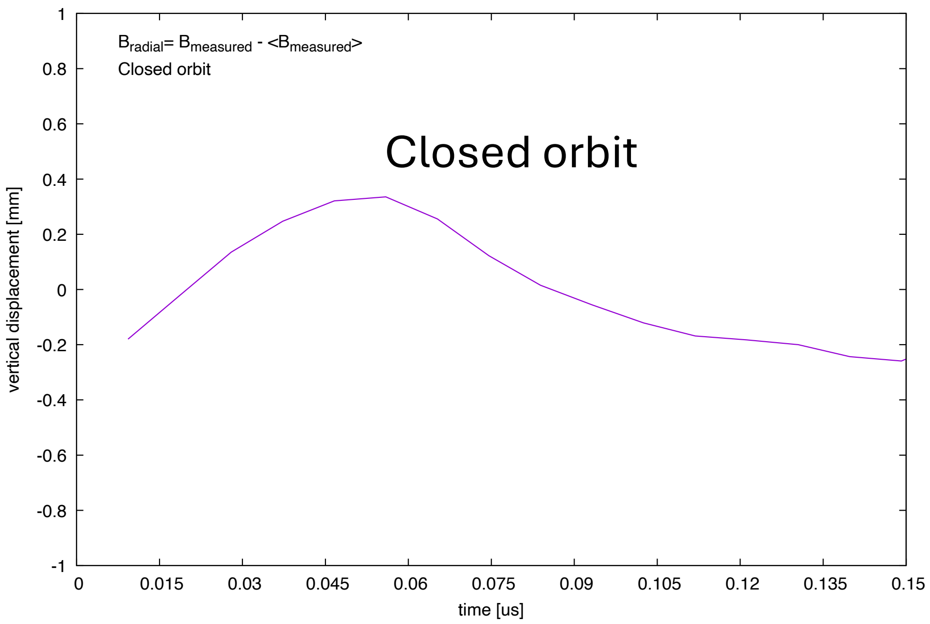
Set average radial field to zero

$$B_{radial} = B_{measured} - \langle B_{measured} \rangle$$

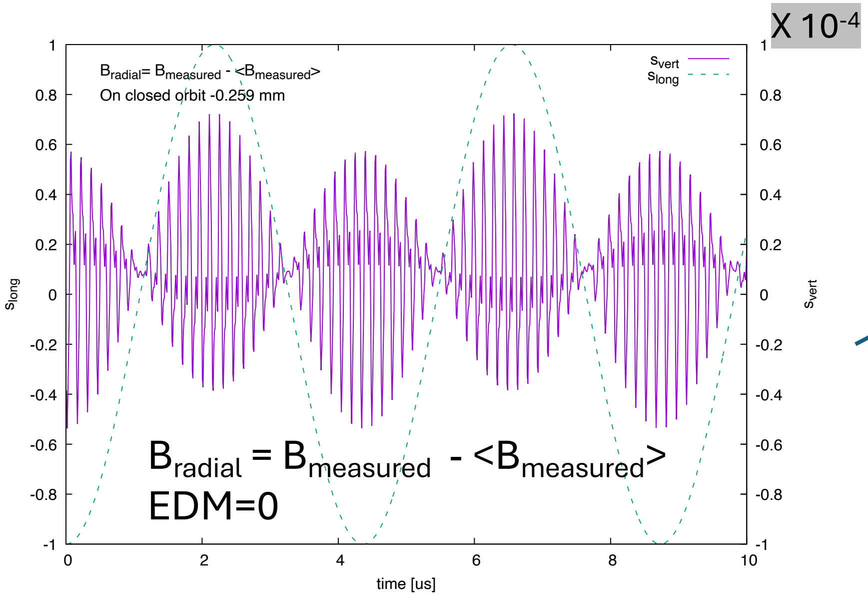


$$B_{radial}(\theta) = \sum_{i=0}^{5} A_i \cos(2\pi i \frac{\theta}{360} + \phi_i)$$

