Chapter 22

Spin Dynamics

22.1 Equations of Motion

The propagation of the classical spin vector $\mathbf{S}$ is described in the local reference frame (§15.1.1) by a modified Thomas-Bargmann-Michel-Telegdi (T-BMT) equation[Hoff06]

$$\frac{\mathrm{d}}{\mathrm{d}s}\mathbf{S} = \left\{ \frac{(1 + \mathbf{r}_t \cdot \mathbf{g})}{c\,\beta_z} \, (\boldsymbol{\Omega}_{BMT} + \boldsymbol{\Omega}_{EDM}) - \mathbf{g} \times \widehat{\mathbf{z}} \right\} \times \mathbf{S} \tag{22.1}$$

where $\mathbf{g}$ is the bend curvature function which points away from the center of curvature of the particle's reference orbit (see Fig. 15.2), $\mathbf{r}_t = (x, y)$ are the transverse coordinates, $c\,\beta_z$ is the longitudinal component of the velocity, and $\widehat{\mathbf{z}}$ is the unit vector in the z-direction. $\boldsymbol{\Omega}_{BMT}$ is the usual T-BMT precession vector due to the particle's magnetic moment

$$\begin{aligned} \boldsymbol{\Omega}_{BMT}(\mathbf{r}, \mathbf{P}, t) &= -\frac{q}{m\,c} \left[\left(\frac{1}{\gamma} + a \right) c\,\mathbf{B} - \frac{a\,\gamma\,c}{1+\gamma} \, (\boldsymbol{\beta} \cdot \mathbf{B})\, \boldsymbol{\beta} - \left(a + \frac{1}{1+\gamma} \right) \boldsymbol{\beta} \times \mathbf{E} \right] \\ &= -\frac{q}{m\,c} \left[\left(\frac{1}{\gamma} + a \right) c\,\mathbf{B}_\perp + \frac{(1+a)\,c}{\gamma} \, \mathbf{B}_\parallel - \left(a + \frac{1}{1+\gamma} \right) \boldsymbol{\beta} \times \mathbf{E} \right] \end{aligned} \tag{22.2}$$

and $\boldsymbol{\Omega}_{EDM}$ is the precession vector due to a finite Electric Dipole Moment (EDM) [Silenko08][1]

$$\boldsymbol{\Omega}_{EDM}(\mathbf{r}, \mathbf{P}, t) = -\frac{q\,\eta}{2\,m\,c} \left[\mathbf{E} - \frac{\gamma}{1+\gamma} \, (\boldsymbol{\beta} \cdot \mathbf{E})\, \boldsymbol{\beta} + c\,\boldsymbol{\beta} \times \mathbf{B} \right] \tag{22.3}$$

Here $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ are the electric and magnetic fields, $\mathbf{B}_\perp$ and $\mathbf{B}_\parallel$ are the components perpendicular and parallel to the particle's momentum, γ is the particle's relativistic gamma factor, q, and m are the particle's charge and mass, $\boldsymbol{\beta}$ is the normalized velocity, $a = (g - 2)/2$ is the particle's anomalous magnetic moment (values given in Table 3.2), and η is the normalized electric dipole moment which is related to the dipole moment $\mathbf{d}$ via

$$\mathbf{d} = \frac{\eta}{2} \, \frac{q}{m\,c} \, \mathbf{S} \tag{22.4}$$

Note: Some authors define η without the factor of c is the denominator.

It is important to keep in mind that the a and g-factors used here are defined using Eq. (3.2) which, in the case of nuclei and other composite baryonic particles, differs from the conventional definition Eq. (3.4). See the discussion after Eq. (3.2).

[1] Note: The value for the EDM is set by `bmad_com[electric_dipole_moment]` (§10.2).