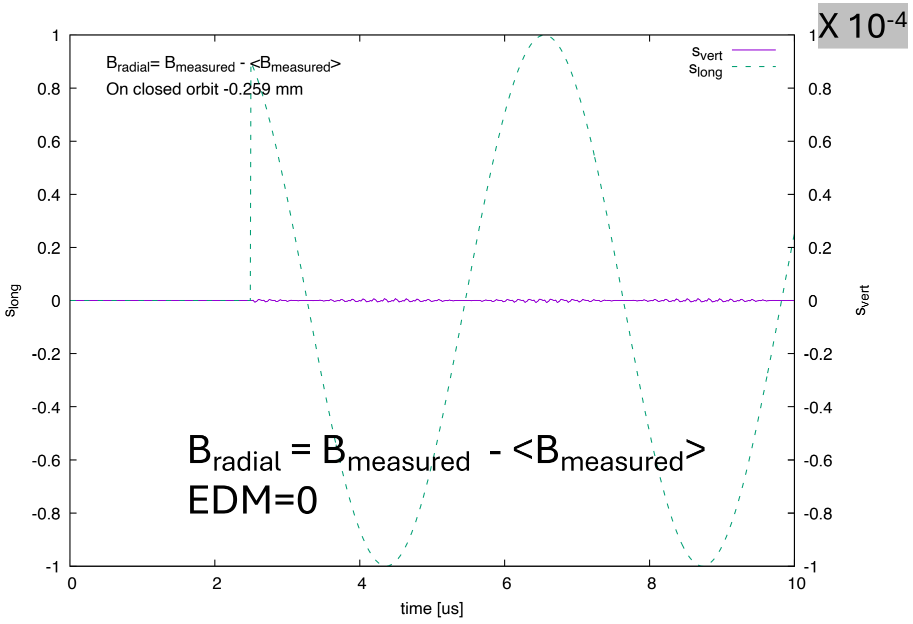
Set $A_0 = 0 \Rightarrow \langle B_{rad} \rangle = 0$

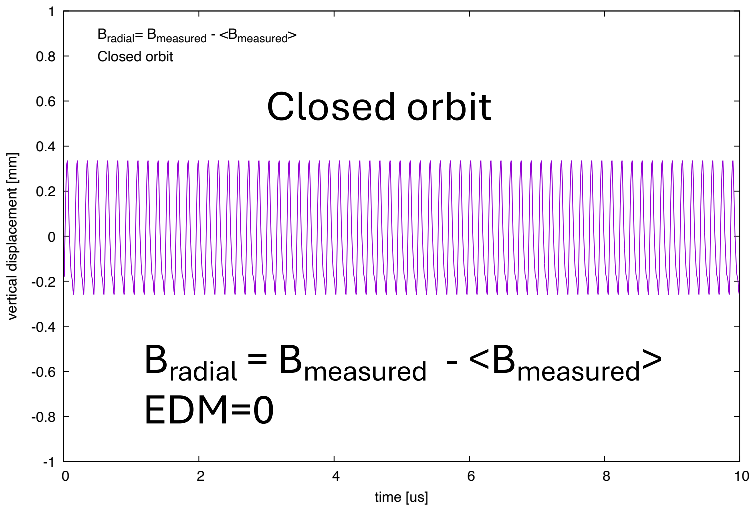


Radial magnetic field - $\langle B_{radial} \rangle = 0$

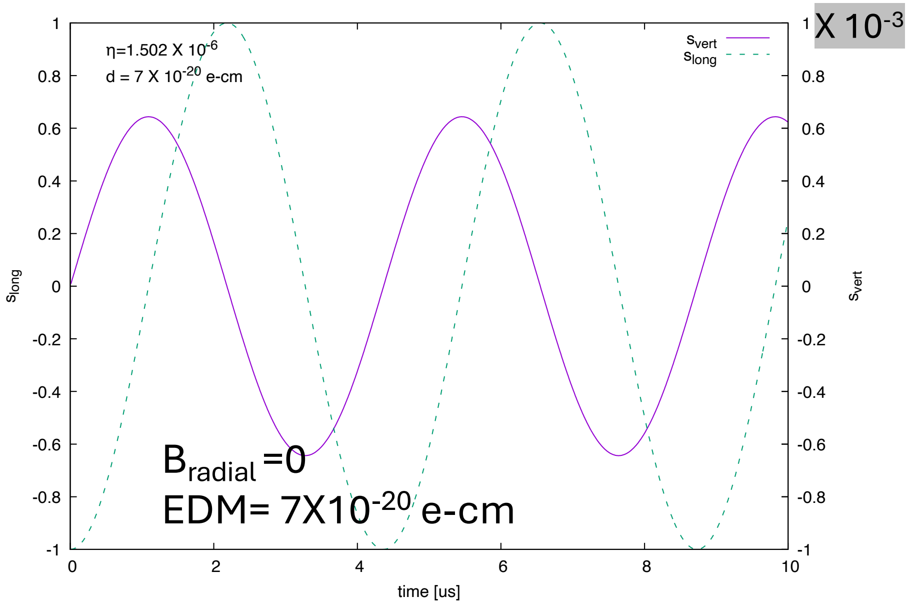


Lowpass filter

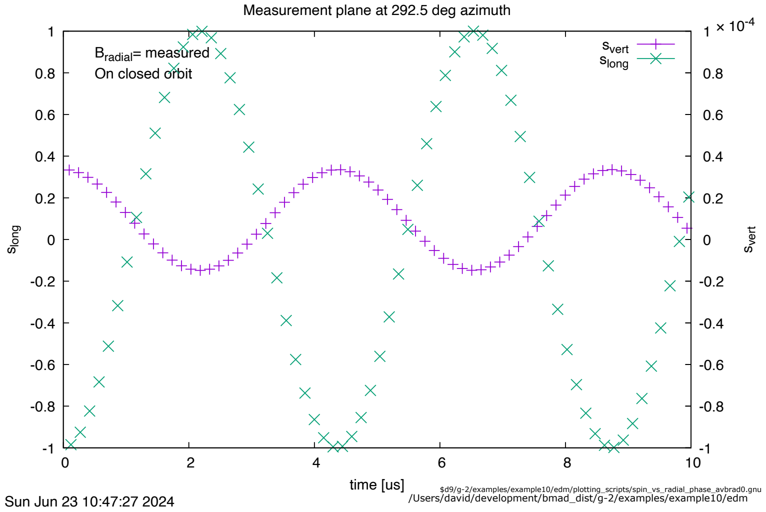


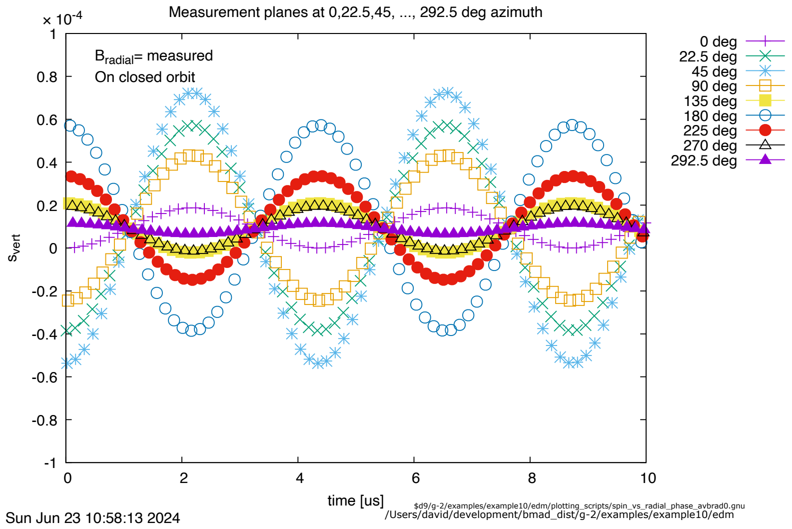


Target EDM limit

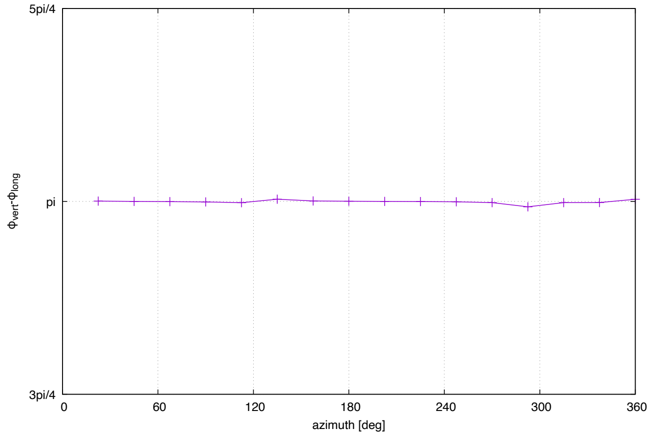


Radial magnetic field - $\langle B_{radial} \rangle = 0$

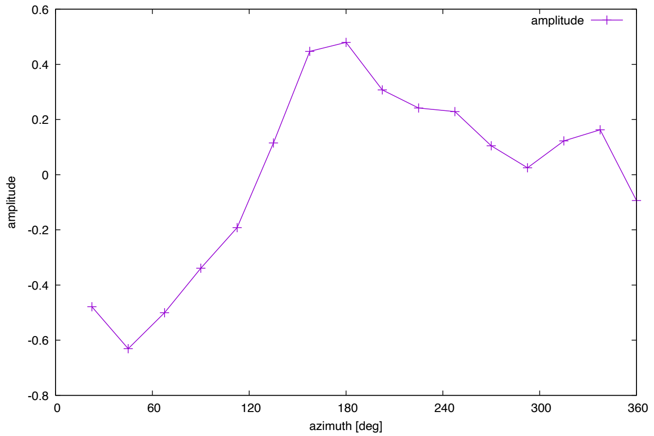




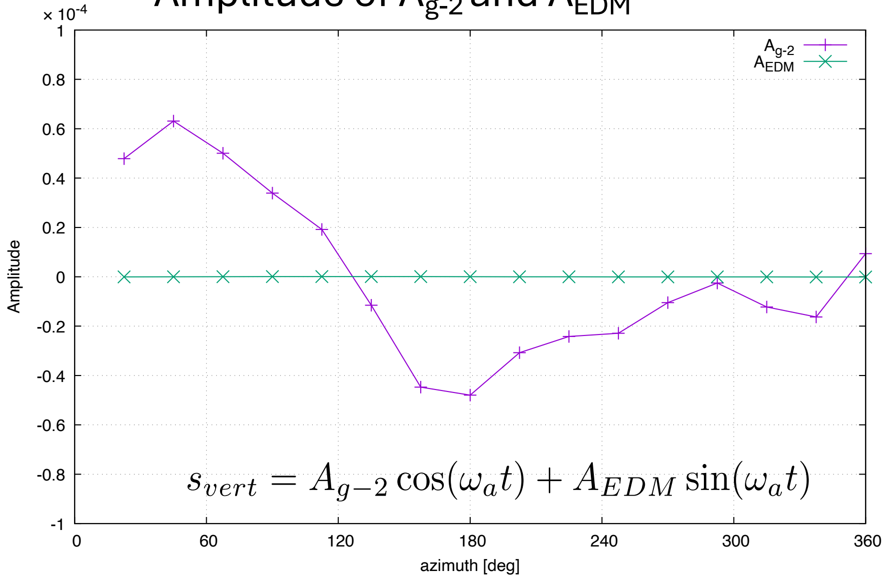
Phase of vertical polarization vs azimuthal 'measurement' plane



Amplitude of vertical polarization vs azimuthal 'measurement' plane



Amplitude of $A_{g\text{-}2}$ and A_{EDM}



$$s_{vert} = A_{g-2}\cos(\omega_a t) + A_{EDM}\sin(\omega_a t)$$

EDM-like signal from radial field depends only on $\langle B_{radial} \rangle$ and is independent of detector location

Conclusions

- There is no contribution to an EDM-like signal from pitch or longitudinal B-field
- Contribution of radial field to EDM-like signal is reduced by $1/\gamma$ by the electric field that provides vertical focusing (and counteracts vertical force from B_{radial})
- The residual EDM-like signal from the radial field depends only on the ring average $\langle B_{radial} \rangle$

Qualifiers

- *Continuous quads*
- *Linear fields*
- *Maxwell violated with addition of nonuniform radial field*